RESEARCH STATEMENT OF MATTEO VIALE

The concept of infinity has been a subject of speculation throughout history and has always held a central role in mathematics and in philosophy. Nonetheless there are deceptively simple mathematical questions regarding the infinite which may not have an answer, at least in a broadly accepted sense. The most prominent undecidable question is Cantor's continuum problem, which was shown to be independent of the standard first order axioms of mathematics (ZFC) by the combined work of Gödel and Cohen. This led Cohen to develop the method of forcing, the basic tool for all current researches in set theory. This method has found applications in virtually all fields of pure mathematics: in the last forty years natural problems of group theory, functional analysis, operator algebras, homotopy theory, and many other subjects were shown to be undecidable by means of forcing. Embracing a platonistic standpoint, Gödel proposed the search for new axioms of mathematics (such as the axioms of large cardinals) and for non-standard methods of proof able to settle all natural problems of mathematics, even when these have been shown to be independent with respect to ZFC: this became known as "Gödel's program". In the last twenty years Woodin has vindicated Gödel's expectations and has shown that forcing cannot be used to prove the independence with respect to ZFC supplemented by large cardinals axioms of all problems of second order number theory (a broad class of problems which includes analytic number theory, large portions of real analysis, differential geometry, and many other subjects). This is a great achievement, since forcing is the unique known method to prove the independence of any problem of second order (or even of third order) number theory with respect to ZFC. Moreover there is a general consensus among set theorist on the existence of large cardinals or, at least, on the consistency of their existence.

My current researches focus on the possibility to extend these absoluteness results to problems of third order number theory (among which is Cantor's continuum problem). I plan to divide this research project in three steps: The first step is the search for simple criteria to classify the family of forcing notions that can be used to prove an independence result in third order number theory. The second step is the search of a "universal axiom" for this family, i.e. a statement with the property that any two models of this axiom, obtained by two distinct forcing notions in the above family, will satisfy the same statements of third order number theory. The third step will try to establish whether two distinct universal axioms can give a different solution to some problem of third order number theory. I can already show that the forcing axiom MM^{++} is a universal axiom for the Π_2-theory of $H_{\aleph_2}$ with respect to stationary set preserving forcings. This is strictly related to Woodin's work on axiom $(*)$ (which is a universal axiom for the full first order theory of $H_{\aleph_2}$ with respect to all forcings). Recently Aspero and Schindler have announced that MM^{++} implies $(*)$, this would create an unexpected bridge between Woodin's work on Ω-logic and the theory of the forcing axioms MM, PFA.