RESEARCH STATEMENT - Boriša Kuzeljević

Most of my current research deals with the posets of elementary substructures of certain Fraissé limits (in the sequel if R is Fraissé structure by $E(R)$ we denote the poset of its elementary substructures ordered by inclusion). Our research is the continuation of the work that my supervisor, professor Miloš Kurilić, has done in this area and will be the core of my doctoral thesis.

In [4] Kurilić proved the following theorem:

Theorem 1. For each linear order $\langle L, < \rangle$ the following conditions are equivalent:
(a) L is isomorphic to a maximal chain in the poset $\langle E(\mathbb{Q}) \cup \{\emptyset\}, \subset \rangle$;
(b) L is an $\mathbb{R}$-embeddable complete linear order and 0_L has no successor;
(c) L is isomorphic to a compact set $K \subset [0,1]_{\mathbb{R}}$ such that $0 \in K'$ and $1 \in K$.

In the proof he essentially uses arguments from [3]. As the set of rational numbers is the first example of Fraissé limit it is natural to ask if the same holds for another Fraissé structure. In our joint work, [5], in our main theorem we show that the answer is affirmative for the random graph R.

Theorem 2. For each linear order $\langle L, < \rangle$ the following conditions are equivalent:
(a) L is isomorphic to a maximal chain in the poset $\langle E(R) \cup \{\emptyset\}, \subset \rangle$;
(b) L is an $\mathbb{R}$-embeddable complete linear order and 0_L has no successor;
(c) L is isomorphic to a compact set $K \subset [0,1]_{\mathbb{R}}$ such that $0 \in K'$ and $1 \in K$.

Although it may seem that Theorem 1 and Theorem 2 are very similar it is not the case with their proofs. Easier part, (b)⇔(c) is completely settled in Theorem 1, and the case (a)⇒(b) is only a slight modification of the same result in Theorem 1. To prove part (b)⇒(a) for an uncountable linear order L we introduce a new representation of the random graph where the set of vertices is given as the set of rational numbers. In this construction we use a generic filter on some specially chosen partial order so that we can get that for example the chain $\mathcal{L} = \langle (-\infty, x) \cap \mathbb{Q} : x \in \mathbb{R} \rangle$ is one maximal chain in $E(R)$. When countable linear orders L are considered, we find such a chain in $E(R)$ using some combinatorial arguments depending only on the structure of the random graph.

One direction for our future research will certainly be to obtain results similar to Theorem 1 and Theorem 2 for some other Fraissé structures. In particular, we hope to prove that the same holds for the generic poset, the Fraissé limit of the class of all finite partial orders. In the case of uncountable linear order our generic construction of the random graph might be useful, whereas in the case of countable linear order we still do not have the appropriate technique.

The other direction involves investigating the factor algebra $P(R)/I$, where R is the random graph and I is the ideal of the sets not containing isomorphic copy of R. This algebra is different from $P(\omega)/\,\mathrm{Fin}$. Take for example an I-mad family $\mathcal{A}$ on $P(R)$. Paper [6] shows that it is possible for $\mathcal{A}$ to be countable. We will mostly be interested in preserving maximality of I-mad families in forcing extensions.

References

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