Internship Proposal 2014

ZAMOLODCHIKOV TETRAHEDRA EQUATIONS AND CATEGORIFICATION

Supervision. Philippe Malbos, `malbos@math.univ-lyon1.fr`.

Host team. Institut Camille Jordan, Université Claude Bernard Lyon 1, Lyon, `http://math.univ-lyon1.fr`.

Duration. Three months.

Subject. Given a complex vector space V, a linear operator $R : V \otimes V \longrightarrow V \otimes V$ is called a *Yang-Baxter operator*, if the equation

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$$

holds in $\mathrm{End}(V \otimes V \otimes V)$, where $R_{12} = R \otimes 1_V$, etc. This equation, called the *Yang-Baxter equation*, or the *triangle equation*, has many interesting applications in mathematics, in particular for classical and quantum integrable systems.

The *Zamolodchikov tetrahedra equation* is a three-dimensional generalisation of the Yang-Baxter equation leading to a construction of three-dimensional classical and quantum integrable systems. In [KV94], Kapranov and Voevodsky give an overview of the Yang-Baxter and Zamolodchikov equations and develop a conceptual framework underlying Zamolodchikov equations using the theory of 2-categories.

The aim of this internship is to understand the article [KV94] and the necessary background, in particular

- the notions of Yang-Baxter and Zamolodchikov equations and their geometric interpretation via statistical models,

- the interpretation of the Zamolodchikov equation in terms of 2-category,

- the formulation of the Zamolodchikov equation using the notion of 2-vector spaces, a categorification of the notion of vector space as a state-space of a system.

Expected skills. Basic knowledge in general algebra (groups, rings, fields), in linear algebra and category theory (see [ML98] for example).

—"—

References.

[KV94] M. M. Kapranov and V. A. Voevodsky. 2-categories and Zamolodchikov tetrahedra equations. In *Algebraic groups and their generalizations: quantum and infinite-dimensional methods (University Park, PA, 1991)*, volume 56 of *Proc. Sympos. Pure Math.*, pages 177–259. Amer. Math. Soc., Providence, RI, 1994.

[ML98] S. Mac Lane. *Categories for the working mathematician*, volume 5 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1998.