

# Elliptic Problems in Smooth and Non Smooth Domains

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## SUMMARY

We are interested here in questions related to the **regularity** of solutions of **elliptic** problems with **Dirichlet** or **Neumann** boundary condition (see ([1])). For the last 20 years, lots of work has been concerned with questions when  $\Omega$  is a **Lipschitz domain**.

We give here some complements for the case of the **Laplacian** (see [3]), the **Bilaplacian** ([2],[6]) and the operator  $\operatorname{div}(A\nabla)$  (see ([5])), when  $\mathbf{A}$  is a matrix or a function, and we extend this study to obtain other regularity results for domains having an adequate regularity.

Using the duality method, we will then revisit the work of Lions-Magenes [4], concerning the so-called **very weak solutions**, when the data are less regular. Thanks to the **interpolation theory**, it permits us to extend the classes of solutions and then to obtain new results of regularity.

**Keywords:** Elliptic problems, Lipschitz domains, regularity

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