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SHORT RESEARCH STATEMENT  
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In my current research, I focus on theory of Borel equivalence relations with connection to various  $\sigma$ -ideals on Polish spaces. I am motivated by the recent work of Vladimir Kanovei, Marcin Sabok and my supervisor Jindřich Zapletal that will be presented in the book *Canonical Ramsey Theory on Polish Spaces*.

The general problem of that work can be stated as follows: let  $X$  be a Polish space,  $I$  a  $\sigma$ -ideal on  $X$  and  $B \subseteq X$  an  $I$ -positive Borel subset,  $E$  a Borel (analytic) equivalence relation on  $X$ . Can we find an  $I$ -positive subset of  $B$  on which  $E$  is simpler, i.e. has a lower complexity?

So far I have investigated equivalence relations Borel bireducible to equivalence relations given by analytic  $P$ -ideals on  $\omega$  (that includes relations such as  $E_2$ ,  $E_{\ell^p}$  for  $p \in [1, \infty)$  and  $E_{c_0}$ ), measure equivalence relation, with connection to  $\sigma$ -ideals such as Silver ideal or Laver ideal. Currently, I am trying to generalize the results for orbit equivalence relations given by actions of a sufficiently general class of Polish groups.

Let me state a sample theorem:

**Theorem** *Let  $E$  be an equivalence relation on  $\omega^\omega$  that is Borel reducible to an equivalence relation  $E_{\mathcal{I}}$  on  $2^\omega$  given by some analytic  $P$ -ideal  $\mathcal{I}$ . Let  $B \subseteq \omega^\omega$  be an analytic set positive in the Laver ideal, i.e. it contains all branches of some Laver tree  $T$ . Then there is a Laver subtree  $S \subseteq T$  ( $[S]$  is a positive subset of  $B$ ) such that either  $E \cap [S] \times [S] = [S] \times [S]$  or  $E \cap [S] \times [S] = \text{id}([S])$ .*

More generally, I am interested in applications of mathematical logic, especially descriptive set theory to mathematical analysis.