YST 2012 Research Statement of Sean Cox

My research focuses on the following topics, which often overlap:

(1) Compactness properties of small cardinals, especially of $\aleph_2$ under forcing axioms;
(2) Generic large cardinals (e.g. saturated and precipitous ideals);
(3) Applications of forcing outside set theory

Regarding topic 1: motivated by some results of Viale-Weiss (under PFA) and Foreman (under MM), I proved that PFA^{++} implies the *Diagonal Reflection Principle (DRP)*, a highly simultaneous form of stationary set reflection. I also proved that MM implies a weaker form of DRP; this was independently shown by Viale. DRP can also be viewed as a natural weakening of the statement: "For every regular $\theta \geq \omega_2$ there is a normal ideal over $\wp_{\omega_2}(\theta)$ whose positive-set forcing is proper" (this is also closely related to a weak version of Foreman's notion of a *decisive ideal*). There are several open problems related to MM, DRP, and $FA^{++}(\sigma\text{-closed})$ that I have been discussing with Matteo Viale and Paul Larson.

Regarding topic 2: given a normal ideal $\mathcal{I}$ on a regular uncountable cardinal, Martin Zeman and I introduced the notion $ProjectiveCatch(\mathcal{I})$; this property is equivalent to precipitousness for ideals on ω_1 (this was pointed out to us by Schindler), but is strictly stronger than precipitousness for ideals on ω_2. If $\mathcal{I}$ is on ω_2, then $ProjectiveCatch(\mathcal{I})$ has especially high consistency strength, even though $ProjectiveCatch(\mathcal{I})$ does **not** imply that generic ultrapowers by $\mathcal{I}$ have strong closure properties (e.g. the closure properties associated with presaturated ideals).

Here is an example of the overlap between topics 1 and 2. Foreman and Magidor had shown that PFA and MM place several constraints on the existence of generic embeddings with critical point ω_2 (e.g. that $(\omega_3, \omega_2) \twoheadrightarrow (\omega_2, \omega_1)$ fails; that there is no presaturated ideal on ω_2; that certain stationary towers with critical point ω_2 cannot be presaturated). Matteo Viale and I proved that various stationary reflection principles place constraints on the existence of presaturated towers of ideals with critical point ω_2. We also have some preliminary results suggesting that these constraints are sharp; Neeman's new method of forcing a model of PFA with finite conditions plays a central role in proving this sharpness.

I am still learning about topic 3, especially those for which MM/PFA seem to give useful information (e.g. the effect of OCA on the separable quotient problem in Banach space theory). I think this is an exciting area and hope to make contributions to it.