Contrôle Continu de math2 du 6 novembre 2008 — Corrigé — Ale Frabetti

Ex. 1 $L:\mathbb{R}^2\to\mathbb{R}^2$ $\quad L\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2x-y\\x\end{pmatrix}=A\cdot\begin{pmatrix}x\\y\end{pmatrix}$, $A\in\mathcal{M}_{2,2}(\mathbb{R}) \Leftrightarrow A=\begin{pmatrix}2&-1\\1&0\end{pmatrix}$

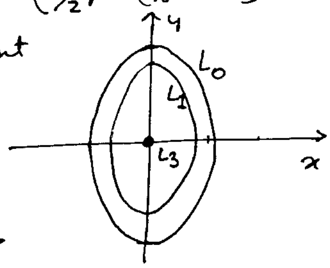
$$\det\begin{pmatrix}2&-1\\1&0\end{pmatrix} = 2\cdot 0-1\cdot(-1)=1.$$

Ex. 2 $f(x,y)=\sqrt{9-4x^2-y^2}$, $\quad 9-4x^2-y^2\geq 0 \Leftrightarrow 4x^2+y^2\leq 9 \Leftrightarrow \left(\frac{x}{3/2}\right)^2+\left(\frac{y}{3}\right)^2\leq 1$ intérieur ellipse

1. Lignes de niveau: $L_0=\{(x,y)\in\mathbb{R}^2,\ \sqrt{9-4x^2-y^2}=0\}=\{(x,y)\ \text{t.q.}\ \left(\frac{x}{3/2}\right)^2+\left(\frac{y}{3}\right)^2=1\}$ ellipse

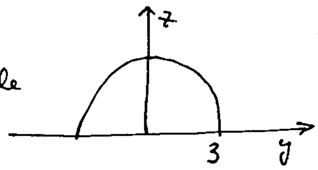
$L_1=\{(x,y)\in\mathbb{R}^2,\ \sqrt{9-4x^2-y^2}=1\}=\{(x,y)\ \text{t.q.}\ 4x^2+y^2=8\}=\{(x,y)\ \text{t.q.}\ \left(\frac{x}{\sqrt{8}/2}\right)^2+\left(\frac{y}{\sqrt{8}}\right)^2=1\}$ ellipse

$L_3=\{(x,y)\in\mathbb{R}^2,\ \sqrt{9-4x^2-y^2}=3\}=\{(x,y)\ \text{t.q.}\ 4x^2+y^2=0\}=\{(0,0)\}$ point

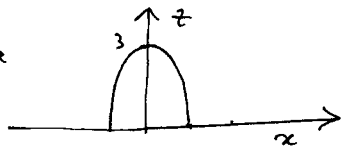


2. Fcts partielles:

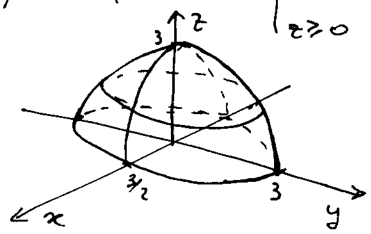
$x=0 \Rightarrow f(0,y)=\sqrt{9-y^2}=z \Leftrightarrow \begin{cases}y^2+z^2=9\\ z\geq 0\end{cases}$ demi-cercle



$y=0 \Rightarrow f(x,0)=\sqrt{9-4x^2}=z \Leftrightarrow \begin{cases}4x^2+z^2=9\\ z\geq 0\end{cases}$ demi-ellipse



3. Graphe:



Ex. 3: $f(x,y)=\dfrac{\sin x}{y}$ $\qquad D=\{(x,y),\ x\in[0,2\pi],\ y>0\}$

1. $\vec{\nabla}f(x,y)=\begin{pmatrix}\dfrac{\cos x}{y}\\ -\dfrac{\sin x}{y^2}\end{pmatrix}$

2. Pour $(x,y)\to(\frac{\pi}{2},1)$: $f(x,y)\sim f(\tfrac{\pi}{2},1)+\dfrac{\partial f}{\partial x}(\tfrac{\pi}{2},1)\,(x-\tfrac{\pi}{2})+\dfrac{\partial f}{\partial y}(\tfrac{\pi}{2},1)\,(y-1)$

$$=\frac{\sin(\frac{\pi}{2})}{1}+\frac{\cos(\pi/2)}{1}(x-\pi/2)-\frac{\sin(\pi/2)}{1^2}(y-1)$$

$\Longrightarrow f(x,y)\sim 1+0\cdot(x-\pi/2)-1\cdot(y-1)=1-y+1=2-y.$

3. $\operatorname{Hess} f(x,y)=\begin{pmatrix}-\dfrac{\sin x}{y} & -\dfrac{\cos x}{y^2}\\ -\dfrac{\cos x}{y^2} & \dfrac{2\sin x}{y^3}\end{pmatrix}.$

Ex. 4 $f(x,y)=y\,\mathrm{sh}\,x$ $\quad$ où $\begin{cases}x=u^2+v^2\\ y=uv\end{cases}$

$$\frac{\partial f}{\partial u}(x(u,v),y(u,v))=\frac{\partial f}{\partial x}\Big|_{\substack{x=u^2+v^2\\ y=uv}}\cdot\frac{\partial x}{\partial u}+\frac{\partial f}{\partial y}\Big|_{\substack{x=u^2+v^2\\ y=uv}}\cdot\frac{\partial y}{\partial u}=y\,\mathrm{ch}\,x\Big|_{\substack{x=u^2+v^2\\ y=uv}}\cdot 2u+\mathrm{sh}\,x\Big|_{x=u^2+v^2}\cdot v$$

$$=2u^2v\,\mathrm{ch}(u^2+v^2)+v\,\mathrm{sh}(u^2+v^2)$$

$$\frac{\partial f}{\partial v}(x(u,v),y(u,v))=\frac{\partial f}{\partial x}\Big|_{\substack{x=\dots\\ y=\dots}}\cdot\frac{\partial x}{\partial v}+\frac{\partial f}{\partial y}\Big|_{\substack{x=\dots\\ y=\dots}}\cdot\frac{\partial y}{\partial v}=y\,\mathrm{ch}\,x\Big|_{\substack{x=\dots\\ y=}}\cdot 2v+\mathrm{sh}\,x\Big|_{x=\dots}\cdot u$$

$$=2uv^2\,\mathrm{ch}(u^2+v^2)+u\,\mathrm{sh}(u^2+v^2).$$