Stability of the characteristic vector field of a Sasakian manifold

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Dedicated to Bang-Yen Chen for its sixtieth birthday

Abstract. – The *Volume* of a unit vector field is the volume of its image in the unit tangent bundle. On the standard odd-dimensional spheres, the Hopf vector fields – that is, unit vector fields tangent to the fiber of any Hopf fibration – are critical for the volume functional, but they are not always *stable*. In fact, stability depends on the radius r of the sphere : for every odd dimension n there exists a "critical radius" such that, if r is lower than this radius the Hopf fields are stable on $\mathbb{S}^n(r)$ and conversely. In this article, we show that this phenomenon occurs for the characteristic vector field of any Sasakian manifold. We then derive two invariants of a Sasakian manifold, its E-stability and its stability number.

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1 General Introduction

Let (M, g) be a oriented Riemanniann manifold, its tangent bundle TM can be endowed with a natural Riemannian metric g^S, known as the Sasaki metric.This metric is defined by :

$$\forall \tilde{X}, \tilde{Y} \in TTM : \quad g^S(\tilde{X}, \tilde{Y}) = g(d\pi(\tilde{X}), d\pi(\tilde{Y})) + g(K(\tilde{X}), K(\tilde{Y}))$$

where $\pi : TM \longrightarrow M$ is the projection and $K : TTM \longrightarrow TM$ is the connector of the Levi-Civita connection ∇ of g. Let $V : M \longrightarrow TM$ be a vector field, the *volume* of V is the volume of the image submanifold $V(M)$ in (TM, g^S) :

$$Vol(V) := Vol(V(M)).$$

It can be expressed by the formula :

$$Vol(V) = \int_M \sqrt{det(Id +^T \nabla V \circ \nabla V)} dvol.$$

In particular, $Vol(V) \geq Vol(M)$ with equality if and only if $\nabla V = 0$, or, in other words, if V is parallel.

The study of the volume of vector fields begins with the pionneering work of Gluck and Ziller [6]. The motivation of this work was to find, according to their own words, the "*visually best organized*" unit vector fields on M. Formally, Gluck and Ziller ask for the infimum of the volume functionnal over all *unit* vector fields :

$$\inf_{V \in \Gamma(T^1 M)} Vol(V).$$

This last restriction is of course necessary to avoid the trivial solution $V \equiv 0$. So, if M is compact, connected, oriented and without boundary, its Euler number must vanish. Thus, in dimension two, M must be a 2-torus, and if this 2-torus is endowed with a natural metric, that is a flat metric, then it admits unit parallel vector fields and there is no problem to find the infimum. The first non trivial question arises in dimension three, when considering the 3-sphere endowed with its standard metric. One remarkable family of unit vector fields on $\mathbb{S}^{2m+1}$ is given by the Hopf fields, that is any unit vector field tangent to the fibers of an Hopf fibration $\mathbb{S}^1 \longrightarrow \mathbb{S}^{2m+1} \longrightarrow \mathbb{C}P^m$. If $\mathbb{S}^{2m+1}$ is thought as the unit sphere of $(\mathbb{C}^{m+1}, J)$ then, a Hopf field is given by $H(x) = Jx$ where $x \in \mathbb{S}^{2m+1} \subset \mathbb{C}^{m+1}$.

Theorem (Gluck and Ziller, [6]). – *The unit vector fields of minimum volume on $\mathbb{S}^3$ are precisely the Hopf vector fields and no others.*

The proof uses the method of calibrations and could not be extended to higher dimensions, so Gluck and Ziller opened the question of whether that result was still true for all odd-dimensional spheres. This was shown to be inaccurate, Pedersen [8] exhibited smooth vector fields on the unit sphere of dimension greater than five with less volume than Hopf vector fields. They are obtained from parallel vector fields, that is parallel translation along geodesics of a given vector at a point p. These parallel vector fields have a singularity of index 0 at $-p$ but a C^∞-pertubation on a small neighborhood of $-p$ gives smooth vector fields on the whole sphere. The volume of these vector fields approaches the volume of the parallel vector field P.

Conjecture (Pedersen, [8]). – *Let $m > 1$, there exists no unit vector fields of minimum volume on $\mathbb{S}^{2m+1}$, moreover :*

$$\inf_{V \in \Gamma(T^1 \mathbb{S}^{2m+1})} Vol(V) = Vol(P).$$

Besides Hopf fields and parallel fields, there is an other family of fields which plays an important role in that problem, namely the radial fields. A radial field R is a field obtained by taking the unit tangent vectors to the radial geodesics issuing from a given point p. It has two singularities at points p

and $-p$ of opposite index ± 1.

Theorem (Brito, Chacon and Naveira, [3]). $-$ *Let V be any non-singular unit vector field on $\mathbb{S}^{2m+1}$, $m > 1$ then $Vol(V) > Vol(R)$.*

Due to the nonvanishing of these index, it is impossible to obtain a smooth unit vector field from it by a C^∞-perturbation on small neighbourhoods of p and $-p$. One has to realize this perturbation on a small tubular neighbourhood of an arc joining p and $-p$. In [2], a construction following these lines is performed to obtain a family R_ϵ of smooth unit vector fields. Unfortunately, as ϵ tends to 0, the volume of R_ϵ does not converge to the volume of R :

$$\lim_{\epsilon \longrightarrow 0} Vol(R_\epsilon) = Vol(R) + \pi vol(\mathbb{S}^{2m}(1)).$$

Moreover, a direct computation shows that if $m > 1$:

$$Vol(R) < Vol(P) < \lim_{\epsilon \longrightarrow 0} Vol(R_\epsilon).$$

Thus the parallel field is a better candidate than the perturbed radial fields family.

There is two obvious conditions that an absolute minimal vector fields of $\mathbb{S}^{2m+1}$ must satisfy : it must by *minimal* that is critical for the volume and *stable*. It is easy to show that Hopf fields are minimal but the stability question is more involved. It was first realized by O. Gil-Medrano and E. Llinares-Fuster [5] that this stability *could* depend on the radius r of the sphere. Indeed, if V is a unit vector field on $\mathbb{S}^{2m+1}(1)$ and V^r the corresponding unit vector field on $\mathbb{S}^{2m+1}(r)$, the function $r \longmapsto Vol(V^r)$ is not homogeneous in r, precisely :

$$Vol(V^r) = \int_{\mathbb{S}^{2m+1}(1)} \sqrt{r^{2n} + r^{2n-2}\sigma_1(M) + ... + r^2\sigma_{2m}(M)}\, dvol$$

where $n = 2m+1$, $M = ^T \nabla V \circ \nabla V$ and the σ_i's are the elementary symmetric polynomial functions of the eingenvalues of M. It turns out that Hopf vector fields remain minimal for any radius but the stability *actually* depends on r.

Theorem [4]. $-$ *Let $m > 1$, unit Hopf vector fields on $\mathbb{S}^{2m+1}(r)$ are stable if and only if $r \leq \dfrac{1}{\sqrt{2m-3}}$. (In $\mathbb{S}^3$, the stability occurs whatever the radius).*

Thus, for every odd-dimensional sphere, there is a "critical" radius which determines the stability of the Hopf vector fields, (if m is 1, we admit that this critical radius is $+\infty$). In fact, as we shall see, this phenomenon always occurs for the characteristic field of any Sasakian manifold $(M^{2m+1}, \eta, \xi, \varphi, g)$.

In this writing, we have denoted by η the contact form, that is a 1-form such that $\eta \wedge (d\eta)^{2m} \neq 0$, by ξ the characteristic field defined by $\eta(\xi) = 1$ and $i_\xi d\eta = 0$, by φ the (1,1)-tensor field satisfying $\varphi^2 = -Id + \eta \otimes \xi$ and by g the metric defined by $g(\xi, X) = \eta(X)$ and $g(X, \varphi Y) = d\eta(X, Y)$. Recall that $(M^{2m+1}, \eta, \xi, \varphi, g)$ is K-contact if ξ is a Killing vector field and that it is Sasakian if, moreover, the following curvature relation holds :

$$R(X,Y)\xi = g(X,\xi)Y - g(Y,\xi)X$$

where R is the (3,1) curvature tensor. For example, the standard Sasakian structure of the sphere $\mathbb{S}^{2m+1}(1) \subset \mathbb{C}^{m+1}$ is the following : g is induced by the Euclidean metric of $\mathbb{C}^{m+1}$, φ is induced by the complex structure J of $\mathbb{C}^{m+1}$, ξ is the Hopf field $-H$ and $\eta = g(\xi, .)$. Due to the curvature condition, if we change the radius, the resulting sphere does not admit any standard Sasakian structure but only a K-contact one. Generally, if $(M^{2m+1}, \eta, \xi, \varphi, g)$ is Sasakian then $(M^{2m+1}, \eta_k, \xi_k, \varphi, g_k)$ with :

$$\eta_k = \frac{1}{\sqrt{k}}\eta, \quad \xi_k = \sqrt{k}\xi, \quad g_k = \frac{1}{k}g,$$

is no longer Sasakian but remains K-contact. It was observed by J. C. González-Dávila and L. Vanhecke that the characteristic field ξ of a K-contact manifold is minimal [7]. More, the characteristic field is also *harmonic*, that is critical for the energy functional (see [9]) :

$$V \longmapsto E(V) = \frac{1}{2}\int_M (\|\nabla V\|^2 + 2m + 1)\, dvol_g.$$

Unlike volume, the functional $k \mapsto E(V_k)$, where V is a unit vector field on a Sasakian manifold and $V_k = \sqrt{k}V$ is the corresponding unit vector field on the K-contact manifold induced by the dilatation of the metric, is homogeneous in k. So the stability of ξ_k as an harmonic unit vector field does not depend on k. We say that a K-contact manifold is *E-stable* if its characteritic vector field is stable for energy. For example, it follows from the works of G. Wiegmink [11], O. Gil-Medrano and E. Llinares-Fuster [5] that the standard K-contact sphere $\mathbb{S}^{2m+1}(r)$ is E-stable if and only if $m = 1$.

We are now in position to state the stability number theorem.

Theorem. – *Let $(M^{2m+1}, \eta, \xi, \varphi, g)$ be a compact Sasakian manifold and $(M^{2m+1}, \eta_k, \xi_k, \varphi, g_k)$ be the family of K-contact manifolds induced by the dilatation of the metric $g_k = \frac{1}{k}g$, $k > 0$.*
a) If M^{2m+1} is E-stable then there exists $k_s \in [0, \infty]$ such that :

- *if $k \leq k_s$, ξ_k is a stable minimal field,*
- *if $k > k_s$, ξ_k is an unstable minimal field.*

b) If M^{2m+1} is not E-stable then there exists $k_s \in]0, \infty]$ such that :

- *if $k < k_s$, ξ_k is an unstable minimal field,*
- *if $k \geq k_s$, ξ_k is a stable minimal field.*

Thus, given a Sasakian manifold, there is a "real" number $k_s(M) \in [0, \infty]$ which cuts the half line $[0, \infty]$ in two segments and the stability of ξ_k depends on which of the two segments k belongs (one of the two segments can be empty). We call $k_s(M)$ the *stability number* of the Sasakian manifold M. For example, the results stated above concerning the sphere simply say that $\mathbb{S}^{2m+1}$ with $m > 1$ is not E-stable and its stability number is $k_s(\mathbb{S}^{2m+1}) = 2m - 3$. If $m = 1$ the sphere $\mathbb{S}^3$ is E-stable and $k_s(\mathbb{S}^3) = \infty$.

The remaining part of this article is devoted to the proof of the stability number theorem.

2 Proof of the stability number theorem

Let $\Gamma(\xi^\perp)$ denote the space of smooth sections of the $2m$-planes bundle $\xi^\perp \to M$.

Lemma 1. – *Let $A \in \Gamma(\xi^\perp)$, the Hessian of the volume functional on the K-contact manifold $(M^{2m+1}, \eta_k, \xi_k, \varphi, g_k)$ at ξ_k has the following expression :*

$$(Hess\, Vol)_{\xi_k}(A) = (1+k)^{m-2} \int_M \left(\|\nabla A\|^2_{g_k} - 2mk\|A\|^2_{g_k} \right) dvol_{g_k}$$

$$+k(1+k)^{m-2} \int_M \left(\|\nabla_{\xi_k} A - \sqrt{k}\varphi A\|^2_{g_k} + Ricci_{g_k}(\varphi A, \varphi A) - 2mk\|A\|^2_{g_k} \right) dvol_{g_k}$$

where $Ricci_{g_k}$ denotes the Ricci tensor.

Proof of Lemma 1. – From Theorem 7 and Lemma 9 of [5] we have :

$$(Hess\, Vol\,)_{\xi_k}(A) = \int_M \Big(\|A\|^2_{g_k} \omega_{\xi_k}(\xi_k) + f(\xi_k) tr(L^{-1}_{\xi_k} \circ {}^T\nabla A \circ L^{-1}_{\xi_k} \circ \nabla A)$$

$$+ \frac{2}{f(\xi_k)} \sigma_2(K_{\xi_k} \circ \nabla A) \Big) dvol_{g_k},$$

where $L_{\xi_k} = Id + {}^T\nabla\xi_k \circ \nabla\xi_k$, $f(\xi_k) = \sqrt{det\, L_{\xi_k}}$, $K_{\xi_k} = f(\xi_k) L^{-1}_{\xi_k} \circ {}^T\nabla\xi_k$, $\omega_{\xi_k} = C^1_1 \nabla K_{\xi_k}$ is the tensor contraction of ∇K_{ξ_k}, and $2\sigma_2(C) = tr(C)^2 - tr(C^2)$. For local computations, we denote by $\mathcal{B} = (E_1, E_{1*}, ..., E_m, E_{m*}, \xi)$ an orthonormal frame for g such that, for $1 \leq i \leq m$, $E_{i*} = \varphi E_i$. Therefore, $\mathcal{B}_k = (E^k_1, E^k_{1*}, ..., E^k_m, E^k_{m*}, \xi_k)$ with $E^k_i := \sqrt{k} E_i$ for $1 \leq i \leq m$, is an orthonormal frame for g_k. It is easy to see that, in $\mathcal{B}_k$, $\nabla\xi_k$ has the following expression :

$$\nabla\xi_k = k^{\frac{1}{2}} \begin{pmatrix} 0 & 1 & & & & \\ -1 & 0 & & & & \\ & & \ddots & & & \\ & & & 0 & 1 & \\ & & & -1 & 0 & \\ & & & & & 0 \end{pmatrix}$$

and thus :

$$L_{\xi_k} = \begin{pmatrix} (k+1)Id & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad f(\xi_k) = (1+k)^m.$$

On a Sasakian manifold, $\nabla\xi = -\varphi$ and $(\nabla_X\varphi)A = g(X,A)\xi - \eta(A)X$ (see [1] for instance), thus $\nabla\xi_k = -k^{\frac{1}{2}}\varphi$ and $\varphi(\nabla_X A) = \nabla_X(\varphi A) - k^{\frac{1}{2}} g_k(X,A)\xi_k$ since $A \in \Gamma(\xi^\perp)$. This yields to the following expression for K_{ξ_k} :

$$K_{\xi_k} = k^{\frac{1}{2}}(1+k)^{m-1}\varphi,$$

and

$$(K_{\xi_k} \circ \nabla A)(X) = k^{\frac{1}{2}}(1+k)^{m-1}((\nabla\varphi A)(X) - k^{\frac{1}{2}} g_k(X,A)\xi_k).$$

Now, a direct computation shows that :

$$\sigma_2(K_{\xi_k} \circ \nabla A) = k(1+k)^{2m-2}(\sigma_2(\nabla\varphi A) - k^{\frac{1}{2}} g_k(\nabla_{\xi_k} A, \varphi A)).$$

For the term $\omega_{\xi_k}(\xi_k)$, we have by definition :

$$\omega_{\xi_k}(X) = g_k((\nabla_{\xi_k} K_{\xi_k})(X), \xi_k) + \sum_{i=1}^{2m} g_k((\nabla_{E_i^k} K_{\xi_k})(X), E_i^k).$$

Since K_{ξ_k} is proportional to φ, the expression of the covariant derivative ∇K_{ξ_k} is easy to obtain and a mere computation shows that :

$$\omega_{\xi_k}(\xi_k) = -2mk(1+k)^{m-1}.$$

It remains to determine the term : $tr(L_{\xi_k}^{-1} \circ^T \nabla A \circ L_{\xi_k}^{-1} \circ \nabla A)$. An other direct computation leads to the following expression :

$$tr(L_{\xi_k}^{-1} \circ^T \nabla A \circ L_{\xi_k}^{-1} \circ \nabla A) = (1+k)^{-2}(\|\nabla A\|_{g_k}^2 + k^2\|A\|_{g_k}^2 + k\|\nabla_{\xi_k} A\|_{g_k}^2).$$

Putting this all together we have :

$$(Hess\ Vol\,)_{\xi_k}(A) = (1+k)^{m-2}\int_M \Big(-2mk(1+k)\|A\|^2_{g_k} + 2k\sigma_2(\nabla\varphi A)$$

$$-2k\sqrt{k}g_k(\nabla_{\xi_k}A,\varphi A) + \|\nabla A\|^2_{g_k} + k^2\|A\|^2_{g_k} + k\|\nabla_{\xi_k}A\|^2_{g_k}\Big) dvol_{g_k}.$$

Since :

$$k\|\nabla_{\xi_k}A - \sqrt{k}\varphi A\|^2_{g_k} = k\|\nabla_{\xi_k}A\|^2_{g_k} - 2k\sqrt{k}g_k(\nabla_{\xi_k}A,\varphi A) + k^2\|A\|^2_{g_k},$$

we also have :

$$(Hess\ Vol\,)_{\xi_k}(A) = (1+k)^{m-2}\int_M \Big(\|\nabla A\|^2_{g_k} - 2mk\|A\|^2_{g_k}\Big) dvol_{g_k}$$

$$+(1+k)^{m-2}\int_M k\Big(\|\nabla_{\xi_k}A - \sqrt{k}\varphi A\|^2_{g_k} - 2mk\|A\|^2_{g_k} + 2\sigma_2(\nabla\varphi A)\Big) dvol_{g_k}$$

Now, for any vector field X on a compact Riemannian manifold (M, g_k) we have (see [10], p. 170 for example) :

$$\int_M Ricci_{g_k}(X,X)\, dvol_{g_k} = 2\int_M \sigma_2(\nabla X)\, dvol_{g_k},$$

and this finishes the proof. $\square$

Lemma 2. – *Let $A \in \Gamma(\xi^\perp)$, the Hessian of the energy functional on the Sasakian manifold $(M^{2m+1}, \eta, \xi, \varphi, g)$ at ξ has the following expression :*

$$(Hess\ E)_\xi(A) = \int_M \big(\|\nabla A\|^2_g - 2m\|A\|^2_g\big)\, dvol_g.$$

Proof of Lemma 2. – From Corollary 5 of [5], we have :

$$(Hess\ E)_\xi(A) = \int_M \big(\|\nabla A\|^2_g - \|A\|^2_g\|\nabla\xi\|^2_g\big)\, dvol_g,$$

and from the proof of Lemma 1, we know that : $\|\nabla\xi\|^2_g = 2m$. $\square$

Let

$$\mathcal{C}(A) = \int_M \big(\|\nabla_\xi A - \varphi A\|^2_g + Ricci_g(\varphi A,\varphi A) - 2m\|A\|^2_g\big)\, dvol_g.$$

Proposition. – *Let $A \in \Gamma(\xi^\perp)$, the Hessian of the volume functional on the K-contact manifold $(M^{2m+1}, \eta_k, \xi_k, \varphi, g_k)$ at ξ_k has the following expression :*

$$(Hess\ Vol)_{\xi_k}(A) = \frac{(1+k)^{m-2}}{k^{m+\frac{1}{2}}}\Big((Hess\ E)_\xi(A) + k\mathcal{C}(A)\Big)$$

Proof of the proposition. – It is enough to observe that :

$$\|A\|^2_{g_k} = \frac{1}{k}\|A\|^2_g, \quad \|\nabla A\|^2_{g_k} = \|\nabla A\|^2_g$$

$$\|\nabla_{\xi_k} A - \sqrt{k}\varphi A\|^2_{g_k} = \|\nabla_\xi A - \varphi A\|^2_g, \quad Ricci_{g_k}(\varphi A, \varphi A) = Ricci_g(\varphi A, \varphi A)$$

$\square$

Proof of the theorem. – a) If $\{k \mid \xi_k \text{ is stable for the volume }\} = \emptyset$, we set : $k_s := 0$. If not, we put :

$$k_s := \sup\{k \mid \xi_k \text{ is stable for the volume }\}.$$

Since stability is a closed condition, this *sup* is a *max*. Of course, by definition, if $k > k_s$ then ξ_k is unstable for the volume. Let $k < k_s$ and suppose that ξ_k is unstable. Since :

$$(Hess\ Vol)_{\xi_k}(A) = \frac{(1+k)^{m-2}}{k^{m+\frac{1}{2}}}\left((Hess\ E)_\xi(A) + k\,\mathcal{C}(A)\right),$$

there exists a vector field $A \in \Gamma(\xi^\perp)$ such that :

$$(Hess\ E)_\xi(A) + k\,\mathcal{C}(A) < 0.$$

In one hand : $(Hess\ E)_\xi(A) \geq 0$, since ξ is a stable harmonic field, thus $\mathcal{C}(A) < 0$. In the other hand, ξ_{k_s} is stable for the volume, thus :

$$(Hess\ E)_\xi(A) + k_s\,\mathcal{C}(A) \geq 0,$$

which leads to a contradiction.

b) Since ξ is an harmonic unstable field, there exists $A \in \Gamma(\xi^\perp)$ such that :

$$(Hess\ E)_\xi(A) < 0,$$

thus, if $\mathcal{C}(A) \neq 0$:

$$k < \frac{|(Hess\ E)_\xi(A)|}{|\mathcal{C}(A)|} \Longrightarrow (Hess\ E)_\xi(A) + k\,\mathcal{C}(A) < 0$$

and

$$k_s := \sup\{k \mid \xi_k \text{ is unstable for the volume }\} > 0.$$

(If $\mathcal{C}(A) = 0$, then obviously : $k_s = \infty$). Of course, if $k \geq k_s$ then ξ_k is stable. Let $k < k_s$ and suppose that ξ_k is stable. For every $\epsilon > 0$ there exists $k_\epsilon < k_s$ such that $|k_s - k_\epsilon| < \epsilon$ and ξ_{k_ϵ} is unstable. Therefore one can assume that : $k < k_\epsilon < k_s$. There exists $A_\epsilon \in \Gamma(\xi^\perp)$ such that :

$$(Hess\ E)_\xi(A_\epsilon) + k_\epsilon\, \mathcal{C}(A_\epsilon) < 0 \qquad (1)$$

and moreover we have :

$$\begin{array}{lll} (Hess\ E)_\xi(A_\epsilon) + k\, \mathcal{C}(A_\epsilon) & \geq 0 & (2) \\ (Hess\ E)_\xi(A_\epsilon) + k_s\, \mathcal{C}(A_\epsilon) & \geq 0 & (3). \end{array}$$

Inequations (1) and (2) imply $\mathcal{C}(A_\epsilon) < 0$ and $(Hess\ E)_\xi(A_\epsilon) \geq 0$, and the comparison of (1) and (3) leads to a contradiction. $\square$

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