University of Illinois at Urbana-Champaign | Fall 2006
Math 444 | Group E13

Graded Homework V
Due Friday, October 13.

1 Using the definition of the limit, show that the following sequences converge and compute their limit :
$x_n = \dfrac{1}{n+1} - \dfrac{1}{n}$; $y_n = \sqrt{n+1} - \sqrt{n}$.

2. Using the theorems that we saw in class (Squeeze Theorem, algebraic manipulations of limits), determine whether the following sequences are convergent and, if they are, compute their limit.
$x_n = \dfrac{(n+1)^3}{n^3}$; $y_n = \dfrac{\sin(n)}{\sqrt{n}}$; $z_n = \dfrac{\sqrt{n}}{n + \sin(n)}$.

3.. Using the definition of $E(x)$ given in the last homework, and the fact that $E(x) \leq x < x+1$, prove that, for any $x \in \mathbb{R}$, one has $\lim \frac{E(nx)}{n} = x$.
(Optional) Can you use this to prove the Density Theorem ?

4. Recall that $n!$ is defined by induction by $1! = 1$, $(n+1)! = (n+1)n!$. Said differently, one has $n! = 1.2.3 \ldots n$.
Define now $u_n = \dfrac{n!}{n^3}$.
(a) Prove that there exists some $N \in \mathbb{N}$ such that, for all $n \geq N$, one has $\dfrac{u_{n+1}}{u_n} \geq 2$.
(b) Prove by induction that, for all $n \geq N$, one has $u_n \geq u_N.2^{n-N}$.
(c) Use this to show that the sequence (u_n) is not convergent.