UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Math 380

Fall 2006
Group G1

Graded Homework VIII .
Due Friday, November 3.

1. Use two different methods to compute the circulation of the vector field V on the curve C, in the following cases :
(a) $V(x,y) = (xy, x-y)$, C is the triangle with vertices $(0,0)$, $(0,3)$, $(1,-1)$ and is oriented clockwise ;
(b) $V(x,y) = (xy, e^y)$, and C is the circle of center $(0,0)$ and radius 3, oriented counterclockwise.

2. For each of the following "differential forms" $P(x,y)dx+Q(x,y)dy$, determine whether there exists a function f such that $P(x,y)dx + Q(x,y)dy = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$; if it exists, find such a function.
(a) $P(x,y) = x^2 + y$, $Q(x,y) = 2y$.
(b) $P(x,y) = xy^2$, $Q(x,y) = x^2y$.
(c) $P(x,y) = 2xy\cos(x^2y) + 1$, $Q(x,y) = x^2\cos(x^2y) + e^y$

3. (a) Prove that the integral $\int_\gamma (6x+2y)dx + (6y+2x)dy$ has the same value whenever γ is a curve from $A = (0,0)$ to $B = (1,1)$. Check this by computing this integral in the case where γ is a straight line segment, and γ is an arc of the parabola of equation $y = x^2$.
(b) Find a function f such that its gradient at the point (x,y) is equal to $(6x+2y, 6y+2x)$; explain why this function enables one to compute easily the integrals of the preceding question.

4. A *cardioid* is a curve of equation (in polar coordinates) $r = (1+\cos(\theta))$, $0 \leq \theta \leq 2\pi$. Compute the area of the domain enclosed by a cardioid ; for this, use θ as a parameter, and use trigonometric relations to show that $x(\theta)y'(\theta) - y'(\theta)x(\theta) = 1 + 2\cos(\theta) + \cos^2(\theta)$ (why does this help ?).