RESEARCH STATEMENT

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The most important large cardinal notions can be characterized in terms of combinatorial properties. My current research concerns three principles known as the *tree property*, the *strong tree property* and the *super tree property* that characterize the notion of weakly compact cardinal, strongly compact cardinal and supercompact cardinal, respectively. An inaccessible cardinal is weakly compact if, and only if, it satisfies the tree property; it is strongly compact if, and only if, it satisfies the strong tree property; it is supercompact if, and only if, it satisfies the super tree property. While these properties are typically associated with large cardinals, they can be satisfied by small cardinals as well. This means that, from a combinatorial point of view, when a cardinal satisfies one of these properties, it "behaves like a large cardinal." While the previous characterizations date back to the early 1970s, a systematic study of the last two properties has only recently been undertaken by Weiss [5]. The investigation into the strong and the super tree properties has been proven very fruitful for measuring the consistency strength of forcing axioms and other related principles. For instance, from the fact that PFA implies the super tree property at $\aleph_2$, Viale and Weiss [4, 3] proved that the consistency strength of PFA is "reasonably" a supercompact cardinal. To date a lot of questions remain open:

(i) what regular cardinals can consistently satisfy the strong or the super tree property?
(ii) what are the consequences of those properties in cardinal arithmetic?

Weiss [6] proved that for every $n \geq 2$, the super tree property can consistently hold at $\aleph_n$, if we assume the existence of a supercompact cardinal. In [2], I generalized that result by proving that there is a model of set theory where all the $\aleph_n$ (with $n \geq 2$) simultaneously satisfy the super tree property (see also [1]). Whether we can go further, and prove the consistency of the super tree property for every regular cardinal, is still an open problem. Concerning the second question, the analogy between strongly compact cardinals and the strong tree property suggests a possible connection between this principle and the singular cardinal hypothesis. Solovay showed that, if κ is a strongly compact cardinal, then the singular cardinal hypothesis holds above κ. For this reason, it seems natural to ask whether the strong tree property for a cardinal κ entails the singular cardinal hypothesis above κ. A thorough investigation into these problems promises a wide range of applications and it will give a deeper understanding of the above mentioned large cardinal notions.

References

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[4] M. Viale and C. Weiss. On the Consistency Strength of the Proper Forcing Axiom, *Advances in Mathematics* 228: 2672-2687 (2011).
[5] C. Weiss. Subtle and Ineffable Tree Properties, Phd thesis, *Ludwig Maximilians Universitat Munchen* (2010).
[6] C. Weiss. The Combinatorial Essence of Supercompactness, submitted to the *Annals of Pure and Applied Logic*.

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