Kostas Tsaprounis
University of Barcelona, Spain

During this last year, I have been working on problems related to the general context of the $C^{(n)}$ - cardinals, as they are introduced in [Bag11].

Recall that $C^{(n)}$ is the closed unbounded proper class of ordinals, which are Σ_n - correct in the universe, i.e., $C^{(n)} = \{\alpha : V_\alpha \prec_n V\}$, for any $n \in \omega$. Now, working within the standard framework of elementary embeddings, of the sort $j : V \longrightarrow M$, for the usual large cardinal axioms, we define, for any $n \in \omega$, the $C^{(n)}$ - version of the large cardinal notion at hand, by requiring, in addition to the standard definition, that the image $j(\kappa)$ of the critical point belongs to the class $C^{(n)}$. This gives rise to large cardinal hierarchies such as $C^{(n)}$ - measurables, $C^{(n)}$ - strongs, $C^{(n)}$ - supercompacts, $C^{(n)}$ - extendibles etc.

The results in [Bag11] have highlighted the strong (structural) reflectional nature of (some of) these hierarchies. Nevertheless, there are many problems left unsolved, sometimes even at the lowest levels, e.g., regarding $C^{(1)}$ - supercompacts vs. supercompacts.

I have been working with these ideas during this last period and I have been able to add to the work of J. Bagaria, both by considering certain $C^{(n)}$ - hierarchies which were not dealt with there, e.g., tall (see [Ham09]), Woodin and strongly compact cardinals and, at the same time, by answering some of questions which were left open.

In particular, using the versatile concept of elementary chains, I have derived consistency results for the $C^{(n)}$ - versions of tall, (super)strong, supercompact and extendible cardinals. Moreover, I have studied the interaction of these notions with the forcing machinery.

Currently, I am working on some related problems at the level of supercompactness where, the central open questions, revolve around the internal structure of the $C^{(n)}$ - supercompact hierarchies and annoyingly underline their vague relation to standard supercompactness.

[Bag11] Bagaria, J., "$C^{(n)}$ - *cardinals*". Archive for Math. Logic. To appear.

[Ham09] Hamkins, J.D., "*Tall cardinals*". Math. Logic Q., Vol. 55 (1), pp. 68 - 86, 2009.