University of Illinois at Urbana-Champaign — Fall 2006
Math 380 — Group G1

Graded Homework V
Due Friday, October 13.

1. Compute the derivative of the function $x \mapsto \tan^{-1}(x) = \arctan(x)$; use it to compute $\int_a^b \frac{dx}{x^2+1}$, where $a, b \in \mathbb{R}$ (in terms of $\arctan(a), \arctan(b)$), then to compute $\int_0^1 \frac{dx}{x^2+x+1}$.
With a change of variables, compute the integral $\int_0^{\frac{\pi}{2}} \frac{\cos(x)dx}{2-\cos^2(x)+\sin(x)}$.

2. Compute the area of the domain D in the two following cases :
(a) D is in the quarter-plane $x \geq 0$, $y \geq 0$ and is delimited by the curves $y^2 = x^3$, $y = x$.
(b) D is the set of all $x, y \geq 0$ such that $x^{2/3} + y^{2/3} \leq 1$.
For the second one, you may begin with the change of coordinates $u = x^{\frac{1}{3}}$, $v = y^{1/3}$; you may also use the fact that $\int_0^{\frac{\pi}{2}} \sin^2(\theta)\cos^2(\theta)d\theta = \frac{\pi}{16}$ (Proving this equality will give some extra credit on the homework).

3. Compute the integral $\iint_D f(x,y)dxdy$ in the following cases :
(a) $f(x,y) = e^{x+y}$ and $D = \{(x,y) \in \mathbb{R}^2 \colon |x-y| \leq 1, |x+y| < 1\}$.
(b) $f(x,y) = x^2 - 2y$, D is the interior of the ellipse of equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
(c) $f(x,y) = x^2 + y^2 - 2y$, D is the circle of center $(1,1)$ and radius 1.
(d) $f(x,y) = xy$, D is the domain of all (x,y) such that $x, y \geq 0$ and $x^2 + \frac{y^2}{4} \leq 1$.
(For (a), (b) and (c), you should use a change of variables adapted to the domain you are integrating on)