Flatness of the rotation curves of the galaxies
Exit the recourse to a massive halo

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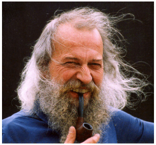

Abstract

The rotation curves of galaxies remain flat to large distances, this fact raises a rather crucial question, and the only explanation is to admit the existence of a massive spherical halo around a galaxy. We show that, within a Newtonian framework, the phenomenon of flatness of the curves is very simply explained without recourse to a possible massive halo. Moreover our method gives also the Einsteinian correction which appears to be non negligible. Our direct method rests on the simulation of a spiral galaxy by a disc of N massive bodies distributed with an axial symmetry. As these bodies follow a given curve of rotation, then the balance of the radial forces between N bodies leads to a set of linear equations (the unknown are the masses of the N bodies) that one reverses and which thus provides the surface density curve. In fact it is an "inverse method of the N-bodies problem", which give results very precise and easy to implement.

Links with others methods

Our method is a Riemannian approximation of the double integral coming from the method which uses the elliptic integrals; it is thus theoretically equivalent to it and also to the method of the Bessel transform. As this last method is invertible, our method provides a surface density whatever the given curve of rotation, and this without recourse to a massive halo.

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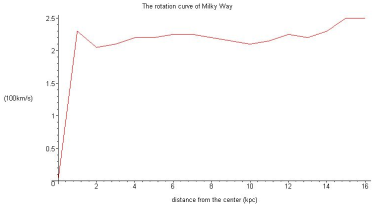

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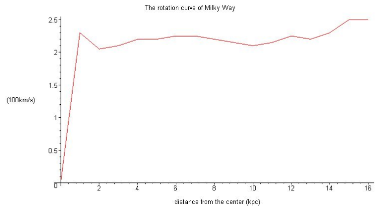

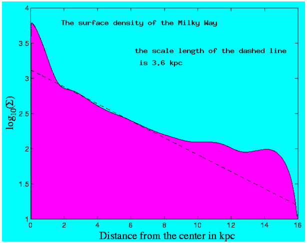

Results for the Milky Way : We derive for the Milky Way a mass of $1.4 \times 10^{11} M_{\odot}$. The exponential profile is well reproduced with a scale length between 3.6 and 5 kpc. The local surface density is found to be $125 \pm 10 M_{\odot}.pc^{-2}$, compatible with other independent determinations.

N body formulation I

Our method is based on a numerical resolution of the Poisson equation. For a maximal disk, given the surface density $\Sigma(R)$, the force acting on a given point of the disk reads :

$$\vec{F} = \int \frac{\Sigma(R)}{R^3} \vec{R} d\vec{R}. \tag{1}$$

Such an integral is mathematically divergent, but there are in fact symmetric compensations near the singular point, so that the integral is well defined.

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We model a spiral galaxy by a central mass m_o, and a disk of radius R_g which consists of n massive bodies of mass m_i and position $\vec{x_i}$ distributed within an axial symmetry. For each point its distance to the center is denoted d_i, its velocity v_i and its distance to other stars $d_{ij} = \|\vec{x_j} - \vec{x_i}\|$.

Using this discretization to calculate the integral (1), the force $\vec{F_i}$ acting at the point $\vec{x_i}$ reads:

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Now if we suppose that these forces give rise to the rotation curve with Newtonian gravitation, we have

$$\vec{F_i} = m_i \frac{v_i^2}{d_i} \frac{\vec{x_i}}{d_i}.$$

The two former equations yields:

$$\frac{v_i^2}{d_i} \frac{\vec{x_i}}{d_i} = \sum_{j \neq i} G \frac{m_j}{d_{ij}^3} \vec{d_{ij}}. \tag{2}$$

N body formulation II

Because of the symmetry of the problem, the first equation for the center ($i = 0$) reduces to $\vec{0} = \vec{0}$, and the other equations can be projected on the radial axis. Introducing the angle θ_{ij} between the vectors $\vec{x_i}$ and $\vec{x_j}$, then $d_{ij}^2 = d_i^2 + d_j^2 - 2 d_i d_j \cos(\theta_{ij})$ and the set of n linear equations (2) reduces to :

$$\sum_{j \neq i} m_j F_{ij} = v_i^2 / d_i, \tag{3}$$

where $F_{ij} = G(d_i - d_j \cos(\theta_{ij}))/d_{ij}^3$.

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In order to solve such a linear system of n equations with $n + 1$ unknows m_i, we have to fix an arbitrary parameter. The total mass of the galaxy seems to be the natural parameter. Using dimensionless quantities, we use the normalised parameter $\omega = \frac{1}{M_g}$, where $M_g = \sum_i m_i$ is the mass of the galaxy.

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Denoting $\mu_i = m_i / M_g$, we thus have to solve the following $n+1$ equations with $n+1$ variables μ_i for each ω:

$$\sum_{j \neq i} \mu_j F_{ij} = \omega \, v_i^2 / d_i, \tag{4}$$

for $i = 1, \ldots, n$

$$\sum_i \mu_i = 1; \tag{5}$$

with the constraint:

$$\mu_i \geq 0, \tag{6}$$

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We stress that for a spherically symmetric distribution of matter, the Gauss theorem implies that only forces due to the matter *inside* the sphere have to be taken into account. This property is no longer valid for a two dimensional distribution (Binney and Tremaine 1987), and the integration domain of (1) must include not only the matter inside, but also *outside* the sphere. Not doing this gives erroneous results. So a maximal disk can't be considered as a limit of flattened spheroids.

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We used a numerical discretisation of 250 000 points. The points are displayed along 500 rings (500 points on each ring). The ring radii r_i are proportional to i^2, i.e. if the first one has a radius r_1, the second one a radius $r_2 = 4r_1$, $r_3 = 9r_1$ for the third ring and so on (in fact, it is to obtain the same order for all $\mu_i = m_i/M_g$, and so a good accuracy by numerical analysis). The constraints (6) determines ω_{min} and ω_{max}. This method provides a very narrow range $[\omega_{min}, \omega_{max}]$ of solutions, the differences between the given curves are indistinguishable.

Exponential disk, Mestel's disk and Keplerian curve I

In order to test the accuracy of our method, we first apply it to the following three well-known types of velocity curves: an exponential disk, a constant rotation curve (Mestel's disk), and a Keplerian rotation curve.

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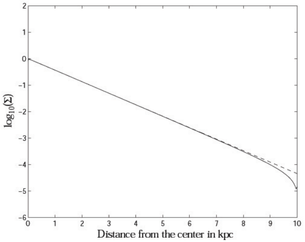

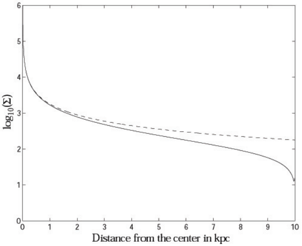

Left: surface density for the rotation curve of an exponential disk (solid curve), compared to the exact profile (dashed curve). The mass of an exponential galaxy of central surface density $1 M_\odot.pc^{-2}$ is $6.2832 \times 10^6 M_\odot$: the mass we derive from the allowed values of ω is $6.279 \times 10^6 M_\odot$, a very goog agreement !
Right: Surface density for a constant rotation curve, assuming a finite disk.

The dashed curve is the density of an infinite Mestel disk.

Exponential disk, Mestel's disk and Keplerian curve II

The rotation curve has the so-called Keplerian shape $v^2(r) = GM/r$. For this model, our numerical method reproduces the analytical result within the computer precision. The total mass is recovered exactly, and the density in the rest of the disk is less than 10^{-14} the central density, as expected for a Keplerian profile.

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How many people have worked after Mestel with the right equivalent methods : the method of the elliptic integrals (based on the disc seen like rings of matter), the method of the Bessel transform (based on asymptotic properties of the gravitational potential created by the disc of matter)? Many. Among them, B. Fuchs and all, astro-ph/0408072 ; A. Pierens and J.-M. Huré, astro-ph/0312529; F.I. Cooperstock and S. Tieu, astro-ph/0610370; L. Marmet http://www.marmet.ca/louis/galaxy/; K. Nicholson astro-ph/0309762; G. Pronko astro-ph/0611303; and so on.
It is very surprising that the wrong method of the spheroids is always used.

For Milky Way and Andromeda

Results for a small Milky Way (16 kpc): a mass of $1.4 \times 10^{11} M_\odot$. The exponential profile with a scale length between 3.6 and 5 kpc. The local surface density is found to be $125 \pm 10 M_\odot.pc^{-2}$.

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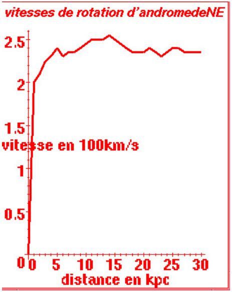

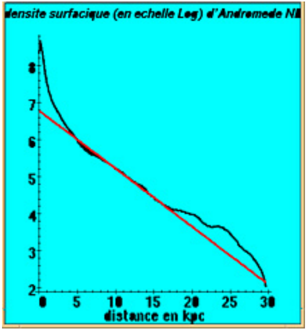

Results for Andromeda N-E : left the rotation curve, right the density curve. The mass is $2.49 \times 10^{11} M_{\odot}$ for the N-E curve (and $2.42 \times 10^{11} M_{\odot}$ for the S-W curve of the galaxy).

Relativistic approach

The Einsteinian study of these curves can be made using the potential coming from a non empty universe. The correction is non negligeable; this can seem surprising, insofar as the gravitational field created by a galaxy is very weak, but on such a scale the Einsteinian correction of the usual Newtonian gravitation proves to be small but non negligeable.

For this, we use the same method but as each point $\vec{x}_i$, is submit to a cosmological accelerating field $\vec{g} = -q_o\, H_o^2\ \vec{x}_i$, where H_o is the Hubble parameter and q_o the decelerating parameter we have to change the equations (2) using

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For all mathematical proofs we refer to

M. Mizony : La relativité générale aujourd'hui ou l'observateur oublié, Editions Aléas, juin 2003. The chapter 9 is on line at :
http://www.univ-lyon1.fr/IREM/michel/pdfch8bis.pdf or in english rotation curves
M. Mizony and M. Lachieze-Rey : Cosmological effects in the local static frame, Astronomy and Astrophysics, Volume 434, Issue 1, April IV 2005, pp. 45-52, gr-qc/0412084.