# Quiz 6
Correction..

NAME ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

1.A five-member evaluating committee votes by approval voting on 10 faculty members for a promotion as indicated by the table below. An "X" indicates an approval vote.

Approval ballots

| Candidates | | | | | |
|---|---|---|---|---|---|
| A | X | | X | X | X |
| B | X | X | X | X | X |
| C | | | X | | X |
| D | | X | X | X | |
| E | X | | X | | X |
| F | X | | | | X |
| G | | X | X | X | |
| H | | X | | X | |
| I | X | | X | X | |
| J | | X | X | X | X |

(a) Which candidate wins if just one of them is to be promoted?
**Answer.** Candidate B is the candidate that wins in this case.
(b) Which candidate(s) receive promotion if 80 percent approval is needed?
**Answer.** Candidates A,B and J receive promotion; the other candidates don't.

2.(a) Calculate $C_2^5$.
**Answer.** $C_2^5 = (5 \times 4)/2 = 10$ (you could also use Pascal's triangle).
(b) If there are 5 voters in a voting system, how many different combinations of YES and NO votes can there be?
**Answer.** There are $2^5 = 32$ different combinations of YES and NO votes.
(c) How many votes are needed for a majority winner if there are 30 voters?
**Answer.** 16 votes are needed for a majority winner (with 15 votes there may be a tie).
(d) In how may different ways can one rank 5 candidates if no ties are allowed?
**Answer.** There are $5! = 5 \times 4 \times 3 \times 2 = 120$ permutations of $\{1, \ldots, n\}$; in other words, one can rank 5 candidates in 120 different ways if no ties are allowed.

3. An eleven-member committee must choose one of the four applicants K,L,M and N for membership on the committee. The committee members have preferences among the candidates as given below. If the committee uses pairwise sequential voting with the agenda K,L,M,N then applicant K wins. Can the three voters who least prefer K vote strategically in some way to change the outcome to one they find more favorable? Why or why not?

### Number of delegates

|               | 6 | 2 | 3 |
|---------------|---|---|---|
| First choice  | K | M | M |
| Second choice | L | L | N |
| Third choice  | N | K | L |
| Fourth choice | M | N | K |

**Answer.** No. The six voters who most prefer applicant K represent a majority of the commitee. No matter how the three voters rank the applicants, candidate K will win.

4. Thirty board members must vote on five candidates X,Y,Z.U and V. Their preference rankings are summarized in the table below. Find the winner (or explain why there is none) if the following systems are applied.

### Number of votes

|               | 12 | 10 | 8 |
|---------------|----|----|---|
| First choice  | X  | Y  | Z |
| Second choice | U  | Z  | U |
| Third choice  | Y  | X  | X |
| Fourth choice | Z  | U  | V |
| Fifth choice  | V  | V  | Y |

(a) Condorcet method.
**Answer.** There is no winner : X beats U 22 to 8, beats V 30 to 0, and beats Y 20 to 10. So we know that U, V and Y cannot win. Hence the only possible winners are X and Z; since Z defeats X 18 to 12 only W can win, but Z loses to Y 22 to 8, so doesn't win either. In other words : every candidates loses some on-on-one duel, so there is no Condorcet winner.
(b) Borda count.
**Answer.** X obtains $12 \times 4 + 10 \times 2 + 8 \times 2 = 84$ points; similarly, the Borda score of Y is $12 \times 2 + 10 \times 4 = 64$, the Borda score of U is $12 * 3 + 10 \times 1 + 8 \times 3 = 70$, the Borda score of V is $12 \times 0 + 10 \times 0 + 8 \times 1 = 8$, and the Borda score of Z is $12 \times 1 + 10 \times 3 + 8 \times 4 = 74$. So $X$ wins the Borda count. (C) Hare system.
**Answer.** First, U and V are deleted; this does not change anything to the first-place votes, so Z is deleted next. In the last round X has 20 votes and Y has 10, so $X$ wins the Hare system. (d) Sequential pairwise voting with agenda X,Y,Z,U,V.
**Answer.** First, X defeats Y by 20 to 10; then Z defeats X by 18 to 12, Z defeats U by 20 to 12, and finally Z defeats V 30 to 0. So in this case candidate Z wins. (e) Plurality voting.
**Answer.** Candidate $X$ has the most first place votes, so wins plurality voting.