I am a first year Ph.D. student, working with Peter Koepke. Currently, we are discussing possible behaviors of the 2^κ-function under the negation of the axiom of choice.

In [AK10], Arthur Apter and Peter Koepke determine various exact consistency strengths of the negation of the Singular Cardinal Hypothesis in Zermelo-Fraenkel set theory. For instance, they prove the equiconsistency of the following two theories, for any $\alpha \geq 2$ fixed: ZFC + "there is a measurable cardinal", and ZF + $\neg$ AC + "GCH holds below $\aleph_\omega$"+ "there is a surjective function $f : [\aleph_\omega]^\omega \to \aleph_{\omega+\alpha}$".

It is possible to avoid the assumption of a measurable cardinal, if one only considers surjections from $\wp(\aleph_\omega)$ (instead of $[\aleph_\omega]^\omega$):

Theorem ([GK12])**.** *Let V be a ground model of ZFC + GCH and λ a cardinal in V. Then there exists a cardinal preserving model $N \supseteq V$ of the theory ZF + "GCH holds below κ"+ "there is a surjective function $f : \wp(\aleph_\omega) \to \lambda$".*

The rough ideas from [GK12] can be described as follows: For every $n < \omega$, $\aleph_{n+1}$-many Cohen subsets are added to $\aleph_{n+1}$. Furthermore, λ-many subsets of $\aleph_\omega$ are adjoined, each of which restricted to any interval $[\aleph_n, \aleph_{n+1})$ is eventually equal to one Cohen subset. Let N be the choiceless submodel generated by certain equivalence classes of the adjoined $\aleph_\omega$-subsets. An isomorphism argument gives that any $X \subseteq \mathrm{Ord}$ located in N must already be contained in a "mild" V-generic extension; consequently, cardinals are N-V-absolute.

In any ZF-model, one can define for cardinals $\aleph_\alpha$:

$$\theta(\aleph_\alpha) := \sup\{\, \xi \mid \text{ there is a surjective function } f : \wp(\aleph_\alpha) \to \xi \}.$$

Concerning the model N constructed above, one can show that, indeed, $\theta(\aleph_\omega) = \lambda^+$. Furthermore, the results from [GK12] can be generalized to arbitrary cardinals $\aleph_\alpha$.

At the moment, we are working on the following question: Given a (sufficiently reasonable) function $E : \mathrm{Ord} \to \mathrm{Ord}$, is there a ZF-model N in which $\theta(\aleph_\alpha) = E(\alpha)$ for all $\alpha \in \mathrm{Ord}$?

My diploma thesis was in the area of forcing and large cardinals. The first part dealt with the question how the folklore *factor lemma* for forcing interations can be strengthened (under reasonable circumstances). The second part was based on Joel Hamkin's article *The lottery preparation* [Ham00], where he introduces a preparatory forcing that makes a variety of large cardinals indestructible by certain types of forcing. Under the assumption that there are no supercompact limits of supercompact cardinals in V, an easy modification leads to a class forcing extension $V[G]$ with the same supercompacts as V, where additionally, every supercompact cardinal κ is indestructible by $<\kappa$-directed closed forcing.

References

[AK10] Arthur Apter and Peter Koepke, *The consistency strength of choiceless failures of SCH*, Journal of Symbolic Logic **75 (3)**, 2010, 1066 – 1080.

[GK12] Moti Gitik and Peter Koepke, *Violating the Singular Cardinal Hypothesis without large cardinals*, 2012, submitted.

[Ham00] Joel D. Hamkins, *The lottery preparation*, Annals of Pure and Applied Logic, **101 (2 - 3)**, 2000, 103 – 146.