University of Illinois at Urbana-Champaign | Spring 2007
Math 181 | Group F1

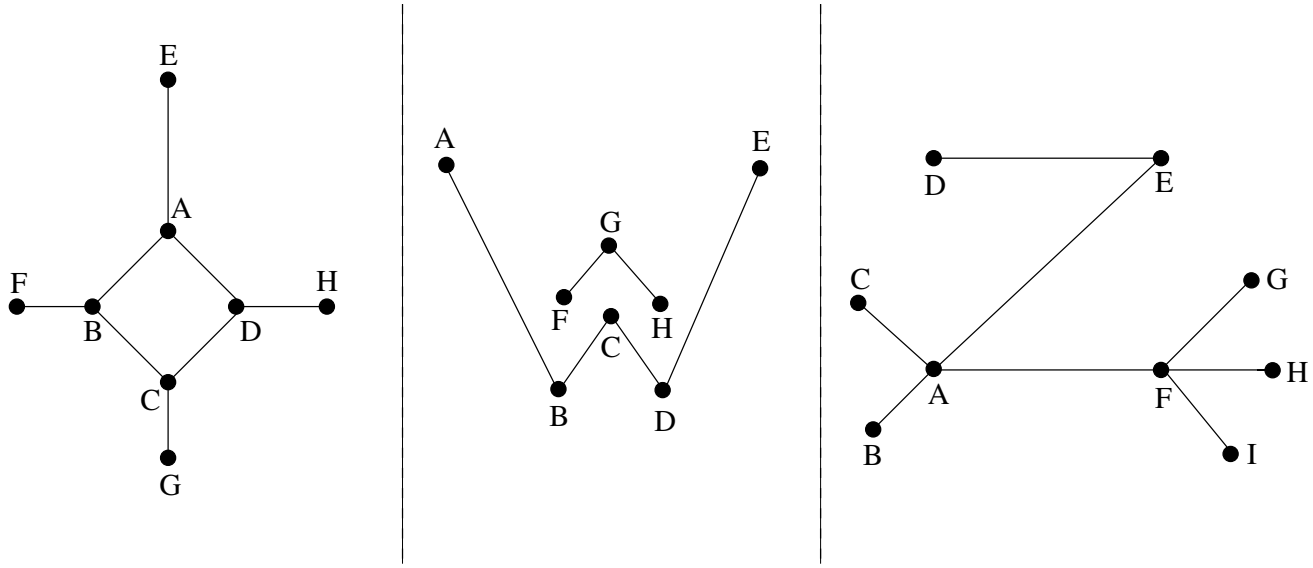
Final Exam.
Wednesday, May 9th, 2007.

Calculators, mobile phones, mp3 players, etc. are prohibited. The only piece of equipment you may use is a basic calculator.
Each exercise is graded out of 10, 15 or 20 points, depending on its difficulty; there are 23 *exercises, and the total number of points is* 300.
You must provide an explanation for all your answers.

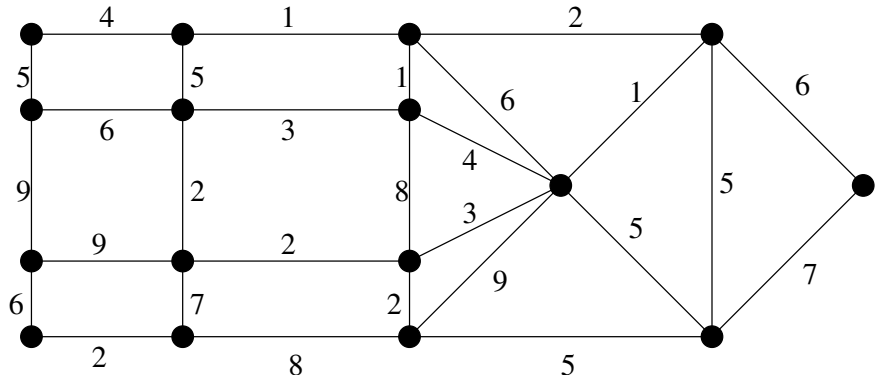
NAME _____

SIGNATURE _____

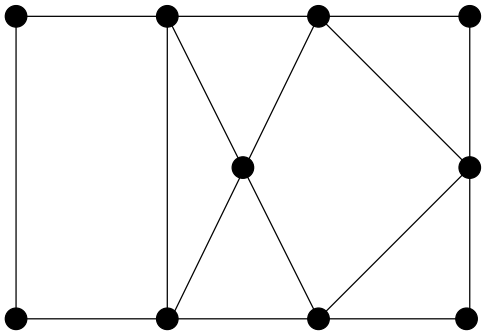
1.(10 points) For each of the following graphs, say whether it is connected or not, and whether it is a tree or not. Each time, give the valence of vertex A.

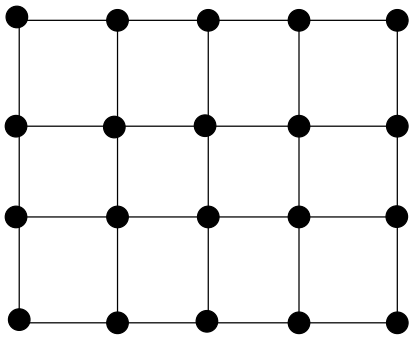


2. (10 points) Apply Kruskal's algorithm to the graph below. Give the cost of a minimum-cost spanning tree for this graph.



3. (10 points) For each of the graphs below, say whether it admits an Euler circuit or not (explain!). If it does, draw such a circuit on the graph; if it doesn't, produce an efficient eulerization (draw a new graph below the original one) and use it to find a circuit that covers all edges while reusing a minimum number of edges (draw that circuit on the original graph).





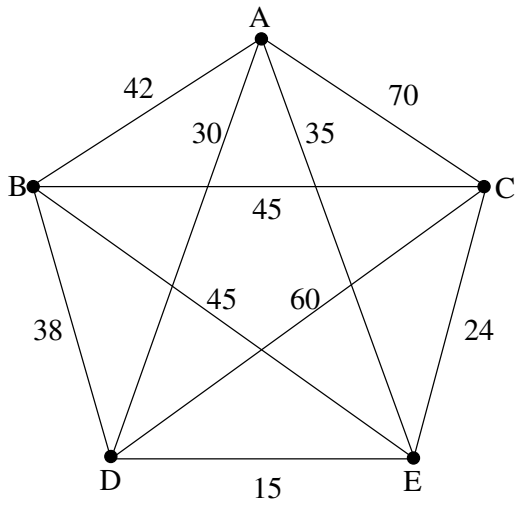
4. (15 points)

(a) A local cafeteria offers a choice of 5 meats, 6 vegetables, and 3 salads. A complete dinner includes 1 meat, 1 vegetable, and 1 salad. How many different dinners can be created?

(b) You're visiting the same cafeteria as above, but this time you want a dinner with 1 meat, 2 vegetables, and no salad. How many different possibilities can you choose from?

(c) You own 10 different shops and a supply depot; each morning a delivery truck leaves the depot, visits each shop and returns to the depot. If it takes 1/3 minute to compute the total length of a tour, how long will it take to apply the brute force algorithm to find the optimal tour for the delivery truck? (just give the formula for the result)

5. (15 points) Consider the graph below. We want to find a minimum-cost hamiltonian circuit.



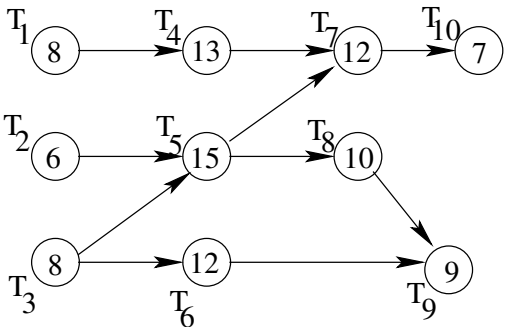
(a) How many hamiltonian circuits would we have to consider if we were to apply the brute force method ?

(b) Find the hamiltonian circuit obtained by applying the nearest-neighbor algorithm starting at B.

(c) Find the hamiltonian circuit obtained by applying the sorted edges algorithm.

(d) Which algorithm is better here ? Is it always the case ?

6. (15 points) Consider the order-requirement digraph below.



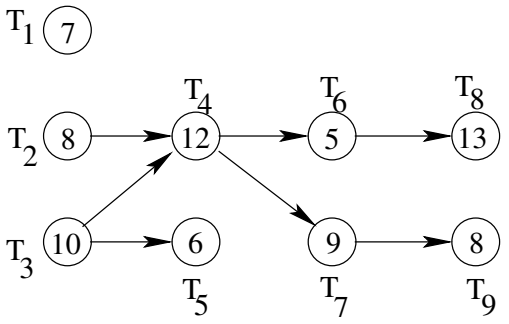
(a) What is (are) the critical path(s) ?

(b) Give an estimate of the minimal amount of time needed to finish the associated job in the following cases :
(i) You don't know the number of processors.

(ii) There are two processors.

(iii) There are three processors.

7. (15 points) Consider the order-requirement digraph below.



Find the priority lists obtained by applying the critical-path scheduling and decreasing-time scheduling algorithms, and give the schedules on two processors obtained by applying the list-processing algorithm to these lists.

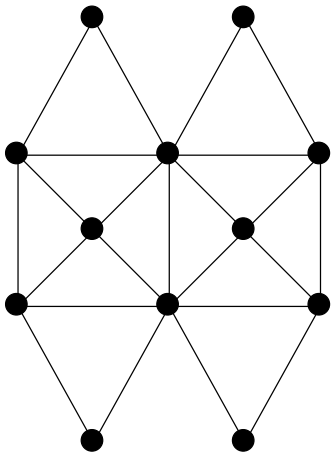
8.(10 points) A student wants to pack data files with the following sizes onto compact discs that can hold no more than 700 MB : 100 MB, 500 MB, 250 MB, 350 MB, 400 MB, 450 MB, 200 MB, 50 MB, 500 MB, 300 MB, 250 MB.

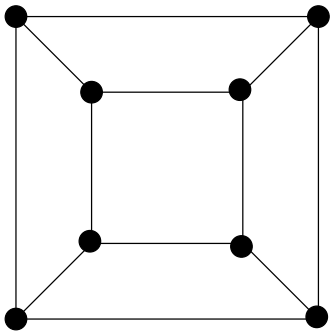
(a) Apply the next-fit algorithm to this list of weights.

(b) Same with the first-fit decreasing algorithm.

(c) Same with the worst-fit decreasing algorithm.

9. (10 points) For each of the graphs below, find a vertex coloring with a minimal number of colors and give the chromatic number.





10. (15 points) When two towns are within 145 miles of each other, the frequency used by a certain type of emergency response system for the towns requires that they be on different frequencies to avoid possible interference with each other. The table below shows the mileage distances between six towns.

	E	F	G	I	S	T
Evansville (E)		290	277	168	303	133
Ft Wayne (F)	290		132	83	79	201
Gary(G)	277	132		153	58	164
Indianapolis(I)	168	83	153		140	71
South Bend(S)	303	79	58	140		196
Terre Haute(T)	133	201	164	71	196	

(a) What would be the minimum number of frequencies that are needed for each town to have its emergency broadcasts not conflict with those of any other town using this system ?

(b) How many different towns would be assigned to each frequency used ?

11. (15 points)

(a) How many ways are there to rank four people without allowing ties ?

(b) How many ways are there to rank four people, allowing ties for first (meaning that two, three or four people can be tied for first place, and there are no other ties ; there may be no tie at all) ?

(c) How many different coalitions of 6 voters are there ?

(d) How many different coalitions of 6 voters are there in which exactly two people vote YES ?

(e) How many different coalitions of 4 voters are there in which at least 2 people vote yes ?

12.(10 points) Match the words on the left to the sentence on the right that best corresponds to them.

Independence of irrelevant alternatives	If everyone prefers A to B then B cannot win
Monotonicity	Ballot changes favorable to one candidate cannot hurt that candidate
Arrow's theorem	The only way for A to go from being a loser of an election to a winner is for at leat one voter to reverse his/her ranking of A and the previous winner.
Pareto condition	Any voting system with more than three candidates can give undesirable outcomes.
Manipulable	A system in which it may be to a voter's advantage to submit one ballot that misrepresents his/her true preferences.

13. (15 points) Use the following election to show that the Hare system is manipulable :

A	B	C	C	D
B	A	B	B	B
C	C	A	A	C
D	D	D	D	A

14. (15 points) Fifty voters who elect one of the five candidates A,B,C,D or E have the preference ballots shown below. For each of the voting systems below, say who is elected (or explain why no one is elected) :

	Number of voters			
	20	14	10	6
First choice	A	B	B	C
Second choice	C	A	A	D
Third choice	E	D	C	B
Fourth choice	B	C	D	A
Fifth choice	D	E	E	E

(a) Borda count

(b) Plurality voting

(c) Hare system

(d) Sequential pairwise voting with the agenda A,B,C,D,E.

(e) Condorcet method

15. (10 points) Use the following ballots to show that the plurality runoff method does not satisfy the Condorcet winner criterion :

	Number of voters		
	2	2	1
First choice	A	B	C
Second choice	C	C	B
Third choice	B	A	A

16. (15 points) Consider the following binary linear code, where four parity-check digits have been added to strings of length 4.

0000 ⟶ 00000000	1000 ⟶ 10000111
0001 ⟶ 00011110	1001 ⟶ 10011001
0010 ⟶ 00101101	1010 ⟶ 10101010
0011 ⟶ 00110011	1011 ⟶ 10110100
0100 ⟶ 01001011	1100 ⟶ 11001100
0101 ⟶ 01010101	1101 ⟶ 11010010
0110 ⟶ 01100110	1110 ⟶ 11100001
0111 ⟶ 01111000	1111 ⟶ 11111111

(a) What is the weight of this code ?

(b) How many single-digit errors can the code detect ? How many errors can it correct ?

(c) Using the nearest-neighbor algorithm, decode (or explain why you can't decode) the words 00100101 and 00011000.

17. (10 points) Recall that a correctly coded ISBN $a_1a_2\ldots a_{10}$ has the property that $10a_1 + 9a_2 + 8a_3 + 7a_4 + 6a_5 + 5a_6 + 4a_7 + 3a_8 + 2a_9 + a_{10}$ is evenly divisible by 11.

(a) Is the number 20-7039-362-3 a valid ISBN ? If not, change the check-digit to correct the error.

(b) Same question with the number 0-7167-4785-0.

18. (15 points) Consider the binary linear code for binary strings of length 5 where one uses the parity-check sums $a_1 + a_2 + a_3$, $a_3 + a_4$ and $a_1 + a_4 + a_5$.

(a) Find the code words for the words 10011 and 10101.

(b) What is the distance between the code words that you obtained in question (a) ?

(c) Assume now that you are given the code $\{010101, 101111\}$. How many errors would have to occur during transmission for a received word to be decoded incorrectly (using nearest neighbor decoding) ?

19. (15 points) Consider the weighted voting system [27 :15,10,8,5].
(a) List all winning and blocking coalitions.

(b) List all minimal winning coalitions.

(c) Is there a dictator ? A voter with veto power ? A dummy voter ? (explain).

20. (10 points)
(a) Are the weighted voting systems [8 :5,4,3] and [15 :8,6,10] equivalent ? (explain)

(b) A committee with one chair, two co-chairs and two other members uses the following scheme : a motion passes if the chair, a co-chair and some other voter vote YES ; or if the two co-chairs and the two members vote YES.
Give a weighted voting system that is equivalent to the system above.

21. (20 points) Compute the Shapley-Shubik and Banzhaf power index of each voter in the weighted voting system [14 :10,6,5,3].

22. (15 points) Consider the weighted voting system [6 :3,1,1,1,1,1,1].
(a) Describe the winning coalitions in which voter A is critical.

(b) Using a counting argument, compute the Banzhaf pozer index of A (**do not make a list !**).

(c) Describe the permutations in which voter A is pivotal.

(d) Using a counting argument, compute the Shapley-Shubik power index of A ; use this to find the Shapley-Shubik power index of a weight-1 voter.

23. (10 points) The table below gives the numerical values of the letters in the Roman alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

(a) Encode the message I LOVE MATH using the Vigenère cipher with the key word SURE.

(b) Given that FINALLY was used as the key word to encrypt YPR HZWGIILS LCC HTBSP with the Vigenère cipher, decrypt the message.