University of Illinois at Urbana-Champaign — Fall 2006
Math 380 — Group G1

Graded Homework VI

Due Friday, October 20.

1. Compute the following integrals :

(a) $\iiint_D xy\,dxdydz$, where $D = \{(x,y,z)\colon 0 \leq x, 0 \leq y, 0 \leq z \leq 1, x^2+y^2 \leq z^2\}$.

(b) $\iiint_D y\,dxdydz$, where $D = \{(x,y,z)\colon 0 \leq x, 0 \leq y, x^2+y^2 \leq z \leq 1\}$.

(c) $\iiint_D \sqrt{x^2+y^2+z^2}\,dxdydz$, $D = \{(x,y,z)\colon 1 \leq x^2+y^2+z^2 \leq 2\}$ (use a change of variables).

2. Compute the coordinates (x_G, y_G) of the center of gravity G of the plane domain D of equation $x^2 \leq 2y \leq x+2$ (recall that x_G is the average value of x in D, and y_G is the average value of y in D).
Note : The term "center of gravity" is used to denote the center of mass of a *homogeneous* solid, i.e one in which density is the same everywhere.

3. Define $H(x) = \int_0^x e^{-t^2}\,dt$, $G(x) = H(x)^2$ and $F(x) = \int_0^1 \frac{e^{-x^2(1+t^2)}}{1+t^2}\,dt$.
(a) Compute $G'(x)$; $F'(x)$. Show that the function $F+G$ is constant.
(b) Find the value of $(F+G)(0)$.
(c) Show that $0 \leq F(x) \leq e^{-x^2}$ for all $x \in \mathbb{R}$. (d) (optional) Find $\lim_{x\to\infty} \int_0^x e^{-t^2}\,dt$.

4.. (a) Find the length of the arc of helix of equation $x(t) = \cos(t)$, $y(t) = \sin(t)$, $z(t) = t$, where $0 \leq t \leq 2\pi$.
(b) A *hypocycloid* is a closed curve in plane of equation $x^{2/3} + y^{2/3} = a^{2/3}$, where a is some positive constant. Find the length of an hypocycloid (first find the length of the part of the curve that is in the first quadrant, then use symmetries of the curve).