Research statement for Scott Cramer:

My research is focused on reflection properties of very large cardinals and their influence on the structure theory of $L(V_{\lambda+1})$. The primary large cardinal of interest is I_0, introduced by Woodin, which states that there is an elementary embedding $j : L(V_{\lambda+1}) \to L(V_{\lambda+1})$ such that $\mathrm{crit}(j) < \lambda$. This axiom sits just below the axiom shown inconsistent by Kunen, that under ZFC there is a non-trivial elementary embedding $V \to V$. A central open question in the area is whether the theorem holds under ZF. In attempting to tackle this problem, understanding the structure of $L(V_{\lambda+1})$ becomes a key issue.

Assuming I_0 holds at λ then $\mathrm{cof}(\lambda) = \omega$ and so the structure $L(V_{\lambda+1})$ is similar to $L(\mathbb{R}) = L(V_{\omega+1})$. In fact, as Woodin first showed, many of the structural properties of $L(\mathbb{R})$ assuming the Axiom of Determinacy holds in $L(\mathbb{R})$ are very similar to the structural properties of $L(V_{\lambda+1})$ assuming I_0 holds. I have applied the notion of inverse limits to develop this connection even further. Laver first used inverse limits in this context to prove reflection results such as, if there exists an elementary embedding $L_{\omega+1}(V_{\lambda+1}) \to L_{\omega+1}(V_{\lambda+1})$ then there is a $\bar{\lambda} < \lambda$ such that there is an elementary embedding $V_{\bar{\lambda}+1} \to V_{\bar{\lambda}+1}$. I extended this type of result to I_0 and used inverse limits to prove structural properties such as:

Theorem 0.1. *Suppose there exists an elementary embedding $L(V^{\#}_{\lambda+1}) \to L(V^{\#}_{\lambda+1})$ (a slightly stronger axiom than I_0). Then there are no disjoint stationary subsets $S, S' \subseteq \{\alpha < \lambda^+ \mid cof(\alpha) = \omega\}$ such that $S, S' \in L(V_{\lambda+1})$.*

These types of results show the utility of inverse limits in this context, and give hope that they could be useful in solving the above question of Kunen's Theorem without choice, and in further understanding the connection between $L(V_{\lambda+1})$ and $L(\mathbb{R})$.