Acronyme	**SWECF**
Titre du projet en français	Ecoulement de fluides complexes en « eaux peu profondes »: analyse mathématique et applications
Titre du projet en anglais	**S**hallow **W**ater **E**quations for **C**omplex **F**luids: mathematical analysis and applications
CSD principale	5 Mathématiques et interactions
CSD secondaire *(si interdisciplinarité)*	

Aide totale demandée	152067 €	**Durée du projet**	48 mois

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1. Contexte et positionnement du projet / Context and positionning of the proposal

This project concerns the modeling and mathematical analysis of snow avalanches, mud floods and torrential lave flows. These natural hazards causes every year numerous material and human damages (destruction of roads, buildings,...). With the expansion of tourism in mountain areas, the expansion of cities in exposed zones, it is important to have efficient tools to determine the main characteristics of this kind of flow: height, velocity, pressure, impact on defence structures, runout extent...One possible way of modeling such flows is to consider a relatively thin layer of fluid flowing down a complex topography under the effect of gravity: this type of flow is described by the *shallow water equations* which is a system of partial differential equations (P.D.E.). These models are commonly used for land management and in particular the determination of flood zones and the design of defence structures. Shallow water equations are usually derived *heuristically* from the Navier-Stokes equations under simplifying hypotheses: however a careful analysis of the Navier-Stokes system shows that most of the models derived until now are either incomplete or correct under very restrictive assumptions. In the former case, some terms are missing, which may lead to inaccuracies, and in the latter, the model is often used out of its domain of validity requiring small slope and curvature and simple rheology. The aim of this project is the rigorous derivation of shallow water equations from the Navier-Stokes equations taking into account different characteristics of the flow: we will obtain *new terms* involving the influence of topography, fluid rheology, and we will include the cases of multifluid flow and of compressible flow. We will also determine precisely the domain of validity of our equations.

Consider a practical exemple, namely snow avalanches and dense snow flows. The snow is composed of water molecules that are in several physical states, thus snow can be considered as a *complex fluid.* Moreover this « fluid » is *Non Newtonian,* that is the deformation rate of the fluid is not proportional to the strain tensor. In dense snow avalanches, two layers coexist, one composed of powder snow and one composed of dense snow. A complex fluid can be the superposition of several layers of fluids with different physical properties (such as density, rheology, physical state). These properties can change drastically the behaviour of the fluid and it is important to take into account these features to obtain correct predictions. The starting point for modeling the flows that we consider are the incompressible Navier-Stokes equations with a *free surface:* this is a very hard problem from the mathematical point of view due to the presence of the *free surface* and the *complexity* of the fluid considered (snow, muds,...). Thus it is very important to obtain simple models that are easier to handle. The flows under consideration are all shallow i.e. the characteristic fluid height is much smaller than the characteristic length of longitudinal variations of the fluid velocity and fluid height. Under this hypothesis, we will obtain *Shallow Water Equations* that describes the evolution of the fluid height and the average velocity distribution: thus we get rid of the free surface problem and decrease the dimension of the problem.

We will derive new and generalized shallow water equations in physically relevant situations, particularly for multilayer flows and non Newtonian fluids. When possible, we will justify rigorously these computations to validate the models. If this is impossible, we will compare the numerical solutions of our generalized shallow water equations with the numerical solutions of Navier-Stokes equations, in order to identify the domain of validity of our models. In order to understand the limitations of our models and for numerical purposes, we will also study the mathematical properties of our generalized Shallow Water Equations : well-posedness of the Cauchy problem, existence and stability of nonlinear waves: shocks, roll-waves, wave trains.

2. Description scientifique et technique / Scientific and technical description

2.1. État de l'art / Background, state of the art

This project deals with mathematical problems from *fluids mechanics* and we will focus on *shallow* flows. More precisely, we consider flows with a characteristic depth that is much smaller than the characteristic wavelength of fluid height and velocity modulations. This situation occurs usually for river, dense snow avalanches, muds floods or ocean circulation. It is important to determine the main characteristics of avalanches and mud floods, so as to manage land more efficiently in relation to natural hazards. In particular, the determination of hazard zones and defences against these hazards depends on an efficient analysis of these destructive flows. The analysis of shallow water flows that we propose in this project may also have possible applications in industry . For instance the control of thin flows in industrial processes is important: a single decorative layer on packaging, design of car windows, windscreens, food fluids...

The physical properties of the fluid, in particular its mechanical properties, can change drastically the characteristics of the flows and thus must be described carefully. The fluids that are of interest for applications and that will be considered in our project are *complex fluids:* these are *Non Newtonian fluids.* More precisely, the deformation rate of the fluid is not proportionnal to the strain that is applied to the fluid. *Rheology* is the science which determines theoretically or experimentally the constitutive law of a fluid, i.e. the mathematical relation between the strain tensor applied to the fluid and the deformation tensor of the fluid. Among all the fluids with complex rheological properties, let us mention *yield stress fluids* like snow or some industrial pastes composed of polymeres: in order to start the flow of these fluids, the applied stress must exceed a given threshold. Other fluids like some muds (mixture of water and clay) have an *apparent viscosity* (the « ratio between the strain and the deformation ») which decreases or increases as a function of the strain (phenomenon of rheofluidification). The *complexity* of the flows that are under consideration may also be due to the presence of several layers of fluids with different physical properties such as density or physical state. This is the case of a snow avalanche, which contains a layer of powder snow and another layer of dense snow. This also occurs when water bodies of different salinity meet or when muds with disctinc physical properties flow together.

Let us briefly describe here the state of the art concerning the modeling, mathematical analysis and numerical simulations of this class of problems. Further references and a detailed description of the objectives will be given in the next sections.

- **Modeling**. We propose to study *free surface* flows of *complex fluids* with *Shallow Water equations.* Those models are usually proposed in hydrology, since they are more managable from a numerical point of view. They are obtained heuristically from the averaged *boundary layer equations* with an assumption on the distribution of the fluid velocity along the height of the fluid. Most of the models derived until now are not complete (see e.g. Piau for Bingham fluids [38]) and may lead to some inaccuracy (see e.g. Ng and Mei for Power-law fluids [39]). A more direct derivation of the Shallow Water Equations from the Navier-Stokes equations for Newtonian fluids with a free surface has been proposed only recently for horizontal bottoms and small viscosity by J.-F. Gerbeau & B. Perthame [15], and generalized by F. Bouchut and M. Westdickenberg [4] for arbitrary topographies and flows that are almost still (lake at rest). For arbitrary viscosity and slope, we derived a Shallow Water model with a source term including gravity, a drag term that is due to the viscosity and a « small » viscous term (see [24,36] for inclined planes, [23] for arbitrary topographies): this derivation is valid for flows that are close to *Poiseuille flows*. Recently, we have derived new shallow water models for *Bingham* and *Power Law* fluids [28]: this was done through a careful asymptotic expansion of solutions to the Navier-Stokes equations in the presence of a singular apparent viscosity near the free surface for Power Law fluids and in the pseudo plug zone for Bingham fluids. These are the first shallow water models obtained using this methodology: we are only aware of lubrication models and inaccurate shallow water models obtained in this fashion (see Ng and Mei [39], Balmforth and co-workers [40]).
- **Mathematical Analysis.** When writing models, a natural question for a mathematician when writing models is the well posedness of the Cauchy problem. The well posedness of

inviscid shallow water equations relies on the theory of hyperbolic systems and the asymptotic behaviour of solutions in the presence of a source term is analysed through the theory of relaxation in hyperbolic systems. When steady solutions are unstable, one can prove the existence of roll-waves (Dressler [14], Jin & Katsoulakis [41]): these are periodic and discontinuous travelling waves solutions of the shallow water equations. We have recently obtained stability and persistence results for that kind of solutions (Noble [21],[27]) by introducing a general framework to studying stability problems in the presence of an infinite number of shocks. The existence of strong and weak solutions for viscous Shallow Water equations (that are similar to Barotropic compressible Navier-Stokes equations), is now a classical result which has first been proved for a class of unphysical viscous terms (see e.g. Lions [42]). The global existence of weak solutions for realistic viscosities dates back to 2003 and has been obtained in two space dimension by D. Bresch and B. Desjardins [7] with some restrictions (presence of capillarity or a turbulent drag term). There are very few results on the mathematical justification of shallow water equations: let us mention here the mathematical justifications of Shallow Water models obtained recently by D. Lannes and co-workers [13,17,18]: the starting point in this case are the Euler equations for *incompressible and irrotationnal fluids* (water waves). Though the proof uses hard analysis and justifies a hierarchy of models, these authors obtained a shallow water model that only takes into account hydrostatic pressure and convection. This is not enough for the applications we have in mind (e.g. complex fluids, non potential flows). For a mathematical justification of Shallow Water equations *with a source term,* we have justified mathematically only recently the derivation obtained by J.-P. Vila [24,36].

- **Numerical Simulations.** The rigorous justification of formal computations does not provide any insight on the qualitative and quantitative behaviour of solutions and on the limitations of the first models we obtained. Our models can be validated through systematical numerical simulations in order to obtain quantitative results that may be compared to experimental data and to direct numerical simulations of the full Navier-Stokes problem. For the numerical simulations of Shallow Water equations and more generally hyperbolic equations with source terms, the design of high order methods is an extremely active field. These are in particular finite volumes methods such as WENO (see the works of Chi Wang Shu and co workers) and « well balanced » schemes (see the papers of B. Perthame, F. Bouchut and co-authors [2]) . For the full Navier-Stokes system with a free surface, there are now efficient numerical methods for 2d flows and 3d flows (high order Lagrange Galerkin methods and ALE methods for the interface) when the fluid is Newtonian or Non Newtonian (see in particular the papers of P. Saramito). these methods are based either on finite element or finite volume methods, level set or diffuse interface methods for treating the free surface.

We will describe our flows by the Navier-Stokes equations with a *free surface*: it is a very hard problem to analyse mathematically and numerically this kind of system because one has to deal with a free surface but also with *complex rheology* (the fluids are Non Newtonian). However, we plan to study fluid flows in the *shallow water* regime: this hypothesis is realistic for e.g. dense snow avalanches, mud floods or in oceanography. This simplifies the equations: the pressure is almost hydrostatic inside the fluid layer and the fluid velocity is almost parallel to the bottom. We then obtain the so called *Shallow Water Equations,* which is a system of Partial Differential Equations describing the evolution of the fluid height and the discharge rate in the streamwise direction. Furthermore, one can obtain simpler models involving only the fluid height, the so called *lubrication equations.* The main feature of these models is that we get rid of the free surface and we obtain a lower dimensional system: we drop the « vertical » space variable and the « vertical » component of the fluid speed. The mathematical analysis and numerical simulations are then more tractable. Our project will concern modeling, mathematical and numerical analysis of shallow water flows.

2.2. Objectifs et caractère ambitieux/novateur du projet / Rationale highlighting the originality and novelty of the proposal

The shallow water equations give a simple description of the flow of a fluid layer under gravity and their numerical treatment is now standard. However most of the known models are incomplete, which leads to inaccuracy. Consider the case of a layer of Newtonian fluid flowing down an inclined plane. In order to study the stability of Poiseuille flows, the physicists have obtained through long wavelength asymptotic shallow water equations. Linearizing these equations around a stationary solution, one gets a stability criterion. Another procedure consists in linearising Navier-Stokes equations around a Poiseuille flow and then taking long wavelength expansions. A stability criterion can be determined also in this case. If everything has been correctly done, these two stability criteria must coincide. Only recently, Ruyer-Quil and Manneville in [43] and J.-P. Vila in [36] formally derived shallow water equations for Newtonian fluids that yields the correct stability criterion. In the case of Newtonian shallow flows with small viscosity and friction at the bottom, Gerbeau and Perthame [15] derived formally a viscous model under the assumption of flat bottom and small slope: Bouchut et al. [4,5] also derived a formal model relaxing the assumptions to arbitrary topographies. Bouchut et al. [4,5] also gave well balanced schemes for this class of models. However, this analysis has not been carried out yet in more complex situations like non Newtonian fluids and Multilayer flows; we plan to study specifically debris flows, snow avalanches and mud floods. There is very few studies on the quantitative comparison of the models in order to determine their domain of validity. Nonlinear waves will appear in numerical simulations and we would like to understand them theoretically in order to validate our simulations Observe that our knowledge of nonlinear waves includes only some informations on roll-waves, solitions and fronts in viscous and inviscid shallow water models heuristically derived.

Therefore, our project focuses on four mains tasks.

- ***Task 1: Mathematical justification of Shallow Water equations from Navier-Stokes equations (hydrostatic and non hydrostatic) for Newtonian fluid****:* we will try to show mathematically and numerically that in the shallow water regime, shallow water equations provide a good approximation of Navier-Stokes equations, for Newtonian fluids (which have the simplest rheology among fluids) submitted to varied forces (gravity, Coriolis forces, wind traction at the surface) and several kinds of boundary conditions at the bottom (no slip, Navier conditions).

- ***Task 2: Shallow Water equations for non Newtonian fluids.*** We will analyse the flow of non Newtonian fluids that are commonly found in the literature (Bingham, Power law, Carreau and Herschel Bulkley type fluids) and for which experimental data are available (mixtures of water and clay, some polymeric mixture, snow): we will establish a hierarchy of shallow water models for these fluids through a direct asymptotical analysis of Navier-Stokes equations. The shallow water models and full Navier-Stokes equations will be analysed both from the mathematical point of view (well posedness, nonlinear waves,...) and the numerical point of view (for which efficient methods will be implemented) in order to justify mathematically the formal computations.

- ***Task 3: Modeling of multilayer flows with distinct physical properties****.* We plan to derive accurate Shallow Water equations from Navier-Stokes equations for a fluid composed of two layers: one of the motivations is to obtain small but physically relevant viscous terms in classical bilayer shallow water models in order to get rid of the indetermination of non-conservative products in the presence of shocks. The other objective is to study bi-layer flows when erosion and sedimentation occur: this is important for practical applications. We will extend these models to a larger number of fluid layers. The objective is to investigate (mathematically and numerically) whether multi-layer Shallow Water equations are useful to approximate the flow of a fluid with an *arbitrary but finite* depth provided that the number of layers is sufficiently large. Amélie Rambaud has started recently a Ph. D. thesis on this particular aspect of our project.

- ***Task 4: Analysis of hydrodynamic instabilities.*** We aim to describe mathematically some nonlinear waves that are observed in nature (mascaret, roll-waves, wave packets) and propose a description through the shallow water equations: we will try to prove the existence of such waves and analyse their *stability,* and thus whether they are observable, using methods from dynamical systems and the theory of hyperbolic systems. Direct numerical simulations on Navier-Stokes and shallow water equations shall help to explore the limits of these equations in the unstable regime.

- ***Recent developments****.* A previous version of this project was submitted last year to the ANR programme. We point out some recent developments. As a first step, we derived an inviscid model in two special cases of non Newtonian flows: Bingham and Power-Law. This yields new criteria of stability in particular for Bingham flows [28]. For Power-Law fluids, our criterion is the same as the one predicted by a direct analysis of Navier-Stokes equations: this is part of a work in progress (P. Noble, L. Chupin, J.-P. Vila) and we aim to prove the well-posedness of free surface Navier-Stokes equations for Power Law fluids in the neighbourhood of steady solutions. In collaboration with E.D. Fernandez-Nieto, we investigate numerically our Bingham shallow water equations and the influence of the new stability criterion. We have also obtained inviscid shallow water equations for bi-layer flows for an arbitrary slope: we have derived a stability criterion for steady solutions and we are studying the conditions of the onset of roll-waves. More generally, during the last two years, we achieved several important steps towards the understanding of the roll-wave phenomenon (stability in viscous and inviscid shallow water equations for Newtonian fluids [21,25,27], existence in general hyperbolic systems [26]). We have also justified rigorously in [24] the shallow water model obtained by J.-P. Vila for a two dimensional flow on a inclined plane and we have extended formally his computations to *arbitrary* topographies [23].

- ***Innovative aspects.*** The shallow water equations are commonly used to describe free surface flows because they are mathematically and numerically simpler than Navier-Stokes equations. However heuristically derived models may be quite inaccurate and their numerical simulations inherits this inaccuracy. A systematic and mathematically justified derivation method for this class of models has not yet been performed.
  The novelty of our project is (1) to tackle non Newtonian and multilayer fluids (2) to obtain shallow water models as mathematically proved as we can (3) to validate the domain of applications of the approximation by numerical comparison between our models and direct numerical simulations of Navier-Stokes equations (4) to obtain as much qualitative informations on coherent structures as possible also to validate our models.
  Our new shallow water models may improve the physical understanding of natural hazards such as snow avalanches, mud floods, torrential lava and debris flows. The possible applications are likely to answer pratical problems like the land management (determination of the extension of an avalanche) and engineering problems such as the design of defence structures.

3. Programme scientifique et technique, organisation du projet / Scientific and technical programme, project management

3.1. Programme scientifique et structuration du projet / Scientific programme, specific aims of the proposal

We have divided the project into 4 main tasks with 4 persons who will be in charge of the coordination of each task (this shall be made precise in the scientific presentation of the tasks). The 4 main tasks are noted T1,T2,T3,T4. If accepted, the project will last four years: in what follows, we have noted Tij a part of the task Ti that will treated during the year j.

- ***<u>Task 1:</u>*** *Mathematical justification of the derivation of shallow water equations from Navier-Stokes equations (hydrostatic and non hydrostatic) for a Newtonian fluid.* The viscous shallow water equations obtained formally by Gerbeau and Perthame [15] provide a good approximation of Navier-Stokes equations and we will try to understand the quality of this approximation. We plan to analyse first the derivation procedure applied to the 2d hydrostatic Navier-Stokes equations with free surface **(T11)**. Next step will be the 3d problem in the presence of Coriolis forces and wind traction: we will first try to establish the well-posedness of these equations and estimate the quality of approximation by viscous shallow water equations **(T12)**. Finally, we plan to analyse the derivation procedure when applied to the full 2d Navier-Stokes system with free surface **(T14)**. We will also determine the limits of this approximation through a direct numerical investigation of the problem. We will first compare numerical simulations of the inviscid shallow water obtained in [24,36] and direct simulations of the Navier- Stokes system with a no-slip condition at the bottom **(T12, T13)**. Then we will investigate numerically the approximation of Navier-Stokes equations with a Navier slip condition at the bottom by viscous shallow water equations derived in [15] **(T13)**.

- ***Task 2:*** *Shallow Water equations for non Newtonian fluids.* We will analyse the flow of non Newtonian fluids commonly found in the literature and for which experimental data are available (mixtures of water and clay, some polymeric mixture, snow). We will establish a hierarchy of Shallow Water models for non Newtonian fluids : Herschel Bulkley, Carreau, biviscous fluids, and for this purpose we will use a direct asymptotical analysis of Navier-Stokes system. As in the Newtonian case [24], we will first try to derive shallow water equations for non Newtonian fluids **(T21)** and this will give a first stability criterion of stationary solutions. We will also try to understand and analyse Poiseuille flow in the non Newtonian case and if possible we will linearize Navier-Stokes system around Poiseuille flow, look for a stability criterion and hopefully validate thus the above shallow water models **(T22)**. In the simpler cases (e.g. Power-Law fluids), we will investigate the well-posedness of Navier-Stokes equations with free surface, the spectral stability being the first step of the analysis. We hope that, as in the Newtonian case, this analysis will help up to justifiy rigorously the shallow water models for some particular non Newtonian fluids **(T23-T24)**. As in task 1, we will try to determine numerically the quality of approximation by comparing numerical simulations on our shallow water models **(T21)** and direct numerical simulations of Navier-Stokes system **(T23-T24).**

- ***Task 3:*** *Modeling of multilayer flows with distinct physical properties.* We aim to derive shallow water equations from Navier-Stokes equations for a fluid composed of two layers of fluid **(T31)**. The existing models are either inaccurate or inviscid. These inviscid models are hyperbolic non conservative systems, and they are obtained by stopping the expansion at a small numbers of terms. Pares and co-workers [8,29] have worked on shock solutions of these non conservative systems, they basically construct a path in order to determine the shock and they find that their shocks are path dependent. Vanishing viscosity methods allow for a unique shock of the path, but the question is to determine a physically relevant viscosity. This can be accomplished by obtaining more terms in the expansion (see the work of Vila [36] in the single layer case) Then we will simulate non conservative flows with a path justified by physical viscosity or directly the viscous models of the flow **(T32)**. Moreover, we will investigate, at least formally, the mass transfer between two layers (sedimentation, erosion): including this phenomenon in the model and the numerical simulations is of great practical importance if one wants to understand snow or submarine avalanches and their impact on the bottom **(T33)**. Next step is to extend the analysis to a larger number of layers and try to see whether multilayer Shallow Water equations can be used to approximate the flow of a fluid with an *arbitrary but finite* depth provided that the number of layers is sufficiently large. We will try to relate *rigorously* the multi layer Shallow Water models with primitive equations and stratified quasi-geostrophic equations that are commonly used in oceanography and meteorology **(T32-T33)**. We will also work on a numerical validation of this approach similarly as in the single layer case **(T32-T34)**.

- ***Task 4:*** *Analysis of hydrodynamic instabilities.* An interesting property of the shallow water models is that they can describe the transition to instability in free surface flows. One observes coherent structures in these flows: solitary waves (tidal bore in the river Gironde), roll-waves (in spillways of a dam, rivers,...) and more complicated phenomena. We aim to prove that some solutions of our shallow water equations describe these waves and we will study their existence **(T41-T42)** and their stability **(T43-T44)** using the methods of dynamical systems and hyperbolic partial differential equations. Note that the question of existence of roll-waves in our models is not straightforward: for instance, it is a hard problem to prove the existence of inviscid roll-waves in bilayer flows (see e.g. [26] for the problem formulation and mathematical issues). In our project, we will focus on the stability of roll-waves. Indeed there are very few studies on the stability of periodic travelling waves in the conservation laws setting and the roll-waves phenomenon is a very interesting exemple of periodic travelling waves in hyperbolic systems. As a by-product of the analysis, we expect to prove that even in the case where the derivation of shallow water equations is not fully justified mathematically (as in unstable regime), these models are still usefull to describe shallow flow with a free surface. This analysis is an important step towards the validation of the models and of the numerical schemes designed for these models.

3.2. Coordination du projet / Project management

We recapitulate here the organisation of the project described above. Underlined are the coordinator(s) of the task.

T1: Mathematical justification of shallow water equation for Newtonian fluids.
- ➔ **Participants: <u>P. Noble, D. Le Roux</u>, L.-M. Rodrigues, S. Delcourte.**

T2: Shallow water equations for Non Newtonian fluids.
- ➔ **Participants: <u>L. Chupin, J.-P. Vila</u>, P. Noble, S. Delcourte.**

T3: Multilayers Shallow Water equations
- ➔ **Participants: <u>F. Filbet</u>, P. Noble, A. Rambaud, D. Le Roux.**

T4: Mathematical analysis of hydrodynamical instabilities.
- ➔ **Participants: <u>P. Noble, J.-P. Vila</u>, L.-M. Rodrigues, V. LeBlanc.**

➔ <u>First year</u>: **T1** : Mathematical justification of viscous shallow water equations from 2D hydrostatic Navier-Stokes equations **T2**: Derivation of Shallow Water equations for fluids with Herschel-Bulkley law. Numerical simulations of the shallow water models for Bingham and Power Law fluids. **T3**: Derivation of viscous bilayer Shallow Water models and well posedness of the system. **T4:** Spectral analysis of roll-waves in the large wavelength limit. Existence of inviscid roll-waves in bi-layer flows and Bingham/Power law fluids.

➔ <u>Second Year:</u> **T1**: Mathematical justification of Shallow Water equations from 3D hydrostatic equations for Newtonian fluids with/without capillarity and/or in the presence of Coriolis forces. Numerical simulations of free surface Navier-Stokes equations and comparison with Shallow water equations: first tests. **T2:** Stability analysis of Navier- Stokes and Shallow Water equations for complex fluids (Power Law, Bingham, Carreau). **T3**: Numerical simulations of multi-layer shallow water equations. **T4:** Existence and stability of periodic travelling waves for shallow water equations with capillarity.

➔ <u>Third year</u>: **T1:** Comparison of Shallow Water equations with free surface Navier-Stokes eqs: test cases in the instability regime: formation of roll-waves, dam break. **T2:** well posedness of free surface Navier-Stokes equations for Non Newtonian fluids (power-law, Carreau law). **T3:** Derivation of shallow water equations in the presence of sedimentation/erosion and numerical simulations. Mathematical convergence of multilayer Shallow Water equations to primitive equations and stratified quasi-geostrophic equations. **T4**: Stability of roll-waves in the vanishing viscosity limit.

➔ <u>Fourth year</u>*:* **T1***:* Rigorous derivation of viscous shallow water equations (Gerbeau/Perthame model) from the full (2d) Navier-Stokes equations**T2**: Mathematical analysis of Shallow water equations for yield stress fluids. Comparison of numerical simulations of Navier-Stokes and shallow water equations for non Newtonian fluids. **T3:** Numerical comparison of multilayer shallow water equations and primitive equations/stratified quasi-geostrophic equations. **T4:** Nonlinear stability of roll-waves: analysis of slow modulations.

3.3. Description des travaux par tâche / Detailed description of the work organised by tasks

3.3.1 Task 1: Mathematical justification of shallow water equations for Newtonian fluids

The purpose of this study is to **prove rigorously** that one can obtain the Shallow Water equations from the full Navier-Stokes equations with a free surface in the asymptotic « shallow waters » and for a Newtonian and incompressible fluid. This is the most simple situation that we can imagine for the type of flows we study. We shall consider different forces (gravity, Coriolis forces, wind traction) and different kind of boundary conditions (no-slip conditions, Navier conditions, Coulomb friction). We plan to focus on the derivation of shallow water equations from hydrostatic Navier-Stokes equations (also called primitive equations) that are commonly used in oceanography before dealing with the more involved case of the full Navier-Stokes system.

a) Rigorous Derivation of Shallow water equations

- *Existing results.* There are very few results in the literature concerning the derivation of the shallow water equations from Navier-Stokes equations through an asymptotic expansion in the shallow water regime: see e.g. J.-F. Gerbeau and B. Perthame [15] for a formal derivation of a viscous shallow water model in the two dimensional case for a Newtonian fluid with small viscosity and a Coulomb friction term and D. Lannes and co-workers [17,18] for a rigorous derivation of Shallow Water equations **without source terms** from Euler equations.

- *A first step in the justification.* For a Newtonian fluid with arbitrary viscosity flowing downward an inclined plane with a no slip boundary condition at the bottom, we have obtained formally a hierarchy of Shallow Water models with a source term containing the effects of gravity and viscosity (see Vila [36] for more details). This approach yields a stability criterion for steady flows that is consistant with the criterion obtained directly from the analysis of the full Orr-Sommerfeld equations in the long wavelength regime. These computations are justified rigorously for a low Reynolds number (laminar regime) and in the presence of capillarity (see Noble & Bresch [24]). However, there are no numerical simulations in this case, which determine quantitatively the limitations of this model (see next section).

- *Further developments.* The model that we justified rigorously does not take into account many relevant physical situations such as the presence of Coriolis forces for 3-dimensional flows, wind traction at the upper surface and other boundary conditions (Coulomb friction term). The model is limited to flows with a non vanishing slope and for large capillarity. Moreover, the model is inviscid and does not give the form of the relevant viscous terms in shallow water equations. We will attempt the horizontal bottom case and in particular we will try to justify rigourously the 1d model proposed by J.-F. Gerbeau and B. Perthame [15] and generalised by F. Marche [44] in the 2d case. We will first start with the derivation of a viscous shallow water model from 2d hydrostatic Navier-Stokes equations: we plan to extend to thin free surface flows the approach developed by Temam/Ziane in [45] for Navier-Stokes equations in thin fixed domains. When this step is achieved, we shall try to understand the problem in the 3d setting for hydrostatic Navier-Stokes equations and then for the full 2d Navier-Stokes system.

b) Numerical validation of the Shallow Water equations.

This part of the analysis is important since there is very few studies on this subject. Here our purpose is to validate the Shallow Water model that we have rigorously justified [24]. More precisely, we would like to provide numerically a quantitative limit of validity for the Shallow Water model through a direct comparison with Navier-Stokes system. Although such a limit is important, it is rarely taken into account in the area of hydraulic shallow flows. The numerical simulations of shallow water equations in this setting will not present particular difficulties since there are many high order numerical schemes which can be used (finite volume methods for hyperbolic equations like WENO) to perform simulations of hyperbolic Shallow Water equations. We will discretize source terms so as to conserve exactly the steady solutions: here we will use well balanced schemes. There is a huge literature on this kind of methods: let us mention the works of Pares, Castro, Nieto [29] on the analysis and implementation of these schemes and the papers of B. Perthame, F. Bouchut and co-workers on their mathematical properties [5]. Concerning the numerical simulations of Navier-Stokes equations and in order to simplify the problem, we will restrict our attention to the 2d vertical case, for which there exists different numerical methods: finite elements methods (ALE), level set or diffuse interface methods in order to treat the free surface motion. The results of numerical simulations of 2d Navier-Stokes systems with free surface corresponding to the hydrostatic and non hydrostatic cases (with finite elements methods, diffuse interface, level set methods) will be compared with those of the Shallow Water equations. The simulation of dam break flows and roll-waves generation will be performed. A particular attention will be dedicated to the instability threshold in the presence of a varying bottom topography.

Task 1 will be supervised by P. Noble (theoretical part) and D. Le Roux (numerical part) in collaboration with L.M. Rodrigues (mathematical justification) and S. Delcourte (numerical analysis of the problem). P Noble and L.M Rodrigues will be in charge of the mathematical justification of derivation of viscous shallow water equations from Navier-Stokes equations: P. Noble has already an experience in the mathematical justification of shallow water equations and L.M. Rodrigues has studied during his Ph.D. thesis compressible barotropic Navier-Stokes equations that are close in structure to viscous shallow water equations. We plan a collaboration on this particular part of the project with D. Bresch (University of Savoie). D. Le Roux and S. Delcourte will be in charge of the numerical comparison of Navier-Stokes and Shallow Water equations and will supervise a post-doc student on this subject: they have a strong experience in the numerical simulation of Navier-Stokes equations with finite elements methods and we will collaborate with E.D. Fernandez-Nieto who is an expert in the numerical simulation of shallow water equations.

3.3.2 Task 2: Shallow Water equations for non Newtonian fluids

a) Formal derivation of Shallow Water equations for non Newtonian Fluids

- *Position of the problem.* Fluids like muds, dense snow, lava, some paints and polymeres are non Newtonian fluids: in general, the deformation tensor of the fluid is not proportional to the strain tensor. In several cases, the strain must be larger that a given threshold for the fluid to start to flow: these fluids are called *yield stress fluids*. The apparent viscosity of the fluid (the ratio between the norm of strain tensor and the norm of deformation tensor) is an increasing or nonincreasing fonction of the deformation tensor; in other words, the viscosity of such fluids depends on strain. From experimental measurements on simple Poiseuille flows, one can determine the *rheology* of the fluid: this is a mathematical relation between the strain tensor and the deformation tensor. The models that are usually found in the literature are *Bingham model* and *Hershel Bulkley model* for yield stress fluids. For fluids with an apparent viscosity depending on the deformation rate, one usually uses either power-law models or regularized versions of power laws (like Ladyzhenskaia or Carreau laws). For the applications we have in mind, we plan to analyse the flow of non Newtonian fluids in the *shallow water limit.* Most of the models that are used in this situation are lubrication models with one equation on the fluid height. However their domain of validity is restricted to small modulations of the free surface: we have to obtain models with a wider domain of validity. The only models of Shallow Water type we are aware of, which are used for non Newtonian fluids are derived heuristically from the boundary layer equations: they are not incomplete and inaccurate from a physical point of view [38],[39],[40].

- *Aims and methodology.* As a first step, we aim to derive shallow water equations from Navier-Stokes equations using the formal asymptotic method already applied in the Newtonian case. Then we will see what are the new terms due to the non Newtonian rheology. Recently, in collaboration with E.D. Fernandez-Nieto (Univ. Sevilla), we have obtained shallow water models both for *Bingham fluids* and *Power-Law* fluid with similar methods: the main issue here is the singularity of the apparent viscosity either near the free surface (for Power law fluids) or in the interface between the viscous and the plug zone (for Bingham fluids) [28]. As a by-product of this analysis, we have obtained new criteria of stability under long wavelength perturbations. We plan to investigate the physically more relevant case of Herschel Bulkley fluids, whose constitutive law combines a power- law with yield stress, the Carreau type law, which is commonly used to regularize the Power-Law model and bi-viscous models, which are regularized versions of Bingham fluids: the main purpose is to obtain shallow water equations for physically relevant situations and to see whether the resulting models are close to the ones we have obtained for Bingham and Power-Law fluids. We will determine whether it is admissible to use these regularized versions to describe Power-Law and Bingham fluids in the long wavelength limit. We will also develop numerical methods for this new type of Shallow Water flows in order to fit our model to experimental data. We have contacted G.M. de Freitas , University of Sao Paulo (Brazil) who will carry out experiments for this kind of fluids.

b) Well posedness of Navier Stokes equations with free surface and shallow water equations in the non Newtonian case.

- *Position of the Problem.* We aim to perform the most rigorous possible derivation of shallow water equations for non Newtonian fluids from Navier-Stokes equations. To our knowledge, there are no mathematical results on the existence of solutions for Navier- Stokes equations for Non Newtonian fluids **in the presence of a free surface.** The only known results concern **fixed domains** in the non Newtonian case: see the work of L. Chupin, C. Guillopé and J.C. Saut [6,9,10,16] for viscoelastic fluids or the papers of Malek, Necas [46] and co-workers for Power-law fluids and regularized version like Ladyzhenskaia law. One possible way to compare the two models is to consider the linearized equations in the neighbourhood of steady solutions: in the Newtonian case, one can see that the two linearized systems predicts the same linear instability threshold in the long wavelength limit. In the non Newtonian case, the computation of the analogue of Orr-Sommerfeld equations (linearized Navier-Stokes equations) is harder since in general the steady solutions cannot be extended above the free surface (in contrast to Newtonian fluids). The problem is even worse in the case of Bingham fluids where a linearization procedure has to be written rigorously: for this case, we will use the formulation established by D. Bresch and co-workers [37] to derive rigorously linearized equations for Bingham fluids. As of now, the only stability results for Navier-Stokes system are consequence of results for shallow water models! It is important here to provide a mathematical framework for the direct study of the linear stability of steady solutions of Navier-Stokes system. This is an important step towards the well-posedness of free surface Navier-Stokes equations and the mathematical justification of shallow water equations.

- *Purpose and methods.* In order to avoid the singularity due to yield stress, we will first restrict our attention to power-law fluids and generalized version like Ladyzhenskaia and Carreau type laws: the purpose is to obtain well-posedness results in the presence of a free surface and thus to generalize results, which were obtained only in fixed domains. In the particular case of power-law fluids, the viscosity diverges to infinity in the neighbourhood of the free surface: it is important here to give a rigorous meaning to the linearization of Navier-Stokes equations. We will also analyse the case of Carreau laws, for which no such problem occurs in the linearization process, but, of course, the linear problem degenerates when the Carreau fluid is « close » to a power-law fluid. In the case of a Carreau fluid, we will investigate the Orr-Sommerfeld equations both numerically and through asymptotic calculations in the long wavelength limit in two regimes: either almost Newtonian or Power-Law. In order to clarify this description let us observe that the Orr-Sommerfeld equations are linear equations obtained after Fourier transform in the longitudinal space variable and Laplace transform in time. This still contains differentials in the vertical direction. The long wavelength limit corresponds to an approximation for small wave numbers, we will analyse in this limit the spectrum of this equation. We will compare this spectrum with the spectrum of linearized Shallow Water equations. We believe that this process will enable us to validate the asymptotic expansions. When Poiseuille flows are spectrally stable, we expect to prove the well-posedness of Navier-Stokes equations as for Newtonian fluids.This is the first step of a mathematical justification of Shallow Water equations (see [24] for more details in the Newtonian case). We will then focus on the Bingham case and use the weak formulation of Bresch and co-workers [36]. First we will write a linearization of Navier-Stokes equations near a Poiseuille type flow and then we will possibly obtain well-posedness results for Navier-Stokes equations with a Bingham fluids assuming the flow is shalllow and close to a Poiseuille.

Task 2 will be supervised by J.-P. Vila (modeling and numerical simulations) and L. Chupin (theoretical aspects) in collaboration with P. Noble (formal derivation of SW eqs, analysis of linearized Navier-Stokes equations for non Newtonian fluids, well posedness aspects) and E.D. Fernandez Nieto for numerical aspects concerning the numerical simulation of shallow water equations for non-Newtonian fluids. J.-P. Vila will be in charge of the modeling and numerical aspects of that task: he has a strong experience on modeling aspects of shallow water flows and numerical simulations of free surface flows (Navier-Stokes/Shallow Water equations). Sarah Delcourte will also work on the numerical aspect of this task, extending the work done in the Newtonian case. L. Chupin will be in charge of the theoretical aspects of the task especially the well posedness of Navier-Stokes equations for Non Newtonian fluids: his experience in thin films of non Newtonian fluids (Hele Shaw type) will be usefull here to tackle the main issues of this task.

3.3.3 Task 3: Multilayers Shallow Water Equations.

a) Multilayer Shallow Water equations with no mass exchange.

- *Position of the problem.* Snow avalanches, mud floods or torrential lava are usually composed of several layers of fluids or different phases with distinct physical properties (density, viscosity, salinity, powder snow/dense snow, sediments). One also finds this

stratification phenomenon in oceanography and meteorology, where ocean or atmosphere are constituted of several layers of fluid with different densities. It is thus important to take into account this situation in order to obtain realistic models. The simplest case concerns bi-layer flows. Most of the bi-layer Shallow Water models that are commonly used in the literature are non conservative hyperbolic systems. This could be the source of mathematical and numerical problems in the presence of shocks: one has to define an admissible shock (see e.g. the papers of Pares [8,29] for the special case of bi-layer shallow water equations). Admissible shocks are not uniquely defined: one possibility is to regularize the problem by inserting a vanishing viscosity, as in to the conservative case, in order to obtain admissibility conditions. However, the admissibility conditions highly depend on the form of the viscosity. We have to write physically relevant viscous terms. The known extensions of bi-layer models to more layers suffer from the same indetermination.
The second application we have in mind for multi-layer flows is the description of a fluid with a free surface and an *arbitrary depth.* We think particularly of a fluid with vertically varying density. The main idea is to cut the layer into thin layers of fluid of constant density, and use thin layer models (lubrication models, shallow water models): this method has been introduced by Pedloskii to describe density stratified geostrophic flows. Starting from a description with 3d Euler equations, a coupled system of 2d quasi geostrophic equations is obtained. However this kind of models are only valid in the geostrophic limit and it could be useful to consider multilayer shallow water equations that are valid in a wider range. Note that we have to be careful in the approximation process since for an homogeneous fluid (constant density) the resulting system is not hyperbolic (see Audusse [1,2]).

- *Aims ans Methods.* The method that we introduced (see the paper of J.-P. Vila [36] and [24]) has the advantage of giving physically relevant viscous terms in the case of a *single layer of fluid.* We aim to apply this method to obtain a bi-layer shallow water model from Navier-Stokes equations for two fluids with distinct densities and viscosity. We will expand up to second order with respect to the aspect ratio, in order to determine which viscous terms must appear in shallow water models. We plan to study different physical situations: the easiest one is the case with friction at the bottom and at the interface between the two fluids. We expect to obtain in this particular case a natural extension of the single layer model derived by Gerbeau and Perthame [15]. We will try to understand the case of an arbitrary slope with a no slip condition at the bottom and at the interface: as shown by J.-P. Vila [36] in the single layer case, we expect a complicated viscous term and we plan to explore the case of small amplitude solutions in order to simplify the situation.
The second part of the analysis concerns the approximation of a fluid of arbitrary depth as a superposition of thin layers of fluids for which simple models exists. We plan to continue the first modeling step that E. Audusse initiated [1,2]: he obtained a multilayer Shallow water system, which approximates the boundary layer equations. This formal result is « optimal » in the sense that one cannot obtain an approximation of the full Navier-Stokes system with a superposition of layer of fluids modeled by Shallow Water equations: the pressure is necessarily hydrostatic within the whole fluid. The system obtained by E. Audusse is not hyperbolic. Therefore, we will analyse a regularized version of it by adding a physical viscous term in the momentum equations in each layer: actually, we consider here a multilayer approximation of the *primitive equations* with a free surface. We will try to prove the existence of weak/strong solutions for this system following the methods introduced by P.L. Lions [42] for compressible fluids and D. Bresch & B. Desjardins [7] for shallow water equations with a physically relevant viscosity. When this step is achieved, we plan to consider the limit of an infinite number of fluid layers and we will try to prove convergence to the primitive equations. We will investigate the effects of Coriolis forces and explore the geostrophic limit in order to connect the multilayer Shallow water model with the multilayer quasi-geostrophic models written by Pedloskii [47]. This asymptotic analysis will be validated by numerical simulations comparing multilayer models with models possessing a free surface (here the primitive equations). Amélie Rambaud has started a Ph. D. thesis on this particular problem in September 2008.

b) Sedimentation and erosion: multiphase modeling.

- *Position of the problem.* We plan to focus on the special problem of the exchange of mass that occurs e.g. between two layers of snow with an avalanche of powder snow and a snow mantle composed of dense snow. In fact, in such cases the dense snow mantle is eroded at the front of the avalanche, which raises the height of the avalanche. Meanwhile, sedimentation occurs at the tail of the avalanche with deposits of snow on the ground and mass loss. When the mass loss is large enough, the propagation of the avalanche is stopped. One also finds this kind of behavior with the sediment transport within rivers and this can dramatically change the river bed. It is then important to take into account the mass exchange in the equations, when considering multilayer models. Even if the notions of erosion and sedimentation may be intuitively captured, a physical description remains a difficult task (and could be the subject of a full project). However, there exist now phenomenological descriptions like BCRE or BRDG models (see [48], [49], [50] for more details) and shallow water models (with erosion) of Savage Hutter type: the fluid is supposed to satisfy a Mohr Coulomb law. They were derived in [51] assuming smallness of bottom curvature.

- *Purposes.* In order to take into account the transportation of sediments and erosion in shallow water models, we will follow the methodolgy developed by Bouchut and co-workers in [51]. Instead of working with Euler equations, we will work with Navier-Stokes equations (with possibly several rheologies): the first step is to write the equations in a reference frame attached to the bottom, namely the interface between fluid and sediment and then to derive shallow water equations in that setting. We will follow the calculations performed in the Newtonian case for *arbitrary* topographies [23]. In order to close the equations, we need an equation for the evolution of the bottom surface: we will employ here the classical models found in the literature (BCRE, BRDG models). The aim of this study is to extend the study in [51] to more general fluids (Newtonian and Non Newtonian flows) and for arbitrary bottom topography (no restriction on curvature) and to determine precisely the new terms describing the influence of sedimentation and erosion. From the numerical point of view, we plan to the work with E. Fernandez-Nieto who already developed numerical schemes dealing with such models [51]. We will also analyse the mathematical properties of these models, giving a particular attention to the influence of sedimentation/erosion on the stability of steady solutions and possible formation of nonlinear waves.

Task 3 will be supervised by F. Filbet (numerical aspects); who is a specialist of finite volume methods and hyperbolic systems and P. Noble (theoretical aspects): more precisely a Ph. D. thesis on the relation between multilayer flows and primitive equations with free surface and an arbitrary depth has started in september 2008 (Ph. D. student: Amélie Rambaud) with P. Noble and F. Filbet as advisors. J. -P. Vila will also work on the modeling aspects of multilayer flows. We plan also to collaborate with the groups of Sevilla and Malaga (E. Fernandez-Nieto, C. Pares, M. Castro). These groups use very simple paths for the shocks in non conservative bi-layer models and we want to compare their approach with ours.

3.3.4 Task 4: Mathematical Analysis of Hydrodynamic instabilities

a) Existence of travelling waves.

- *Position of the problem.* Many nonlinear wave propagation phenomena at the free surface of a fluid are observed like tidal waves in the river Gironde, roll-waves in river and channels. It is a difficult task to construct solutions of Navier-Stokes equations which possess these behaviours. Therefore simplified models are usually considered. As an exemple, in the problem of water waves, one usually uses the Korteweg deVries equations to describe the propagation of a solitary wave at the free surface. In the situations under consideration, a similar approach would lead to single equation models like Burgers equations or Benney type equation, which do not admit such waves as solutions: in fact Benney equations only predict the correct behaviour of small amplitude solutions in the stable regime. In the unstable regime, it is important to use the shallow water model to obtain roll-waves [14,41] and other travelling waves. The main task here will be to understand the conditions of formation of these particular waves as solutions of the shallow water equations since they are usually undesirable: think of high pressures on protection devices or in fish passes, floodin rivers or artificial channels. It is important to eliminate or at least control the formation of such waves. We plan to study the formation of nonlinear waves in shallow water flows of complex fluids with the models that we will develop. It is also important to understand the main properties of these waves, since they can enable us to validate the numerical schemes for shallow water models. We will focus on two types of travelling waves that are commonly observed in nature: solitary waves (such as tidal wave in the river Gironde) and periodic travelling waves (like roll-waves in channels). This type of solutions is known to exist for a particular class of Shallow Water equations since the seminal work of Dressler [14]: typically, there is a family of periodic solutions parametrized by the discharge rate and the period, which converges to a solitary wave as the period goes to infinity. These waves are *discontinuous;* they admit Lax shocks. Considering a viscous perturbation of the shallow water equations analyzed by Dressler [14], we have proved that an analogous scenario takes place with *continuous* roll-waves and solitary waves that are *close* to

Dressler roll-waves for vanishing viscosity.

- *Aims and methods.* We wish to complete the analysis by considering the shallow water models obtained within this project and possibly to try to understand the existence of *wavelength selection*. We will focus on the models for complex fluids such as *Bingham fluids, Power-law fluids,* fluids with *capillarity and multi-layer fluids.* We plan to separate the study into two parts: (1) roll-waves in inviscid shallow water models and (2) roll-waves in viscous or capillar shallow water equations. Concerning inviscid equations and in particular Bingham fluids, there is very few existence results for periodic travelling waves: let us mention the work of G.M. Maciel and J.-P. Vila [19] on the existence of roll-waves in a restricted range of wavelength for a particular Shallow Water model of Bingham fluid. We plan to extend these results to the Shallow water models for complex fluids that will be obtained within this project: we will also investigate this problem numerically with the numerical methods developed previously. An important task is to see whether at the numerical level, these waves can be observed. A second important case is bi-layer inviscid shallow water models: here the existence of inviscid roll-waves is not so obvious. In this case, the admissibility conditions for the shock is a system of two equations and four unknows. The unknows are the values of the two heights on either side of the shock. In order to find a periodic solution, we have to construct a fixed point of the composition of two successive transformations (1) the before/after transformation defined above and (2) the evolution in space of the solution. Right now we just don't know how to do that in general. However, for small shocks and conservative hyperbolic systems [26], we proved the existence of these fixed points and therefore of periodic solutions. We would like to remove thie conservativity assumption.This should provide a more complete picture of transition to instability than the one given by Kliakhandler [52] for systems of Benney and Kuramoto-Sivashinky models..
  Of particular interest also is the case of regularized shallow water type equations. The regularization could come from viscosity or capillarity. Concerning viscous models, we would like to know whether the bi-layer and mono-layer scenarios are similar for the onset of periodic modulation at the free surface. Moreover, we will also investigate the emergence of internal waves in the presence of an unstable distribution of density. The case of shallow water equations with capillarity is particularly interesting since, until now this is the only case where we have a rigorous justification. The existence of periodic travelling waves is a classical result for the Euler-Korteweg model, which is a particular version of shallow water equations when gravity and friction are neglected: we will try to prove the existence of periodic travelling waves when these source terms are not neglected. In this case, the analysis is harder since the source terms destroy the underlying Hamiltonian form of the ordinary differential system for travelling waves in the Euler-Korteweg model. We will analyse the problem with the help of dynamical systems techniques: center manifold reduction, local bifurcation theory so as to obtain periodic travelling waves in this case. We will also explore the possiblity of getting large amplitude roll-waves when the capillary terms are small, using the theory of singular perturbations in ordinary differential equations.

b) Stability of roll-waves.

- *Statement of the problem.* The stability analysis of solitary waves and periodic travelling waves has known spectacular developments during these last ten years. Stability is a necessary condition for the observativity of waves. Concerning fronts and solitary waves, the theory has been essentially settled now in the reaction diffusion and in the conservative settings: the spectrum is composed of essential spectrum determined by the limits of solutions at infinity and point spectrum studied with an Evans function. There is a huge literature giving estimates on the linearized evolution semi group, and nonlinear stability has been inferred from the strong spectral stability (see e.g. papers of Métivier, Zumbrun in conservation setting, D. Henry in reaction diffusion setting). The theory is far from being as complete in the case of periodic travelling waves: one of the main difficulty is the fact that the spectrum is essential and hard to compute. Since the seminal work of Gardner and co-workers, which introduces an Evans function to study the spectrum in this setting, much effort has been spent to compute the spectrum in several asymptotic regime: a periodic solution close to a soliton (in the reaction diffusion setting) [31] or long wavelength limit (see [32] in the reaction diffusion setting, [33,35] in the conservative case). In the case of reaction-diffusion equations, one can deduce nonlinear stability from strong spectral stability through the analysis of modulation equations introduced by Whitham [53] (a viscous Burgers equation in this case: see [32] for more details). The situation is less clear for conservation laws: Oh and Zumbrun obtained pointwise estimates on the linear semi-group assuming strong spectral stability [35]. However, these estimates are not sufficient to conclude nonlinear stability. On the other hand, Serre [33] established a relation between the well-posedness of modulation equations and the spectral stability of periodic travelling waves. The case of roll-waves for viscous shallow water equations is of particular interest since it is at the intersection of the two fields: source terms and physical viscosity. We obtained spectral stability under long wavelength perturbations [25] and derived pointwise estimates on the semi-group: one of the purpose of this study is to establish stability results in this case. We will explore several asymptotics: in particular the dynamic of slow modulations, following the ideas of Serre [33] and Doelman and co-workers [32]. Of particular interest also is the vanishing viscosity limit: viscous roll-waves converge to inviscid roll-waves. We recently obtained a full description of the spectrum in this case: the main issue is to formulate a stability problem in the presence of an infinite number of shocks [21]. From this analysis, we deduced the persistence of inviscid roll-waves [27]. The aim of this project is also to deduce information on the stability of viscous roll-waves in the vanishing viscosity limit.

Nonlinear stability of roll-waves and slow modulations. Following the strategy introduced in the reaction-diffusion, we analyse directly the question of nonlinear stability of roll-waves through the derivation of *modulations equations*: following the homogeneization strategy introduced by Whitham [53] and Serre [33], one can use a formal asymptotic expansion to prove that these perturbations satisfy, up to leading order, a system of conservation laws. In the case of roll-waves, we expect that this system is hyperbolic and thus well-posed. It is a consequence of [33] that relates the hyperbolicity of the slow modulation system to the spectral stability of periodic travelling waves in the conservation laws setting and [21] where spectral stability of viscous roll-waves under long wavelength perturbations is proved. We will try to construct an approximate solution of the shallow water model, starting from a smooth solution of the slow modulation systems so that this approximate solution will be close to a roll-wave on asymptotically large time. This method has been applied successfully in the reaction-diffusion setting and mimics a centre manifold reduction [32]. We plan also to analyse this problem numerically by direct computations on the Shallow Water system.

- *Vanishing Viscosity.* We will analyse the full nonlinear problem of roll-waves stability in the vanishing viscosity limit. In this case, we proved that continuous roll-waves are close to Dressler roll-waves. We also proved that these discontinuous periodic waves are stable in the following sense: starting from initial data close to a roll-wave and in particular possessing an infinite distribution of shocks, the Cauchy problem is well posed on a sufficiently small time interval [21,27]. We will show that these generalised roll-waves have a continuous counterpart in the vanishing viscosity limit. Then we will obtain a nonlinear stability result of roll-waves. We will adapt the methods of matched asymptotics used by F. Rousset in the case of large amplitude shocks [54].

Task 4 is supervised by P. Noble and J.-P. Vila: both have a strong experience in the analysis of roll-waves, both from an analytical point of view (P. Noble) and numerical point of view (J.-P. Vila). Valérie Le Blanc has staerted a Ph. D. Thesis on the nonlinear stability of viscous and inviscid roll-waves. L.M Rodrigues will also work on stability aspects of the problem since some techniques are quite reminiscent with the one he employed to prove nonlinear stability of Oseen vortices in 2d Navier-Stokes equations. We will also collaborate with S. Benzoni (ICJ) on the mathematical aspects involving capillarity.

4. Stratégie de valorisation des résultats et mode de protection et d’exploitation des résultats / Data management, data sharing, intellectual property and results exploitation

The main purpose of this project is to validate mathematically, numerically (and experimentally for particular situations) the shallow water equations in some well-controlled situations which are elementary components of real flow phenomena. We will contact physicists in CEN (Centre for the study of snow), CEMAGREF (a research institute on engineering in agriculture and environment) in order to prepare the exploitation of our results in realistic situations: we plan to integrate our models in numerical codes that are used to design protection devices and determine the main characteristics (velocity, pressure, extension) of natural hazards such as dense snow avalanches or mud floods, debris flows. Observed that this is a project in mathematics and transfering our knowledge to people performing industrial applications could be in itself a project submitted to ANR in the future, we will first publish our results in high quality journals in direction of applied mathematicians (SIAM J. Math Anal, SIAM J. Scien. Comp., JCP, Comm Part. Diff Eq, Journ Math Pur App, Indiana Univ. Math. Journ, ARMA,...) and physicists (Journal of Fluid Mechanics, Journal of Rheology,..). We also plan to organize regularly workshops gathering specialists in areas of applied mathematics, fluid mechanics (theory and experimentations). The participants to this project are all members of research groups that gather applied mathematicians in the Region Rhone Alpe (JERA) and France (GdR MOAD: research network in partial differential equations): this ensures a real diffusion of our results. We will design a web page that will present our main results, the evolution of the project and the organisation of workshops and conferences.

5. Organisation du projet / Consortium organisation and description

5.1. Description, adéquation et complémentarité des participants / Relevance and complementarity of the partners within the consortium

- *Scientific environment in Lyon.* The *Institut Camille Jordan* of the University of Lyon is a laboratory that gathers more than a hundred mathematicians of different specialities. Within this institute, the group MMCS (Mathematical Modeling and Numerical Analysis) gathers almost all specialists of numerical analysis, applied P.D.E.'s and scientific computations in Lyon that all we join recently (P. Noble, L. Chupin: 2005, F. Filbet: 2006, D. Le Roux, L.M. Rodrigues, S. Delcourte: 2008).
- If accepted, this project would promote, inside MMCS, the **emergence of a research group focused on the dynamic of shallow fluid flows** by the combination of our respective skills in the domains of asymptotical and mathematical analysis (Noble, Chupin, Vila, Rodrigues), numerical simulations (Vila, Le Roux, Delcourte, Filbet) and fluid mechanics and with the recruitment of Ph.D. students and post-doc students that will be involved in this project. Moreover, the presence of S. Benzoni in the team MMCS enforces the skills of the group in the domain of transition of phase, capillar fluids and hyperbolic systems.
  The analysis of complex fluid flows is also a domain that is constantly developed in the other universities of the Region Rhone Alpes inside the applied mathematical laboratory of Grenoble (LJK) and Chambery (LAMA). In Lyon, a seminar dedicated to the *Mechanic of compressible fluids* is organised regularly by D. Bresch, P. Mironescu and C. Villani at the Ecole Normale Supérieure. We have frequent contacts with the applied mathematicians of all these teams especially through the organization, every year, of a workshop JERA (namely « P.D.E. in Rhone Alpes region »), the 2007 edition having been held in Lyon. At the national level, we are all members of the research group « GdR MOAD » (Modeling, Asymptotics and nonlinear Dynamics), managed by S. Benzoni that brings together in particular applied mathematicians in P.D.E., numerical analysis and fluid mechanics (Paris, Bordeaux, Marseille,Toulouse, Lille, Nice, Rennes).

- *Scientific environment in Toulouse.* J.-P. Vila is a specialist of the mathematical and numerical analysis of hyperbolic systems and in particular shallow water equations and has many interactions with industry (he is consultant at ONERA: the French Aerospace Lab) and with physicists specialized in hydrology (contact ONERA: P. Villedieu, contact Fluid Mechanics Institute of Toulouse: O. Thual). Marc Boutounet has started in 2006 a Ph.D. thesis supervised by J.-P. Vila and he is working on shallow water equations over complex topographies with applications to industry and multilayer models.

- *International collaborations.* We are working with a group of applied mathematicians from the Universities of Sevilla and Malaga (Spain), who are interested in numerical simulations of shallow water equations with possible applications to the analysis of submarine avalanches and tsunamis. The researchers that are involved are Enrique Fernandez Nieto (junior researcher) and professors C. Pares and M. Castro. They specialize in the numerical simulations of shallow water equations and their presence will be helpful for the analysis of the problems of multilayer models, sediment transport and dry fronts. On the numerical side of this project, let us also mention that F. Filbet works in collaboration with Chi-Wang

Shu (Brown University, Providence, USA) on finite volume schemes for hyperbolic equations (WENO and well balanced schemes). Concerning the experimental validation of the models, J.-P. Vila works with a laboratory of hydrology in Brazil: the contact is G. F. Maciel (Sao Paulo) who proposed to conduct some experiments on muds (mixture of water and clay). We have also a contact at the EPFL (Polytechnic School of Lausanne), with C. Ancey for experiments in situ for snow avalanches.

5.2. Qualification du porteur du projet / Qualification of the principal investigator

My main research domain is fluid mechanics with a particular focus on shallow water equations. My interest for this kind of models is twofold: analysis of nonlinear waves and mathematical derivation of models. During my Ph.D. thesis, I analysed the hydrodynamical instabilities of these equations, which are called roll-waves. The main issue was the formulation of a stability problem in the presence of an infinite number of shocks: this was done through an original formulation of the equations where the possible perturbations have their shocks fixed. This framework was particularly fruitful since I obtained spectral stability and persistence results for an interesting class of perturbations [21],[27]. Furthermore, this framework provides a general method for obtaining roll-waves solutions in different situations: small amplitude pulsating roll-waves in a fluid flowing down a periodically modulated bottom, small amplitude roll-waves in general hyperbolic equations. After getting a position at the University of Lyon in 2005, I extended my investigation to viscous roll-waves, generalizing the estimates obtained by Oh-Zumbrun [34,35]: in the case of shallow water equations, I had to deal with physical viscosity. This analysis of nonlinear waves naturally lead me to consider also the question of the physical relevance of such equations and the question of their derivation from Navier-Stokes equations. In 2006, I started a collaboration with J.-P. Vila, L. Chupin and E.D. Fernandez Nieto, in view of a formal derivation new Shallow Water equations for different situations of physical interest, using the methodology introduced by J.-P. Vila [36]: Newtonian fluids down arbitrary topographies and Bingham/Power-Law fluids down inclined planes. Meanwhile and in collaboration with D. Bresch, we proved that the mathematical introduced by Vila is rigorous and thus obtained the first justification of shallow water equations from Navier-Stokes equations. Since Vila's methodology is rigorous and efficient, we would like to apply it to a number of core problems forming the basis of the present proposal.

Let me also mention that I have also worked on applications of dynamical systems theory in finite dimension (singular perturbations methods, Lyapounov reduction) and infinite dimension (centre manifold reduction) to the analysis of localized oscillations in discrete (finite or infinite) Hamiltonian systems (FPU chains, spins chains, molecules). This kind of techniques will be particularly useful for the analysis of nonlinear waves, especially the existence of periodic travelling waves (see e.g. the existence of viscous roll-waves close to inviscid roll-waves using singular pertubations theory in O.D.E. [22] and existence of inviscid roll-waves in general hyperbolic systems [26]).

Concerning my abilities to coordinate this project, I participated to the organization of different conferences and workshop (HYP 2006, JERA 2007, GdR MOAD 2008, SIAM Roma 2008). I am also the organiser of the seminar of the group « Mathematical Modeling and Scientific Computing » in Lyon. I am now familiar with the administrative and management aspects of research groups and scientific meetings. Moreover, I have also supervised students writing their master's thesis and I advise two Ph.D. students who work in the present project (A. Rambaud & V. Le Blanc). Finally, I have proposed and started several collaborations with a number of participants of the project, in particular with L. Chupin, J.-P. Vila on non Newtonian fluids: this provides a natural basis to develop the project.

I have published 15 research papers in well know journals (Archive for Rationale Mechnics and Analysis, Communications in Partial Differential Equations, SIAM Journal of Applied Math, SIAM Journal on Mathematical Analysis, Annales of IHP (C): Nonlinear analysis, Nonlinearity, Journal of Nonlinear Science, Physica D) and I have given 16 oral presentations in internation conferences (6) and seminar of applied mathematics (10) .

5.3. Qualification, rôle et implication des participants / Contribution and qualification of each project participant

Nom	Prénom	Emploi actuel	Unité de rattachement et Lieu	Personne . mois	Rôle/Responsabilité dans le projet 4 lignes max
NOBLE	*Pascal*	*Maitre de Conférence s*	Institut Camille Jordan, UMR CNRS 5208, Lyon	36	*Coordination des taches 1 (justification rigoureuse des équations de Saint Venant) et 4 (ondes non linéaires)*
CHUPIN	Laurent	Maitre de Conférences	Institut Camille Jordan, UMR CNRS 5208, Lyon	16	*Coordinateur tache 2: équations Shallow Water pour les fluides non newtoniens*
VILA	Jean-Paul	Professeur	Institut de Mathématique de Toulouse, INSA	24	*Coordinateur tache 2: fluides non newtoniens. Tache 3: modeles bi couches*
FILBET	Francis	Professeur	Institut Camille Jordan, UMR 5208, Lyon	16	*Coordinateur tache 3: modeles Shallow Water multi couches. Aspects Numériques*
LE ROUX	Daniel	Professeur	Institut Camille Jordan, UMR 5208, Lyon	24	*Coordinateur tache 1: aspects numériques de la comparaison Navier-Stokes/ Shallow Water*
RODRIGUES	Luis Miguel	Maitre de Conférences	Institut Camille Jordan, UMR 5208, Lyon	16	*Tache 1: obtention rigoureuse des équations type Shallow Water à partir de Navier-Stokes.*
DELCOURTE	Sarah	Maitre de Conférences	Institut Camille Jordan, UMR CNRS 5208, Lyon	16	*Tache 1: aspects numériques de la comparaison des équations Navier-Stokes/Shallow Water*

6. Justification scientifique des moyens demandés / Scientific justification of requested budget

" *Équipement / Equipment*

As described in the scientific part, the aim of the present project is to develop new numerical alogrithms for the approximation of oceanographic or avalanche models. The computational cost for the numerical simulations of fluid mechanic models is very heavy (2d/3d Navier-Stokes equations with free boundary or Multi-Layer shallow water models). Of course, before performing numerical simulations on parallel super computers, it is necessary to perform preliminary numerical tests on simpler situations on Personal Computers, which will be used in the first steps of algorithm validation. Therefore, we would like to buy 7 or 8 Personal Computers (2 per years) allowing the development of numerical codes (around 5000 euros per years).

" *Personnel / Staff*

In order to achieve the objectives presented in this project, it would be important to obtain a post doc position of 1 year in order to work specifically on the numerical aspects of that project: in particular the comparison between free-surface Navier-Stokes equations and Shallow Water equations. The post doc student will be advised by D. Y. Le Roux and S. Delcourte and would benefit of a particularly stimulating scientific environment since we have also contact with physicists of the CEMAGREF (Grenoble), ONERA (Toulouse) and the CEN (EPFL Lausanne) and the post doc student would benefit of an interdisciplinar formation.

" *Missions / Missions*

Most of the expenses will be dedicated to the invitation of foreign researchers (C.W. Shu, E.D. Fernandez Nieto, C. Pares,...) and local researchers (D. Bresch, F. Bouchut, E. Audusse, J.F. Gerbeau,J.-P. Vila...) at the Institut Camille Jordan for workshop sessions in order to exchange techniques and tools in fluid dynamics, numerical analysis. This will also pay missions in the different laboratories associated to that project through different collaborations (with D. Bresch, Chambery, E.D. Fernandez-Nieto, Sevilla, missions Toulouse/Lyon,...) and expenses for national and international conferences dedicated to Partial Differential Equations, Fluid Mechanics,...to ensure a large diffusion of our results.

" Autres dépenses de fonctionnement (in french)

Nous demandons pour chaque année le montant maximum (10.000 euros) au titre de la dispense d'enseignement de 96h/an: en effet, ceci permettra de dégager en partie du temps pour chaque membre investi dans le projet et coordonnant une tache du projet et pour l'encadrement des étudiants en thèse et post doc. Ces dispenses permettront également de faciliter le départ pour des missions à l'étranger et pour des conférences. Enfin, une partie de ces heures seront réservées au jeunes maitres de conférences qui viennent de démarrer leur carrière (Rodrigues, Delcourte) pour leu r permettre de s'investir au mieux dans le projet que nous proposons.

7. Annexes

7.1. Références bibliographiques / References

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[9] L. Chupin, *Existence result for a mixture of non Newtonian flows with stress diffusion using the Cahn-Hilliard formulation.* Discrete and Continuous Dynamical Systems - serie B, vol. 3, 1 pp. 45-68 (2003)

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[12] L. Chupin, L. I. Palade, *Generalised thin film Newtonian flow behavior near a sharp edge*. submitted

[13] T. Colin, D. Lannes, *Justification of and long-wave correction to Davey-Stewartson systems from quadratic hyperbolic systems. Discrete and Continuous Dynamical Systems,* 11 (2004), no. 1, 83-100.

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[Existence and stability of roll-waves for the Saint-Venant equations] C. R. Math. Acad. Sci. Paris 338 (2004), no. 10, 819—824.
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7.2. Biographies / CV, Resume

***Coordinator of the project**: Pascal NOBLE*

***Members**: Laurent CHUPIN*

Sarah DELCOURTE

Francis FILBET

Daniel LE ROUX

Luis Miguel RODRIGUES

Jean-Paul VILA

***Ph D Students:** Marc BOUTOUNET (ONERA, Toulouse)*

Amélie RAMBAUD (ICJ, Lyon)

Valérie LE BLANC (ICJ, Lyon)

- **Coordinator of the project:** Pascal NOBLE, 32 years old (M)

  web: http://math.univ-lyon1.fr/~noble

- Research interests:

  Nonlinear Hyperbolic Systems with source terms: Sahallow Water equations, analysis of hydrodynamic instabilities like roll-waves.

  Modeling and mathematical analysis of shallow water flows: applications to snow avalanches and mud floods.

  Applied dynamical systems: mathematical and numerical analysis of localized oscillations in finite and infinite systems of oscillators

  Free boundary problem and application to combustion: radiative transfer/combustion models: existence of waves and flame balls.

- Responsabilities: in charge of seminar of the team MMCS, organization of conference of conferences and workshop (HYP 2006, JERA 2007, GdR MOAD, Lyon 2008).

- Cursus: since 2005: Assistant Professor in Applied Mathematics (Univ Lyon)

  sept 2004-février 2005: Post doc at the Vrije Universiteit (Amsterdam)

  sept 2000-dec 2003: Ph-D student at the University of Toulouse: *Invariants manifolds methods for Shallow Water equations and infinite dimensional Hamiltonian systems.* Supervisors: J.-M. Roquejoffre, J.-P. Vila.

- Publications:

  [1] *On the spectral stability of roll-waves*, Indiana. Univ. Math. J. 55 (2006) no. 2, p. 795-848.

  [2] *Existence of pulsating roll-waves for the Saint Venant equations*, Arch. Rat. Mech. Anal. 186 (2007) no. 1, p. 53-76.

  [3] *Linear Stability of viscous roll-waves*, Comm. Par. Diff. Eq. 32 (2007) no. 11, p. 1681-1713.

  [4] *Shallow water flows for arbitrary topography*, in coll. with M. Boutounet, L. Chupin & J.-P. Vila, Comm. Math. Sci. 6 (2008) no. 1, p. 29-55.

  [5] *Mathematical Justification of a shallow water model*, in coll. with D. Bresch, accepted for

publication in Methods and Applications of Analysis 14 (2007) no. 2, p. 87-117.

Member 1: Laurent Chupin, 32 years old (M)

web: http://popocatepelt2.insa-lyon.fr/~chupin/

- Research interests:

  Mathematical modeling in fluide mechanics: diphasic viscoelastic fluids, thin viscoelastic fluids, thin rough domains, Hele Shaw flows for Non Newtonian fluids.

  Mathematical analysis of PDE from fluids mechanics: Navier Stokes equations, Olroyd fluids, Cahn Hilliard equations for diphasic flows, Reynolds equations for viscoelastic fluids

  Numerical analysis and simulations: viscoelastic fluids, diphasic fluids.

- Cursus: since sept 2004: Assistant Professor in Applied Mathematics (INSA Lyon)

  2003- 2004: ATER at the University of Bordeaux

  sept 2000-dec 2003: Ph-D student at the University of Bordeaux: *Mixture of viscoelastic fluids.* Supervisors: P. Fabrie.

- Publications:

  [1] *Some theoretical results concerning diphasic viscoelastic flows of the Oldroyd kind,* Advances in Differential equations, vol.9, 9-10 pp. 1039-1078 (2004)

  [2] *Numerical study of viscoelastic mixtures through a Cahn-Hilliard flow model***,** European Journal of Mechanics - B Fluids, vol. 23, 5 pp. 759-780 (2004), *in collaboration with P. Fabrie and F. Boyer*

  [3] *Viscoelastic fluids in a thin domain,* Quaterly of Applied Mathematics, vol. LXV 4 (2007) p. 625-651, in coll. with S. Martin & G. Bayada.

  [4] *Fast approximate solution of Bloch equation for simulation of RF artifacts in Magnetic Resonance Imaging*, accepted for Mathematical and computer Modelling, in coll. with S. Balac

  [5] *On a coupled free boundary for a piezoviscous fluid in thin film***,** Differential and Integral Equations, Vol. 21, N° 1-2 January/February 2008, in coll. with G. Bayada & B. Grec.

Member 2: Daniel Le Roux (M),

Research interests: Influence of numerical schemes on singular PDE

Analysis of finite-element, finite volume and discontinuous Galerkin schemes

Semi-Lagrangian methods on unstructured meshes

Numerical simulation of free surface flows (shallow-water and Navier-Stokes systems) and

environmental applications.

Cursus:

Since september 2008: Associate Professor in Applied Mathematics (Univ.Lyon)

June 2005-september 2008: Associate Professor, Department of Mathematics and statistic, Laval University, Québec, Canada

January 2001-may 2005 : Assistant Professor, Department of Mathematics and statistic, Laval University, Québec, Canada

June 2000-december 2000: Invited Researcher at the Institute for Computational

Engineering and Sciences (ICES), The University of Texas at Austin, Texas, USA

February 1998 - May 2000: Postdoctoral fellow, Advanced Study Program (ASP), National Center for Atmospheric Research (NCAR), Boulder, Colorado, USA

December 1977: Ph.D. Thesis, McGill University, Montréal, Canada. « A semi-Lagrangian finite-element barotropic ocean model. »

Publications:

[1] Toumbou B., Le Roux D.Y., Sene A.,``A shallow-water sedimentation model with friction and Coriolis: An existence theorem",Journal of Differential Equations, 2008, 244, pp.~2020--2040.
[2] Le Roux D.Y., Rostand V., Pouliot B.,``Analysis of numerically-induced oscillations in 2D finite-element shallow-water models, Part I: Inertia-gravity waves", SIAM J. Sci. Comput., 2007, 29, pp.~331--360.
[3] Mohammadian A., Le Roux D.Y.,``Simulation of shallow flows over variable topography using unstructured grids", Int. J. Numer. Methods Fluids, 2006, 52, pp.~473--498.
[4] Hanert E., Le Roux D.Y.., Legat V., Deleersnijder E.,``An efficient Eulerian finite-element method for the shallow-water equations", Ocean Modelling, 2005, 10, pp.~115--136.
[5] Le Roux D.Y., ``A new triangular finite element with optimum constraint ratio for compressible fluids",SIAM J. Sci. Comput., 2001, 23, pp.~66--80.

Total number of publications in peer reviewed international journals: 35

Proceedings of congress (peer reviewed) : 7

Member 3: Francis Filbet, 32 years old (M)

web: http://math.univ-lyon1.fr/~filbet

- Research interests:

  Numerical simulations of charged particle systems

  Spectral Methods for Boltzmann Equations

  Analysis of Finite Volume Schemes

  Mathematical modeling of Chemotaxis

- Cursus: since oct 2006: Associate Professor in Applied Mathematics (Univ. Lyon, ICJ)

  2005: HdR thesis, coordinator: P. Degond

  2004-2006: Chargé de Recherche CNRS (Univ. Toulouse, MIP)

  2002-2004: Chargé de Recherche CNRS (Univ Orléans, MAPMO)

  2001 PhD Thesis of the University Henri Poincaré. *Contribution à l'analyse mathématique et la simulation numérique de l'équation de Vlasov*. Advisors: S. Benachour, E. Sonnendrucker

Publications

[1] *Derivation of Hyperbolic Models for Chemosensitive Movement,* J. Math. Biol. 50 (2005), p. 189-207, with ph. Laurencot and B. Perthame.

[2] *Approximation oh Hyperbolic Models for Chemosensitive Movement,* SIAM J. Sci. Comput. 27 (2005), p. 850-872 with C.-W. Shu

[3] *A Finite Volume Scheme for the Patlak Keller Segel chemotaxis model,* Numerische Mathematik 104 (2006) p. 457-488.

[4] Asymptotic behaviour of a finite volume scheme for the transient drift-diffusion model IMA J. Num. Anal. 27 (2007) p. 689-716.

[5] *Convergence of a finite volume scheme for the coagulation fragmentation equations,* Math. Comp. 851-882 (2008), with J.-P. Bourgade.

–

Member 4: Jean-Paul Vila, (M)

Research interest: shallow water equations, hyperbolic systems, numerical analysis and SPH methods

Position: Associate professor at the Institut de Mathématiques de Toulouse (INSA)

Consultant at ONERA.

Publications:

[1] SPH renormalized hybrid methods for conservation laws: applications to free surface flows. *Meshfree methods for partial differential equations II,* 207—229, Lect. Notes Comput Sci Eng 43.

[2] Convergence des méthodes particulaires renormalisées pour les systèmes de Friedrichs. C.R. Math. Acad. Sci. 340 (2005) no 6. 465-470. with N. Lanson

[3] Convergence of an explicit finite volume scheme for first order symmetric systems, Numer Math. 94 (2003) no 3. 573-602 with P. Villedieu.

[4] Shape optimal design problem with convective and radiative heat transfer: analysis and implementation, J. Optim Theory Appl. 110 (2001), no 1, 75-117, with D. Chenais, J. Monnier.

[5] Meshless methods for conservation laws. Nonlinear waves: computation and theory, Math. Comput. Simulation 55 (2001), 493-501, with N. Lanson.

Member 5: Luis Miguel Rodrigues (M), 28 years old

Research interests:

Asymptotic analysis of PDE, fluid mechanics. Long-time asymptotic behaviour, convergence to vortices.

Isentropic compressible, inhomogeneous incompressible or constant-density Newtonian flows.

Cursus:

since sept 2008: Assistant Professor in Applied Mathematics (Univ Lyon, ICJ)

sept 2004-dec 2007: Ph-D student at the University of Grenoble,
Long-time asymptotic behavior of bidimensional viscous flows,
advisor: Th. Gallay.

Publications:

[1] *Asymptotic stability of Oseen vortices for a density-dependent incompressible viscous fluid,* to be published in Annales de l'Institut Henri Poincaré (C) - Analyse non linéaire(2008).

[2] *Sur le temps de vie de la turbulence bidimensionnelle,* with Th. Gallay, accepted for publication in Annales de la Faculté des Sciences de Toulouse (2008).

[3] *Vortex-like finite-energy asymptotic profiles for isentropic compressible flows*, accepted for publication in Indiana University Mathematics Journal (2008).

Member 6 Sarah Delcourte (F) 27 years old,

Research Interests:
Finite Volume and Discontinuous Galerkin methods
Elastodynamics in velocity-stress formulation
Div-Curl problem and Helmholtz Decomposition
Fluid Dynamics equations
Unstructured and non-conforming meshes
Preconditioning and numerical Eigenvalues
Adaptative meshes near singularities

Cursus:

since october 2008: Assistant Professor in Applied Mathematics, University Claude Bernard (LYON)

2007-2008: Post doc position at INRIA Nice (NACHOS project) under the supervision of N. Glinsky-Olivier

2004-2007: Ph D Thesis « Développement de méthodes de volumes finis adaptatives en mécanique des fluides under the supervision of K. Domelevo and P. Omnes (University Toulouse III)

Publications:

[1] S. Delcourte, K. Domelevo and P. Omnes, "*A discrete duality finite volume approach to Hodge decomposition and div-curl problems on almost arbitrary two-dimensional meshes*", SIAM Journal on Numerical Analysis, 45 (3), 1142-1174, 2007.

[2] S. Delcourte and P. Omnes, "*A finite volume method for the Stokes problem on two-dimensional arbitrary grids*", in preparation.

[3] S. Delcourte, "*A discrete duality finite volume method for elliptic problems with corner singularities*", submitted.

[4] S. Delcourte, K. Domelevo and P. Omnes, "*Discrete duality finite volume method for second order elliptic problems*", Proceedings of Finite Volumes for Complex Applications IV (F. Benkhaldoun, D. Ouazar and S. Raghay eds., Hermes Science publishing), pp. 447-458, 2005.

[5] S. Delcourte and D. Jennequin, "Saddle point preconditioners for linearized Navier-Stokes equations discretized by a finite volume method", submitted. **CANUM 2008 - Prix poster**

7.3. Implication des personnes dans d'autres contrats / Involvement of project particpants to other grants, contracts, etc...

Nom de la personne participant au projet	Personne. mois	Intitulé de l'appel à projets Source de financement Montant attribué	Titre du projet	Nom du coordinateur	Date début & Date fin
FILBET	9	ANR SYSCOMM 2008	MANIPHYC	BOCQUET	2009-2012
CHUPIN	9	ANR SYSCOMM 2008	MANIPHYC	BOCQUET	2009-2012
CHUPIN	9	ANR Jeunes Chercheurs 2008	RUGO	GERARD-VARET	2009-2012
VILA	9	ANR Programme Blanc 2008	APAM	BENDALI	2009-2012