Research statement

Thilo V. Weinert
weinert@math.uni-bonn.de

Currently I am a PhD student of Stefan Geschke at the Hausdorff Research Center in Bonn. I work on a project concerned with cardinal charactaristics that are derived from continuous Ramsey theory. A theorem of Andreas Blass states that if one colours the n-tupels of elements of the Cantor space continuously with m colours, m and n both being finite, then there exists a perfect weakly homogeneous set meaning that the colour of an n-tupel in this set only depends on the order of the levels where the branches separate. As an example a tripel can be such that the two leftmost branches separate before the two rightmost do or vice versa. In general an n-tupel has one of $(n-1)!$ possible splitting types. Now these weakly homogeneous sets generate a σ-ideal and one can ask for its covering number, i.e. the minimal size of a family of weakly homogeneous sets covering the whole space. This is a cardinal characteristic and indeed one that tends to be large. There are two respects in which it is large, its cardinal sucessor has size at least continuum and it is always at least as large as the cofinality of the null ideal and hence at least as large as any cardinal characteristic from Cichoń's diagram. Currently it is known that the characteristic for pairs is small, i.e. $\aleph_1$ in the Sacks model. Much more is unknown however.

Another topic besides cardinal characteristics of the continuum which I consider to be very interesting are partition relations between countable ordinals, this endeavour started some time ago with papers of Erdős, Rado and Specker. Recently I was able to calculate some transfinite Ramsey numbers. More precisely I was able to prove $r(\omega^2 2, 3) = \omega^2 10$, $r(\kappa\lambda 2, 3) = \kappa\lambda 6$ and $r(\kappa\lambda 3, 3) = \kappa\lambda 15$ for κ weakly compact and $\lambda < \kappa$ a cardinal. In this context, $r(\alpha, \beta)$ is the least ordinal γ such that $\gamma \to (\alpha, \beta)$.

References

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