University of Illinois at Urbana-Champaign — Fall 2006

Math 380 — Group G1

Graded Homework III

Due Friday, September 29.

1. Compute the directional derivative of the mapping f defined by $f(x,y) = xy + \ln(x^2+1)$ in the direction given by $u = (\frac{\sqrt{3}}{2}, \frac{1}{2})$.

2. Given unit vectors $u = (u_x, u_y)$ and $v = (v_x, v_y)$, and a function $z = f(x,y)$ with continuous second-order partial derivatives, find a formula (involving the second-order partial derivatives of f and the coordinates of u, v) for the mixed second directional derivative $\nabla_u \nabla_v z$.

3. Show that y is defined implicitly as a function of x in the neighborhood of the point P in the following equations :
• $x\cos(xy) = 0$, $P = (1, \frac{\pi}{2})$;
• $xy + \log(xy) = 1$, $P = (1,1)$ (log stands for the natural logarithm).
Use implicit differentiation to compute $y'(1)$ and $y''(1)$ in the first case, and $y'(1)$ in the second case.

4. Show that $2xy - z + 2xz^3 = 5$ can be solved implicitly for z as a function of x and y near $(1,2,1)$. Compute the first-order partial derivatives of z at $(1,2)$, as well as $\frac{\partial^2 z}{\partial y^2}(1,2)$.

5. Determine whether the function $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by $f(x,y,z) = (e^x\cos(y) + z, x\sin(y)\sin(z), xz\cos(y))$, admits a differentiable inverse g near $(1, \frac{\pi}{2}, \pi)$. If so, give the value of the Jacobian determinant of g at the point $= (\pi, 0, 0)$.