State-dependent sweeping processes and evolution problems

Manuel D. P. Monteiro Marques *

November 13, 2017

Abstract

The basic state-dependent sweeping process in a Hilbert space may be written in short as

$$-\frac{du}{dt}(t) \in N_{C(t,u(t))}(u(t)),$$

where $u : I = [0,T] \to H$ is an absolutely continuous function, $C(t,u)$ are subsets of H and $N_{C(t,u)}(x)$ denotes the outward normal cone to $C(t,u)$ at x. The r.h.s. may also contain standard $f = f(t,u)$ terms.

Basic results for this problem will be presented, along the lines of the papers [1]-[4]. In their simplest form [1], the sets $C(t,u)$ are closed and convex and the dependence $(t,u) \mapsto C(t,u)$ is Lipschitz-continuous w.r.t. Hausdorff distance h

$$h(C(t,u), C(s,v)) \leq L_1 |t-s| + L_2 |u-v|_H,$$

with $L_2 < 1$. An example of application is given in [2]. It is also possible to work with prox-regular sets [3] and in ordered Hilbert spaces [4].

More generally, the previous study is extended by replacing the normal cone to $C(t,u)$ by a maximal monotone operator $A(t,u)$. Consider the quasi-variational problem: find $u : I \to H$ such that

$$-\frac{du}{dt}(t) \in A(t,u(t))(u(t))$$

so that, for all $v \in D(A(t,u(t)))$ and $z \in A(t,u(t))v$, one has

$$\langle \frac{du}{dt}(t) + z, v - u(t) \rangle \geq 0.$$

If the variation of the m.m.o. is measured by Vladimirov's pseudo-distance, one obtains a basic existence result as above.

This is part of joint program of work with D. Azzam-Laouir (Jijel) and C. Castaing (Montpellier).

*CMAF-CIO, Departamento de Matemática, Faculdade de Ciências da Universidade de Lisboa, Campo Grande, 1749-016 Lisboa, Portugal; mdmarques@fc.ul.pt

REFERENCES

[1] Kunze, M, Monteiro Marques, MDP, On parabolic quasi-variational inequalities and state-dependent sweeping processes, *Top. Methods Nonlinear Anal.* 12 (1998) 179-191.

[2] Kunze, M, Monteiro Marques, MDP, A note on Lipschitz continuous solutions of a parabolic quasi-variational inequality, in *Nonlinear evolution equations and their applications (Macau, 1998)*, 109-115, World Sci. Publ., 1999.

[3] Chemetov, N, Monteiro Marques, MDP, Non-convex quasi-variational sweeping processes, *Set-Valued Analysis* 15 (2007) 209-221.

[4] Chemetov, N, Monteiro Marques, MDP and Stefanelli, U, Ordered non-convex quasi-variational sweeping processes, *J Convex Analysis* 15 (2008) 201-214.