

**Midterm 2.**  
Friday, March 30.

*No documents allowed. Mobile phones, mp3 players, etc., are also forbidden. The one and only piece of equipment you may use is a basic calculator- and you won't need it.  
You must provide explanation for all your answers.*

NAME \_\_\_\_\_

1. The table below shows chemical compounds which cannot be mixed without causing dangerous reactions. Draw the graph that would be used to facilitate the choice of disposal containers for these compounds ; what is the minimal number of containers needed ?

	A	B	C	D	E	F
A		X		X	X	
B	X		X		X	X
C		X		X		
D	X		X			X
E	X	X				
F		X		X		

2. (a) In designing a security system for its accounts, a bank asks each customer to choose a five-digit number, all the digits to be distinct and nonzero. How many choices can a customer make?

(b) A restaurant offers 4 soups, 10 entrees and 8 desserts. How many different choices for a meal can a customer make if one selection is made from each category? If 3 of the desserts are pie and the customer will never order pie, how many different meals can the customer choose?

(c) You want to create a mileage grid showing the distance between every pair of the 50 U.S state capitals. How many numbers will you have to compute?

3. (a) In how many ways can a voter rank three candidates, without allowing ties? And if ties are allowed?

(b) Use Pascal's triangle to compute  $C_2^4$  and  $C_3^5$ .

(c) How many different combinations of YES and NO votes can there be with 5 voters?

4. (a) Explain the Pareto condition.

(b) Explain independence of irrelevant alternatives.

(c) Explain why majority rule is not a good way to choose between four alternatives.

(d) Arrow's impossibility theorem states that any voting system can give undesirable outcomes. Explain.

(e) Explain the Chair's paradox.

5. Twenty-nine voters must choose from among three alternatives : A,B and C. The voters preference schedules are shown below. The method used is the Borda count.

	Number of voters			
	12	8	6	3
First choice	B	C	A	C
Second choice	C	A	B	B
Third choice	A	B	C	A

- (a) What is the winning alternative?
  
- (b) Can the voters who least prefer the winning alternative change their preference list to produce an outcome they like better (explain why/why not)?
  
- (c) Which alternative would have won if the Hare system had been used?

6. Fifty voters who elect one of the five candidates A,B,C,D or E have the preference schedule shown below.

	Number of voters			
	20	14	10	6
First choice	A	B	B	C
Second choice	C	A	A	D
Third choice	E	D	C	B
Fourth choice	B	C	D	A
Fifth choice	D	E	E	E

- (a) Which candidate (if any) will be elected using plurality voting?
  
- (b) Same question for the Condorcet method.
  
- (c) Same question for sequential pairwise voting with the agenda A,B,C,D,E.
  
- (d) Can you find an agenda that would make candidate A win?

7. Eight board members vote by approval voting on four candidates A,B,C and D. An “X” in the table below indicates an approval vote.

Approval ballots								
A	X	X	X	X		X	X	X
B		X	X		X	X		X
C		X		X	X		X	X
D	X		X	X	X	X	X	

(a) Which candidate will be chosen by the board if just one of them is to be elected?

(b) Which candidates would be chosen if 75 percent approval was needed?

8. Use the following election to show that the plurality runoff voting system is manipulable.

A	A	C	C	B
B	B	A	A	C
C	C	B	B	A

9. True or false?

Any graph admits a vertex-coloring using 4 colors or less.

The plurality voting system is not manipulable.

The Borda count voting system satisfies the independence of irrelevant alternatives criterion.

Sequential pairwise voting satisfies the Condorcet Winner criterion.

Every set of voters' preference ballots produces a Condorcet winner.

