

Research Statement

Daisuke Ikegami

I am currently working on the following two topics: 1. Gale-Stewart games and Blackwell games, 2. Ω -logic and Boolean valued second order logic.

1. Blackwell games are infinite games with imperfect information generalizing the game “Rock-Paper-Scissors” and Blackwell determinacy is an extension of von Neumann’s minimax theorem for Blackwell games while Gale-Stewart games are infinite games with perfect information generalizing the game “Chess” and the determinacy of Gale-Stewart games has been deeply investigated in set theory.

With Hugh Woodin, I showed that the Axiom of Real Blackwell Determinacy ($\text{Bl-AD}_{\mathbb{R}}$) and the Axiom of Real Determinacy ($\text{AD}_{\mathbb{R}}$) are equivalent under $\text{ZF}+\text{DC}$. [2] We are working on whether $\text{ZF}+\text{Bl-AD}_{\mathbb{R}}$ and $\text{ZF}+\text{AD}_{\mathbb{R}}$ are equiconsistent using the technique of Core Model Induction.

2. Ω -logic is a logic of forcing absoluteness introduced by Hugh Woodin which describes the phenomena of forcing absoluteness implied by large cardinals in terms of universally Baire sets of reals. Boolean valued second order logic is a second order logic with Boolean valued truth values which sits between the Henkin semantics and the full semantics of second order logic.

With Jouko Väänänen, I showed that the validity of Boolean valued second order logic is as complex as that of Ω -logic and it is Δ_2 in ZFC assuming Ω -conjecture while the validity of full second order logic is Π_2 -complete in ZFC . [1] Also we showed that the Hanf number and the cardinals related to compactness of Boolean valued second order logic are strictly smaller than those of full second order logic under some assumptions of Ω -logic and large cardinals. The same could hold for Löwenheim-Skolem number consistently. All of these results indicate that Boolean valued second order logic is less complex than full second order logic.

References

- [1] Daisuke Ikegami and Jouko Väänänen. Ω -logic and Boolean valued second order logic. Preprint in 2012.
- [2] Daisuke Ikegami and W. Hugh Woodin. Real Determinacy and Real Blackwell Determinacy. Preprint series in Institut Mittag-Leffler, available at <http://www.mittag-leffler.se/preprints/files/IML-0910f-32.pdf>, 2010.