RESEARCH STATEMENT

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My research focuses on set-theoretic topology. I am especially interested in non-metrizable homogeneous compacta. Erik Van Douwen asked about 40 years ago whether there is a homogeneous compactum with a family $\mathfrak{c}^+$-many pairwise disjoint open sets. This is still open in all models of ZFC. Haar measure witnesses that a compact group cannot have an uncountable family of pairwise disjoint open sets. On the other hand, the set of all binary sequences of length ω^2, with the topology induced by its lexicographic ordering, is compact and homogeneous and has a family of $\mathfrak{c}$-many pairwise disjoint open intervals. However, the Erdös-Rado Theorem prevents a homogeneous compact ordered space from having $\mathfrak{c}^+$-many distinct points. Another major obstruction is Kenneth Kunen's result that products of infinite compact F-spaces are not homogeneous. Yet, based on my recent progress for the special case of openly generated compacta, I conjecture that the answer is "yes," and, moreover, that every compactum is a continuous image of a homogeneous zero-dimensional compactum. Equivalently, I conjecture that every boolean algebra A extends to a boolean algebra B such that for every two maximal ideals I and J of B, there is an automorphism of B sending I to J.

While Van Douwen's Problem is my primary motivation, most of my research has approached the problem indirectly by considering order-theoretic cardinal invariants in topology, a topic I am interested in for its own sake. Call a poset κ-flat if every subset of size κ lacks an upper bound. Every point in every known homogeneous compactum (including all compact groups) has a local base that is ω-flat when ordered by reverse inclusion, and I conjecture that this is true of all homogeneous compacta. A test question is whether there is a homogeneous compactum with a local base Tukey equivalent to $\omega \times \omega_1$. The conjecture implies "no." If we replace $\omega \times \omega_1$ with a product order that has a "gap," such as $\omega \times \omega_2$ or $\omega \times \omega_1 \times \omega_3$, then the answer is "no." Of the known connections to Van Douwen's Problem, the strongest is that GCH implies that if κ is a cardinal, X is a homogeneous compactum, and X does not have a κ-flat local base, then X has a family of κ^+-many pairwise disjoint open sets.

Dropping the requirement of homogneity and/or compactness leads to many other interesting questions. For example, does the countably supported box product topology on $2^{\aleph_\omega}$ have an ω_1-flat π-base? The answer is "yes" if $\mathfrak{c} > \aleph_\omega$. Moreover, ZFC proves that this space has an ω_4-flat base. However, it is consistent, relative to a 2-huge cardinal, that this space does not have an ω_1-flat local base in any ccc forcing extension. For another example, if X is a topological space, κ is a cardinal, and X^2 has a κ-flat base, then does X have a κ-flat base? There are non-compact spaces X and Y such that $X \times Y$ has an ω-flat base, but neither X nor Y does. On the other hand, GCH implies that if X is compact homogeneous, n is finite, and X^n has a κ-flat base, then so does X.

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