Vendredi
Corrigé

Exercice 1

$$|8(\sqrt{3}+i)| = 8\sqrt{4} = 16$$

$$8(\sqrt{3}+i) = 16\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = 16\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$= 16\, e^{i\pi/6} \quad \leftarrow (1)$$

$$z = \rho e^{i\theta}, \quad z^4 = \rho^4 e^{4i\theta}$$

$$\Leftrightarrow \begin{cases} \rho^4 = 16 \\ 4\theta = \pi/6 + 2k\pi, & k \in \mathbb{Z} \end{cases} \quad \leftarrow (1)$$

$$\Leftrightarrow \begin{cases} \rho = 2 \\ \theta = \frac{\pi}{24} + k\pi/2, & k \in \mathbb{Z} \end{cases} \quad \leftarrow (0,5)$$

$$\Leftrightarrow \rho = 2, \quad \theta = \frac{\pi}{24}, \frac{13\pi}{24}, \frac{25\pi}{24}, \frac{37\pi}{24} \quad \leftarrow (1,5)$$

(4)

Exercice 2

1) $\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} - \frac{\sqrt{3}}{2} = 1 - \frac{\sqrt{3}}{2}$ ← 0,5

2) $X = \cos x$ ← 0,5

$X^2 - \frac{1+\sqrt{3}}{2} X + \frac{\sqrt{3}}{4} = 0$

$\Delta = \frac{1+3+2\sqrt{3}}{4} - \sqrt{3} = 1 - \frac{\sqrt{3}}{2}$ ← 0,5

$= \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2$ ← 0,5

$X = \frac{\frac{1+\sqrt{3}}{2} \pm \left(\frac{1-\sqrt{3}}{2}\right)}{2}$

$X = \frac{1}{2}$ ou $\frac{\sqrt{3}}{2}$

$\cos x = 1/2 \iff x = \pm\frac{\pi}{3} + 2k\pi$ ← 1

$\cos x = \frac{\sqrt{3}}{2} \iff x = \pm\frac{\pi}{6} + 2k\pi$ ← 1

4

Exercice 3

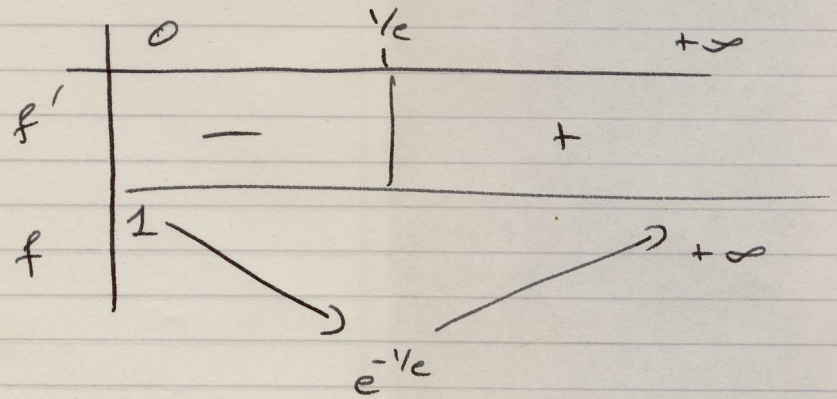
$f(x) = x^x = e^{x \ln x}$ ← 0,5

- est définie sur $]0, +\infty[$ ← 0,5
- $f'(x) = (\ln x + 1)\, e^{x \ln x}$ ← 1
- $\lim\limits_{x \to +\infty} e^{x \ln x} = +\infty$ ← 0,5

  $\lim\limits_{x \to 0^+} e^{x \ln x} = 1$ car ← 0,5

  car $\lim\limits_{x \to 0^+} x \ln x = 0$ ← 0,5

  et exp est continue ← 0,5
- Signe de f' = signe de $\ln x + 1$



1

5

Exercice 4

$f(x) = x^4 + 1$, $f'(x) = 4x^3$

1) Eq tangente :

$$y = f(x_0) + f'(x_0)(x - x_0)$$ ← (1)

en $(0, f(0)) = (0, 1)$, $f'(0) = 0$

$$y = 1$$ ← (1)

2) $y = f(x_0) + f'(x_0)\,x - f'(x_0)\,x_0$

passe par $(0,0)$ $\Leftrightarrow$ $f(x_0) - f'(x_0)\,x_0 = 0$ ← (1)

$x_0^4 + 1 - 4x_0^4 = 0$

$x_0^4 = 1/3$

$x_0 = \pm 1/3^{1/4}$ ← (1)

(4)

(V4)

Exercice 5

$$\int_0^{\sqrt{\pi/2}} 2x^3 \cos(x^2)\, dx$$

$y = x^2$ est une bijection dérivable sur $[0, \sqrt{\pi/2}]$ ← (0,5)

$dy = 2x\,dx$ $\left(\text{ou } dx = \frac{1}{2\sqrt{y}}\,dy\right)$ ← (1)

$$= \int_0^{\pi/2} 2y \cos y\, dy$$ ← (1)

I.P.P. $= [y \sin y]_0^{\pi/2} - \int_0^{\pi/2} \sin y\, dy$ ← (1)

$= \pi/2 + [\cos y]_0^{\pi/2}$

$= \frac{\pi}{2} - 1$ ← (0,5)

(4)

Grand total : (21)