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Des spécificités de la modélisation en mécanique du contact

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Abstract of the contents

- **Friction and contact problems relate to non-smooth mechanics**

- **Consequences on the formulations**

- **Regularization … an artefact !?**

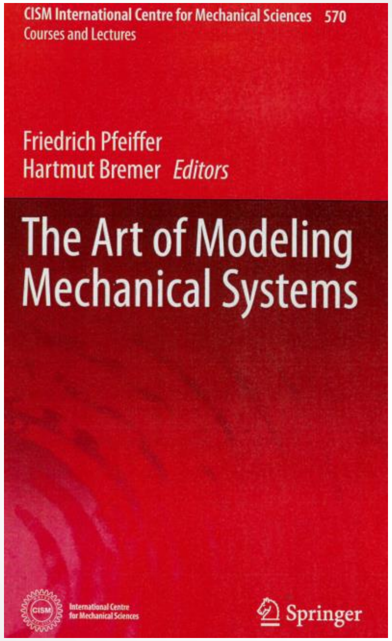
- **Overview of the main mathematical results**

- **Consequences on the numerical methods**

- **Extension to adhesion**

Reference :

*[1] M. RAOUS, **The art of modelling in contact mechanics,** in "The art of modelling mechanical systems", F. Pfeiffer – H. Bremer,(eds.), CISM Courses and Lectures, n°570, Springer Verlag, Wien-New York, 2017, 203-276.*



1 – Basic models and formulations

Friction and contact problems relate to <u>non-smooth mechanics</u>

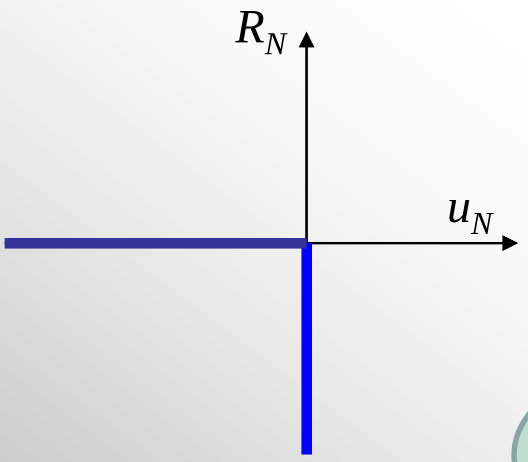
- Unilateral conditions – non penetration

$$u = u_N\,\boldsymbol{n} + \boldsymbol{u_T} \qquad R = R_N\,\boldsymbol{n} + \boldsymbol{R_T}$$

Normal-tangential decomposition

Complementarity problem (Signorini)

$$u_N \leq 0\,,$$
$$R_N \leq 0$$
$$u_N R_N = 0$$



Not a function

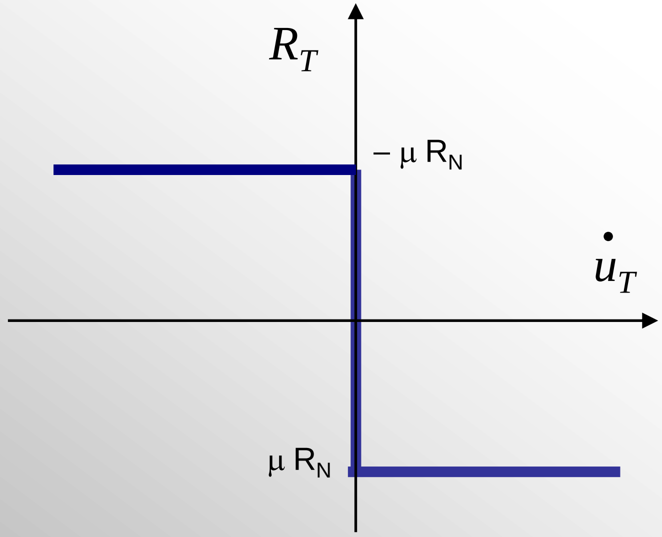
A multivalued application !

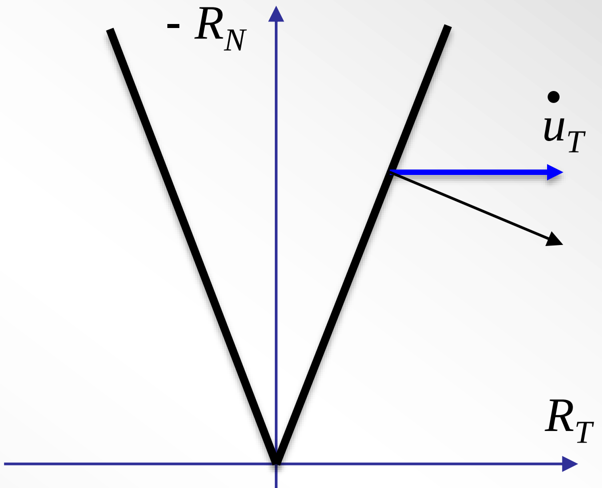
- Friction : Coulomb friction - Armonton

$\| R_T \| \leq \mu \; | R_N |$

If $\| R_T \| < \mu \; | R_N |$ then $\dot{u}_T = 0$

If $\| R_T \| = \mu \; | R_N |$ then $\boxed{\dot{u}_T = - \lambda R_T}$





Cône de Coulomb

Not a function
A multivalued application !

Non associate law !...
No normality rule

Consequence
on the thermodynamics formulation

The potentials (free energy and potential of dissipation) include non differentiable terms and indicator functions of sets characterizing the inequality contraints

The state laws and the complementary laws have to be written in term of differential inclusions

See [1]

Consequence
on the variational formulation

Elasticity problem

Variational equation

⇨ *u in U* *such that* *a (u, v) – L(v)* = 0 for any *v* in *U*

Elasticity problem with unilateral contact and friction

Implicit variational inequality

⇨ $$a(u, v-u) + j(u,v) - j(u,u) \geq L(v-u) \qquad \forall v \in K$$

Variational form of the unilateral problem with Coulomb friction

An implicit variational inequality

(or a quasi-variational inequality on the dual form)

Let K be the convex of the admissible displacements :

$$\boxed{K = \{v \in U / \; v_N \leq 0 \text{ on } \Gamma_C\}} \quad \text{with} \quad U = \left\{v \in \left[H^1(\Omega)\right]^2 / \; v = 0 \text{ on } \Gamma_D\right\}$$

Problem $(\boldsymbol{P_v})$: let Φ_1, Φ_2 be given as previously defined in $(\boldsymbol{P_c})$,

find u $\in$ K such that :

$$a(u, v-u) + J_1(u,v) - J_1(u,u) \geq L(v-u) \qquad \forall v \in K \qquad (2)$$

with :

$$\begin{aligned} a(u,v) &= \int_\Omega \sigma(u)\varepsilon(u)\, dx && (3)\\ &= \int_\Omega K_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(v)\, dx \qquad \forall u,v \in U && (4) \end{aligned}$$

$$L(v) = \int_\Omega \Phi_1 v\, dx + \int_{\Gamma_F} \Phi_2 v\, ds \qquad \forall v \in U \qquad (5)$$

$$\boxed{J_1(v,w) = \int_{\Gamma_C} \mu\, |R_N(v)| \; ||w_T||\, ds} \qquad (6)$$

Ref :

- Duvaut G., Lions J. L., 1972, ***Les inéquations en mécanique et en physique****, Dunod, Paris*

- P.D. Panagiotopoulos, ***Inequality problems in Mecahnics, convex and non Convex energy functions****, Birkhäuser Verlag, Boston Basel (1985) and Hemivariational inequalities*

Static and quasi-static problem

The static problem has no sense for frictional problem but it will be helpfull for solving the **quasi-static problem**

Problem (P_{temps}) : Find u(t) $\in$ K and F_N such that :

$$\begin{cases} a(u(t), v - \dot{u}(t)) + j(u(t), v) - j(u(t), \dot{u}(t)) \geq L(v - \dot{u}(t)) \\ \qquad + < R_N\,(u(t)), v_N - \dot{u}_N(t)) > \qquad \forall v \in V \\ \\ < R_N\,(u(t)), z_N - u_N(t)) > \;\geq 0 \qquad \forall z \in K \end{cases}$$

Coupling of two variational inequalities

Reference:

M. Cocou, E. Pratt, M. Raous, ***Formulation and approximation of quasistatic frictional contact****, International Journal for Engineering Sciences, 34, n°7, 783-798, 1996*.

Dynamics

Non derivability versus time (shocks)

The classical motion equation has no anymore sense

$$M\ddot{U} + C\dot{U} + KU = F$$

(here the discrete form)

Consequence of discontinuity of the velocities

The equation of motion has to be written in term of **differential measure**

Problem P_h: Find U such that $\forall t \in [0,T]$ $U(t) \in V_h$, $U(0) = U_0$, $\dot{U}(0) = V_0$ and :

$$M.d\dot{U} + K.U + C.\dot{U} = F + Rd\nu \tag{79}$$

and for the contact nodes:

$$U_N(t) \leq 0 \; R_N(t) \leq 0 \text{ and } R_N(t)U_N(t) = 0$$

$$\|R_T(t)\| \leq \mu|R_N(t)| \text{ and } \begin{cases} \text{if } \|R_T(t)\| < \mu|R_N(t)| & \dot{U}_T = 0 \\ \text{if } \|R_T(t)\| = \mu|R_N(t)| & \exists\lambda > 0 \,\text{t.q.}\, \dot{U}_T = -\lambda R_T(t) \end{cases}$$

where $d\dot{U}$ is a differential mesure representing the discretized acceleration and $d\nu$ is a nonnegative real mesure relative to which $d\dot{U}$ happens to possess a density function.

Reference (many other works):
M. Jean, J.-J. Moreau, 1987, ***Dynamics in the presence of unilateral contact and dry friction: a numerical approach****, in « Unilateral problem in structural analysis II, CISM Lectures collection, 304, Del Piero& Maceri (Eds), Springer Verlag Wien.*

About numerical methods for dynamics problems

Newmark is not convenient because of no derivability of the solution

Let's have a look on the numerical method NSCD

Non Smooth Contact Dynamics method (M. Jean – J.-J. Moreau)

The system on differential measure can be written in the following form $\forall t \in [0, T]$

$$M(\dot{U}(t) - \dot{U}(0)) = \int_0^t (F - K.U - C.\dot{U})ds + \int_{[0,t]} R d\nu$$

$$U(t) = U(0) + \int_0^t \dot{U} ds$$

Time discretization: $i = 0...N$, $t_i = i.h$ (h is the time step)

$$M(\dot{U}(t_{i+1}) - \dot{U}(t_i)) = \int_{t_i}^{t_{i+1}} (F - K.U - C.\dot{U})ds + \int_{[t_i,t_{i+1}]} R d\nu$$

$$\bar{R}^{i+1} = \frac{1}{h}\int_{[t_i,t_{i+1}]} R d\nu.$$

Combination of θ-Methods

We have used the following three methods:

• θ-Method : both integrals are approximated by the classical θ-method

$$\int_{t_i}^{t_i+1} f ds \approx \; h(\theta f(t_{i+1}) + (1-\theta) f(t_i))$$

• θ-Euler-Method: the first integral is approximated by the θ-method and the second one by the Euler implicit method,

• modified θ-Method: both integrals are approximated by the θ-method but in the contact relations the displacement $u(t_{i+1})$ is replaced by

$$\hat{u}(t_{i+1}) = u(t_{i+1}) + h(1-\theta)\dot{u}(t_{i+1}).$$

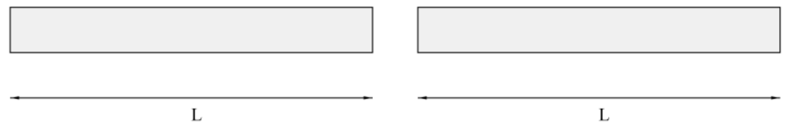


Figure 8. Impact between two bars.

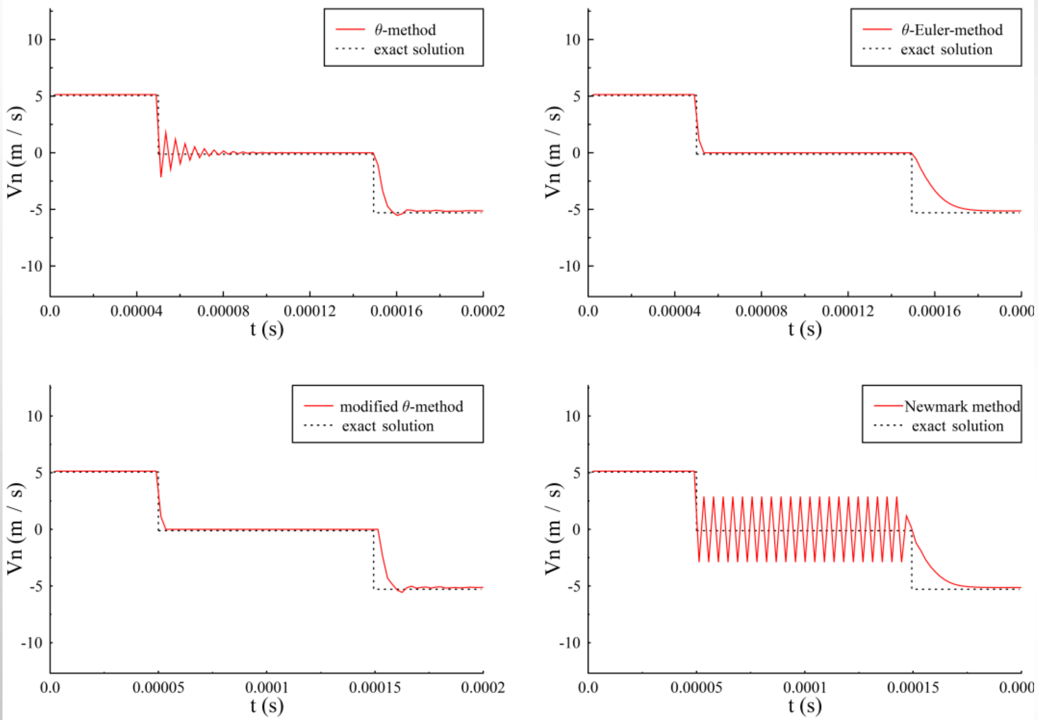


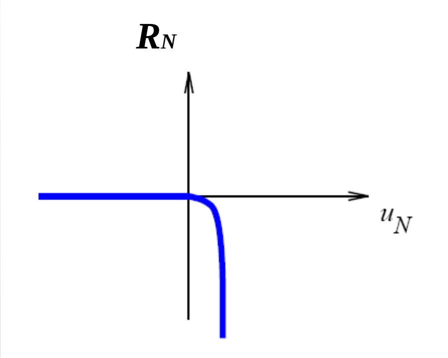
Figure 9. Normal velocity at the center of the contact zone.

2 – Regularization : *a comfort or an artefact ??*

Replace the multivalued applications by functions in order to get a classical non linear (but smooth!) problem.

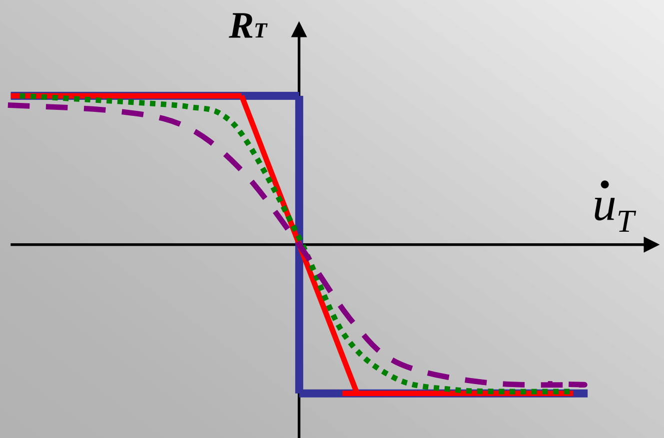
But … the regularized problem is <u>another problem</u> (very different of the initial one)

- compliance or penalization for the contact



But penetration into the obstacle !.....

- tanh (·······), sqrt (– – -), polynom (——) for friction



But always sliding !.....

We have to stress that when using regularization
we get other models ! The behaviors are different !

We have replaced the multivalued application by functions …
- very comfortable mathematical aspects
- very comfortable for computations (regularization is often used in computer codes)

- **unilateral contact** : with normal compliance such that : $-R_N = C_n(u_N)_+^{m_n}$

"squeeze of the asperities" … any doubt !
But when one computes the squeeze of an asperity in large plastic deformations (see reference below), it turns out that the coefficient C_n and m_n are huge and do not correspond to the convenient values for the computations.

When compliance is used in the computational code, it has to be checked if the penetration is convenient for the problem under consideration.

Reference :
*M. Raous, M. Sage, **Numerical simulation of the behavior of surface asperities for metal forming**, in "Numerical Methods in Industrial Forming Processes, Chenot-Wood & Zienkiewicz (eds), 1992, Balkema, pp. 175-180.*

- **Friction** : when the friction law is regularized, that means that the solid is always moving … (except if the tangential force is zero !)

With a very small force you can get the refrigerator going across the kitchen if you wait for a sufficient long period of time !…

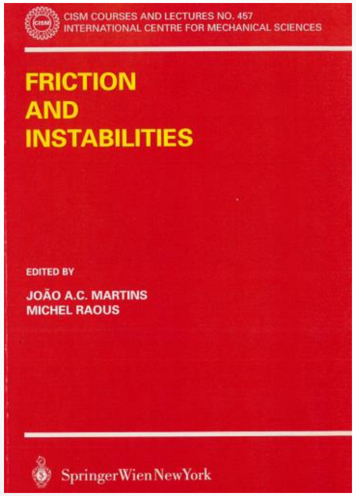
- The choice of the parameters of the regularization has an important influence on the tangential forces …

- Regularization does not fit subtle analysis as the **study of instabilities** or squeal and other ones ... The results of the analysis will depend on the values of the regularization parameters.

  Instabilities can be characterized when using the strict Coulomb friction with a constant friction coefficient because of the non smooth character of the law.

Reference

*J.A.C. Martins, M. Raous (Eds), **Friction and instabilities**, CISM Courses and Lectures, Springer Verlag, n°457, Wien-New York, 2002. (310 pages)*



Conclusion :
Regularization is very comfortable for the mathematics and the computations but it deals with another problem and precautions have to be taken (choice of the parameters, verifications a posteriori, …). It is not always convenient !

Consequence of no normality rule

Non associated law ⇒ No equivalence with the minimization of potential energy

No minimization problem

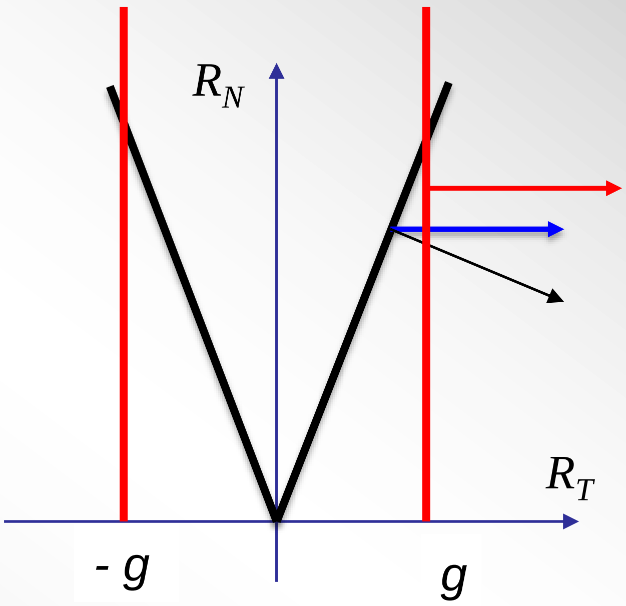
Elasticity problem

Find u in U such that for any v in U	$J(u) \leq J(v)$
with	$J(v) = \frac{1}{2}\, a\,(v, v) - L(v)$

How to get a minimization problem … the Tresca friction

The problem can be set as a Tresca problem included in a fixed point process

$$\left.\begin{array}{l} ||R_N|| \leq g \text{ with} \\ \textit{If } ||R_N|| < g \text{ then } \dot{u}_T = 0 \\ \textit{If } ||R_N|| = g \text{ then } \exists \lambda \geq 0 \text{ such that } \dot{u}_T = - R_T \end{array}\right\}$$



Associate law
(normality rule for the sliding !)

Problem $\boldsymbol{P_{fp}}$: Find the fixed point of the application S :

$$S(g) = -\,\mu\, R_N\,(u_g)$$

with u_g solution of the following problem $P_{varTresca}$:

Problem $\boldsymbol{P_{varTresca}}$: For a given g, find $u_g \in$ K such that :

$$a(u_g, v - u_g) + j(v) - j(u_g) \geq L(v - u_g) \qquad \forall v \in K$$

with: $j(v) = \int_{\Gamma_C} g\|v_t\|\, ds$

which is shown to be equivalent to the following minimisation problem

Problem $(\boldsymbol{P_m})$: For a given g, find $u_g \in K$ such that

$$\boxed{J(u_g) \leq J(v) \qquad \forall\, v \in K}$$

with:

$$\boxed{J(v) = \frac{1}{2} a(v, v) + j_g(v) - L(v)}$$

3 – Outlines of some mathematical results

Mathematical framework

Duality product

Contact Displacement space $U_C = H^{1/2}(\Gamma_C)^3$ $\quad (R, w) \quad$ $\mathscr{F} = H^{-1/2}(\Gamma_C)^3$ *contact force space*

Trace operator

displacement space $U = H^1(\Omega)^3$ $\quad < \phi, v > \quad$ $\Phi = L^2(\Omega)^3 \otimes L^2(\Gamma_f)^3$ *load space*

Deformation operator

$\mathcal{E}$ = *Grad*$_s$ *u*

Equilibrium equations

$div\ \sigma = -\varphi_1$ in Ω

$\sigma . n = \varphi_2$ on Γ_2

Deformation space $E = L^2(\Omega)^9$ $\quad << \sigma, \varepsilon >> \quad$ $S = L^2(\Omega)^9$ *Stress space*

$$<< \sigma, \varepsilon >>= \int_\Omega \sigma(u)\varepsilon(v)\, dx \qquad \forall v \in U$$

$$< \Phi, v >= \int_\Omega \phi_1 v\, dx + \int_{\Gamma_F} \phi_2 v\, ds \qquad \forall v \in U$$

$$(R, w) = \int_{\Gamma_C} \mu |R_n(u)| \|w_t\|\, ds \qquad \forall w \in U_c$$

$H^1(\Omega) = \{ u \in L^2(\Omega)^3 ;\ \partial u / \partial x_i \in L^2(\Omega)^3 \ {}_{i=1,3} \}$

Panorama of the mathematical results

In classical elasticity, existence and uniqueness theorems are based on coercivity and continuity of the operators and the demonstration is based on Lax-Milgram theorem and Korn inequality (equivalence of norms).

The (given) applied forces were chosen to be in $L^2(\Omega)^3 \otimes L^2(\Gamma_f)^3$. In contact mechanics, the first difficulty which arises is that the **contact force which is unknown is a distribution belonging to in $H^{-1/2}(\Gamma)$**. This implies some compactness difficulties and a regularization using a convolution product is used in most of the mathematical results.

New definition for the contact force (introduction of non local friction)

$R^* = R \star \varphi$ where φ is a function with compact support and « very smooth »

R is in $H^{-1/2}(\Omega)$ is a distribution

R^* is in $L^2(\Omega)$ is a function in $L^2(\Omega)$

References:

G. Duvaut, *Equilibre d'un solide élastique avec contact unilatéral et frottemement de Coulomb*, CRAS, Paris, 290A, 263-265, 1980.

Cocou, M., Existence of solutions of Signorini problems with friction, Int. J. Engng. Sci., Vol. 22, N° 5, 1984, pp. 567-575.

Existence and uniqueness of the solutions

a - Static problem

(no mechanical meaning but interesting intermediate problem)

An implicit variational inequality

Problem : Find $u \in K$ such that for any $v \in K$

$$a(u,v-u) + j(v,u) - j(u,u) - (f,v-u) \geq 0$$

where K is the convex of the admissible displacements u_N

Mathematical results for the static problem

- **Signorini problem (no friction)** : existence and uniqueness
  Fichera (1964)

- **Signorini + Coulomb** : existence if μ is small and no uniqueness
  Necas-Jarusek-Haslinger (1980), Jarusek (1983), Eck-Jarusek (1998)

- **Signorini + Coulomb (non local friction)** : existence and uniqueness if μ is small
  Cocou (1984), Duvaut (1972), Demkowicz-Oden (1982)

- **Compliance + Coulomb** : existence and uniqueness if μ is small
  (Klarbring-Mikellic-Shillor (1989)

b – Quasi-static problem

Two coupled variational inequalities (one of them is implicit)

Problem (P_{temps}) : Find u(t) ∈ K such that :

$$\begin{cases} a(u(t), v - \dot{u}(t)) + J_1(u(t), v) - J_1(u(t), \dot{u}(t)) \geq L(v - \dot{u}(t)) \\ \qquad + < \sigma_N(u(t)), v_N - \dot{u}_N(t)) > \qquad \forall v \in V \\ \\ < \sigma_N(u(t)), z_N - u_N(t)) > \; \geq 0 \qquad \forall z \in K \end{cases}$$

with $$J_1(v, w) = \int_{\Gamma_C} \mu \, |R_N(v)| \; ||w_T|| \, ds$$

Mathematical results for the quasi-static problem

- **Signorini + Coulomb** : existence if μ and grad μ are small (condition in L^{∞} and in $H^{-1/2}$) and no uniqueness

  Andersson (2000), Cocou-Rocca (2000, 2001, 2001)

- **Signorini + Coulomb (non local friction)** : existence if μ is small (condition only in L^{∞}) and no uniqueness

  Cocou-Pratt-Raous (1995, 1996)

- **Compliance + Coulomb** : existence if μ is small and no uniqueness (only a few works)

  Andersson (1991), Klarbring-Mickelic-Shillor (1989)

c – Dynamics formulation (J.-J. Moreau)

Differential measures (shocks = discontinuity of the velocities)

Problem P_h: Find U such that $\forall t \in [0,T]$ $U(t) \in V_h$, $U(0) = U_0$, $\dot{U}(0) = V_0$ and :

$$M.d\dot{U} + K.U + C.\dot{U} = F + Rd\nu \tag{79}$$

and for the contact nodes:

$$U_N(t) \leq 0 \; R_N(t) \leq 0 \text{ and } R_N(t)U_N(t) = 0$$

$$\|R_T(t)\| \leq \mu|R_N(t)| \text{ and } \begin{cases} \text{if } \|R_T(t)\| < \mu|R_N(t)| & \dot{U}_T = 0 \\ \text{if } \|R_T(t)\| = \mu|R_N(t)| & \exists\lambda > 0 \,\text{t.q.}\, \dot{U}_T = -\lambda R_T(t) \end{cases}$$

where $d\dot{U}$ is a differential mesure representing the discretized acceleration and $d\nu$ is a nonnegative real mesure relative to which $d\dot{U}$ happens to possess a density function.

Mathematical results for the dynamic problem (quasi nothing !…)

❑ Continous problem

➢ **Frictionless in elasticity**

- **Normal compliance**: existence - Martins-Oden (1987, 1988)
- **Signorini** : a few results on specific geometries (axial symetry) – Munoz-Rivera-Racke (1998)

➢ **Viscoelasticity**

- **Normal & tangential compliance :** existence and uniqueness - Martins-Oden (1987, 1988), Kuttler (1997)
- **Signorini + non local friction**: existence - Cocou(2002), Cocou-Scarella (2006)
- **Signorini + Tresca friction**: existence – Jarusek (1996)

❑ Discrete problem

- Existence and uniqueness for analytical loading in 1D
  - **frictionless** – Ballard (2000)
  - **with friction** – Ballard-Basseville (2005)

and works of Michèle Schatzman

In most of the cases, there is no uniqueness.

Some results of uniqueness have been obtained if μ is small

μ small ? Is it sufficient or also necessary ? What does “small” mean ?

For the simplest case of the static problem, it is possible to construct examples showing the existence of multiple solutions as the famous simple example of Anders Klarbring with a few degrees of freedom

References:
Klarbring A., Examples of non uniqueness and non existence of solutions to quasistatic contact problem with friction, Ingenieur-Archiv, 60, 1990, pp. 529-541.

see also
- *Janovsky V. (1980 and 1981)*
- *Alart P. – Curnier A. (1986)*
- *Mitsopoulos E.N., Doudoumis I.N. (1987)*
- *François Hild*
- *etc …*

3 – Outline of some numerical methods

A brief overview not exhaustive !!!!

At least 4 classes for quasi-static problems:

- Regularization – penalization :	non linear problem, Newton and other
- Lagrangian :	Mixed formulations - Uzawa
- Minimization :	SORP and other
- Mathematical Programming method : (direct method)	Complementarity formulation Lemke

For dynamics problem

- NSCD Non Smooth Contact Dynamics method

a - Regularization – Penalization

As presented before, **a non linear problem** is obtained and classical methods can be used (Newton, etc …)

Special care has to be dedicated to the choice of the penalization parameters and to the control of the solution

b - Lagrangian formulation

Mixed formulation *(u, R)*

both displacement (or velocity) and contact force

Saddle point (min-max problem) ⇨ **Uzawa**

Augmented Lagrangian are very used

c – Sequence of minimization problems

Fixed point method associated to the Tresca problem :

⇒ **minimization under contraints of a non linear and not differentiable functional**

SORP, Aitken, Conjugate Gradient (but a regularization is needed !)

They solve the **initial non smooth problem** !

Very powerfull when multigrid methods are used (good smoothers)

d - Mathematical Programming method (direct method)

Complementariry problem

$\boldsymbol{Problem\ P_{compl}}$: Find $F \in \Re^p,\ u \in \Re^p$ such that

$$\left.\begin{array}{l} Mu = F^* + R \\ R_i \leq 0,\ u_i \leq 0 \qquad i = 1 \ldots p \\ R_i u_i = 0 \qquad i = 1 \ldots p \end{array}\right\}$$

- M and F are respectively a non-symmetric matrix and a loading vector deduced from the FEM problem by **condensation** (including a change of variables for the friction conditions)
- R and u are the contact forces and the contact displacements,
- p is the number of contact degrees of freedom (small !)

Direct methods

- **LEMKE (Mathematical Programming method) – pivoting techniques similar to Simplex Method**
- **Interior points method**

4 – Extension to adhesion

Unilateral contact, friction … and

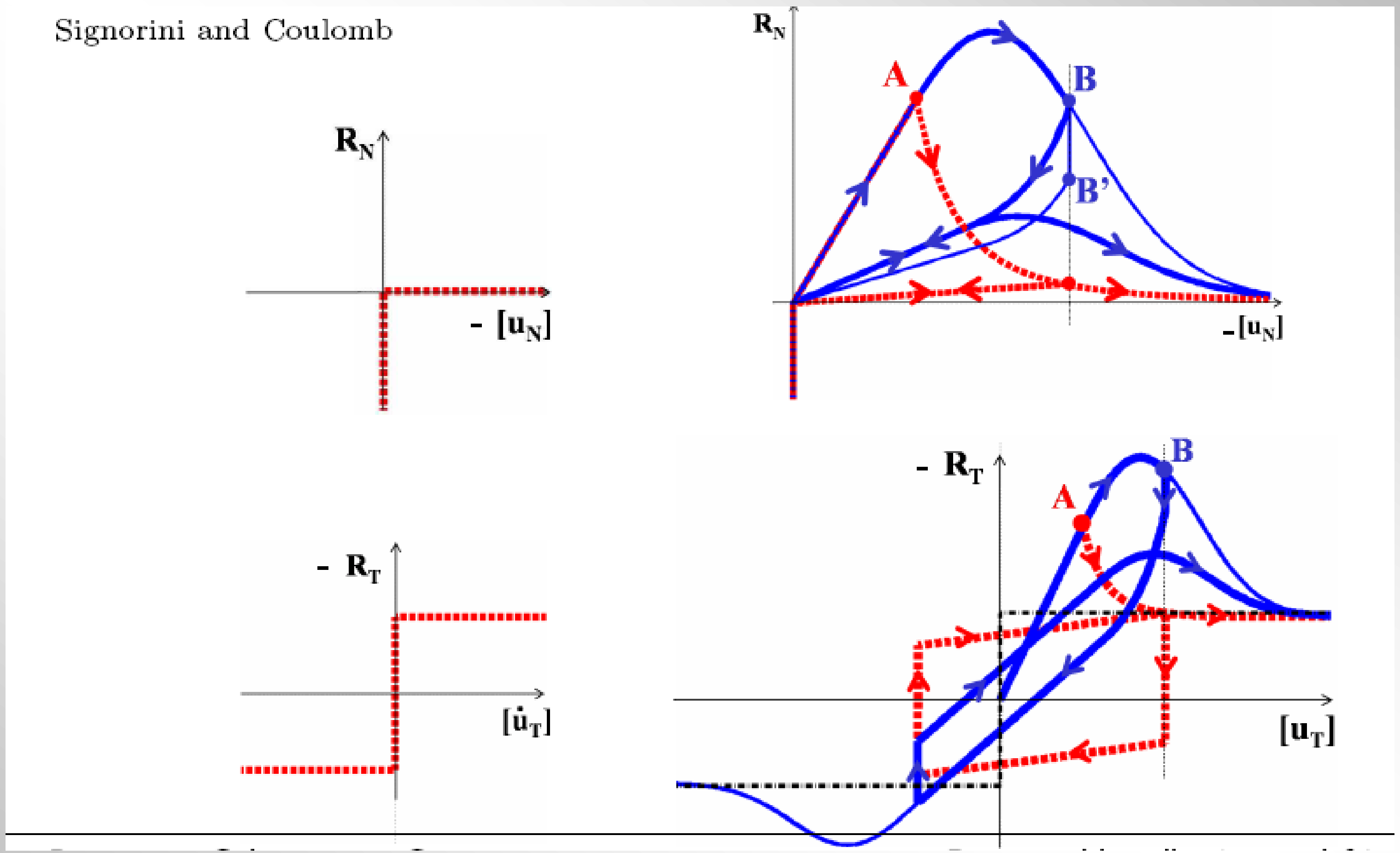
- **normal resistance when traction is applied**

- **tangential resistance before sliding**

- **damage of the interface: adhesion forces disappear when the contact forces are strong enough**

- **eventually viscosity effects (dependence on the loading velocity)**

The RCCM model *(Raous-Cangémi-Cocou-Monerie)*



Reference:

M. Raous, L. Cangemi, M. Cocou, ***A consistent model coupling adhesion, friction and unilateral contact,*** *Computer Methods in Applied Mechanics and Engineering, 177, n°3-4, 1999, pp. 383-399.*

$$[u_N] \leq 0,\ \sigma_N + C_N [u_N] \beta^2 \leq 0,\ (\sigma_N + C_N [u_N] \beta^2) [u_N] = 0$$

Unilateral contact

$$\left| R_t - \beta^2 C_t u_t \right| \leq \mu(1-\beta) \left| R_n^- \right|$$

where

$$\text{if } \left| R_t - \beta^2 C_t u_t \right| < \mu(1-\beta) \left| R_n^- \right| \quad \text{then} \quad \dot{u}_t = 0$$

$$\text{if } \left| R_t - \beta^2 C_t u_t \right| = \mu(1-\beta) \left| R_n^- \right| \quad \text{then } \exists \lambda \geq 0 \quad \dot{u}_t = -\lambda (R_t - \beta^2 C_t u_t)$$

Friction and adhesion

$$\dot{\beta} = -\frac{1}{b}(\omega - \beta(C_n u_n^2 + C_t \left| u_t \right|^2)^-$$

Evolution of the intensity of adhesion

Parameters of the model :

- μ friction coefficient
- C_n, C_t initial stiffness of the interface
- ω adhesion energy
- b viscosity of the interface

Variables:

- u_n, u_t normal and tangential displacements
- R_n, R_t normal and tangential forces
- β adhesion intensity (damage)

Material boundary assumption :

A surfacic energy E and a specific entropy S are associated to Γ_C.

Free energy of Helmoltz : $\Psi = E - ST$ defined onΓ_C.

Choice of the state variables

$Variables$	$[u_N]$	$[u_T]$	β
$Thermodynamic\ forces$	$R_N = R_N^r + R_N^{ir}$	$R_T = R_T^r + R_T^{ir}$	G_β

Reversible parts of the behaviour

Choice of the free energy Ψ

$$\boldsymbol{\Psi}([\mathbf{u_N}],[\mathbf{u_T}],\beta) = \underbrace{\frac{C_N}{2}[u_N]^2\beta^2 + \frac{C_T}{2}[u_T]^2\beta^2 + (1-\beta)\,w}_{\boldsymbol{\Psi}^{\mathbf{d}}} + \underbrace{I_K([u_N]) + I_{[0,1]}(\beta)}_{\boldsymbol{\Psi}^{\mathbf{c}}} \qquad (1)$$

The state laws (differential inclusions)

$$\left\{ \begin{array}{l} \left. \begin{array}{ll} (a) & R_T^r = \dfrac{\partial \Psi^d}{\partial [u_T]} \\ (b) & R_N^r - \dfrac{\partial \Psi^d}{\partial [u_N]} \in \partial I_K([u_N]) \end{array} \right\} \\ (c) \quad -G_\beta - \dfrac{\partial \Psi^d}{\partial \beta} \in \partial I_{[0,1]}(\beta) \end{array} \right\} \quad on\ \Gamma^c$$

Indicator function

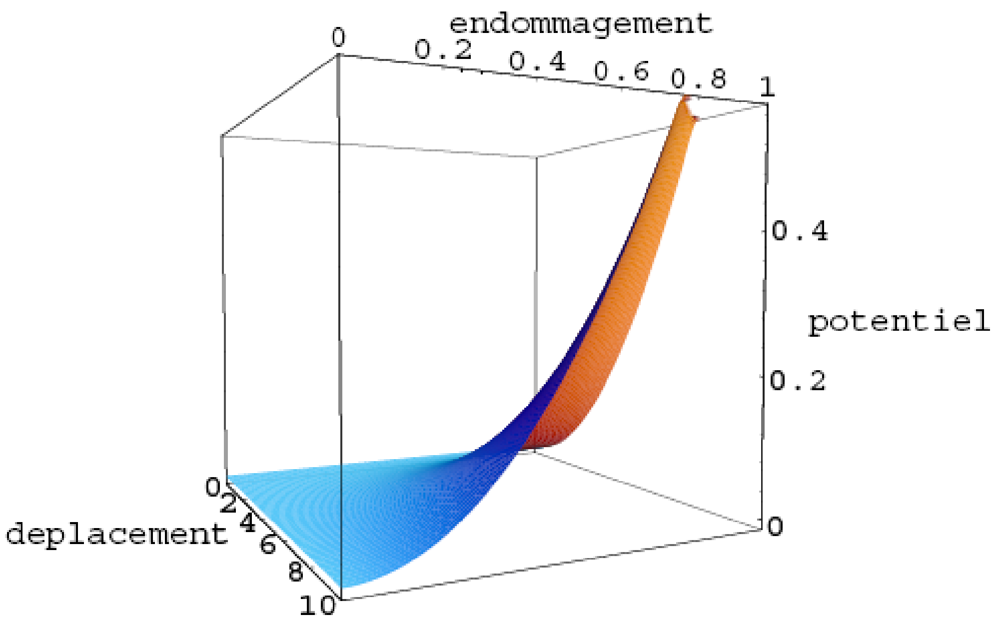
$$I_K(\upsilon_N) = \begin{cases} 0 & \text{if} \quad \upsilon_N \in K \\ +\infty & \text{if} \quad \upsilon_N \notin K \end{cases}$$

$$K = \{v \,/\, v \leq 0\}$$

State variables	Forces and reversible laws
$[u_N]$	$R_N^r - C_N\,[u_N]\beta^2 \geq 0\,, \quad [u_N] \leq 0\,,$ $\left(R_N^r - C_N\,[u_N]\beta^2\right)\,[u_N] = 0\,.$
$[u_T]$	$R_T^r = C_T\,[u_T]\beta^2\,.$
β	$G_\beta \geq w \quad if \quad \beta = 0\,,$ $G_\beta = w - \left(C_N\,[u_N]^2 + C_T\,[u_T]^2\right)\beta \quad if \quad \beta \in]0,1[\,,$ $G_\beta \leq w - \left(C_N\,[u_N]^2 + C_T\,[u_T]^2\right) \quad if \quad \beta = 1\,.$

Ψ^c is convex but non differentiable

Ψ^d is differentiable but not convex versus (u, β)
(but convex versus u and convex versus β)



Irreversible parts of the behaviour

Choice of the potential of dissipation Φ

$$\Phi = \Phi([\dot{u_T}], \dot{\beta}) \text{ such that : } \begin{cases} \Phi \text{ convex relatively to } (v,\gamma) \\ \Phi(v,\gamma) \geq 0 \quad \forall (v,\gamma) \in V \times H \\ \Phi(0,0) = 0\,. \end{cases}$$

$$\Phi\left([\dot{u_T}], \dot{\beta}; \chi_N\right) = (1-\beta)\,\mu\left|R_N - C_N\,[u_N]\beta^2\right|\,||[\dot{u_T}]|| + \frac{b}{p+1}\left|\dot{\beta}\right|^{p+1} + I_{C^-}(\dot{\beta}) \tag{4}$$

The complementarity laws

Irreversibility concerning $[\dot{u_T}]$ et $\dot{\beta}$

$$\sigma = \sigma^r \quad R_N = R_N^r$$

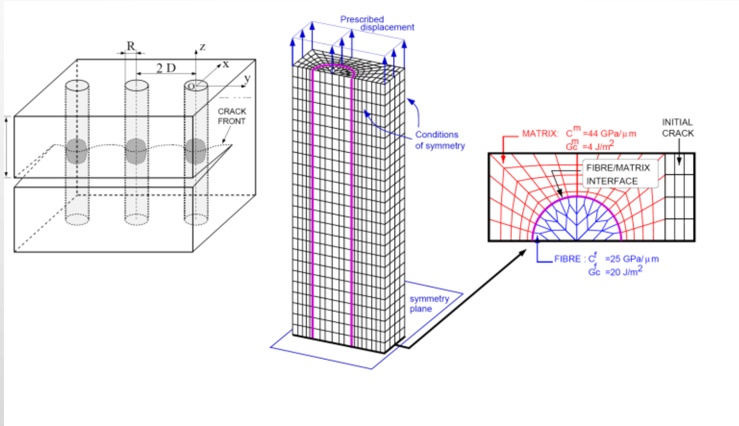
$$(R_T^{ir}, G_\beta) \in \partial\Phi([\dot{u_T}], \dot{\beta})$$

$$\|R_{\mathrm{T}}^{ir}\| \leq g(\chi_{\mathrm{N}}), \quad \text{avec} \quad \begin{cases} \|R_{\mathrm{T}}^{ir}\| < g(\chi_{\mathrm{N}}) \Rightarrow \dot{u}_{\mathrm{T}} = 0, \\ \|R_{\mathrm{T}}^{ir}\| = g(\chi_{\mathrm{N}}) \Rightarrow \exists \lambda \geq 0, \dot{u}_{\mathrm{T}} = \lambda R_{\mathrm{T}}^{ir}\,. \end{cases}$$

$$\widetilde{G_\beta} \in \partial_{\dot{\beta}}\Phi_S \iff b|\dot{\beta}|^p = -\left(\widetilde{G_\beta}\right)^-$$

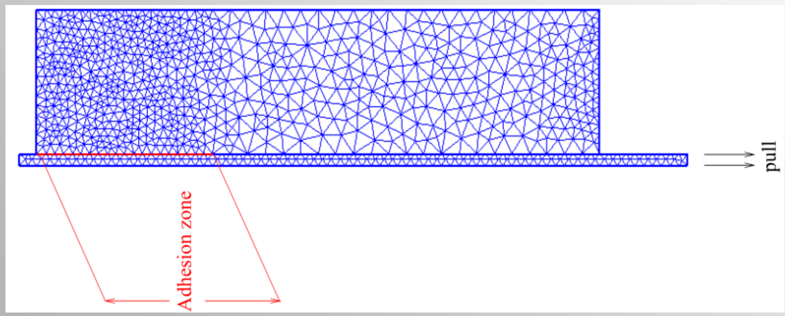
A few examples of application

Influence of adhesion and friction between fiber and matrix on the crack progression in a composite material

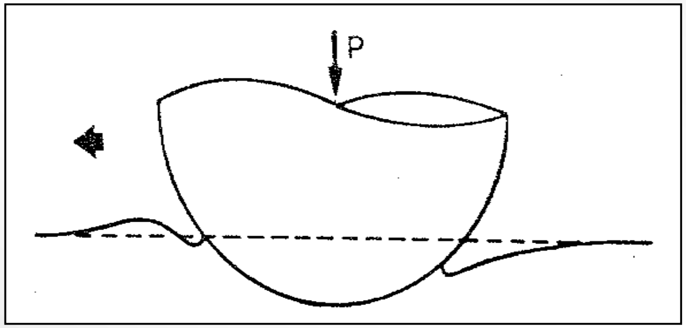


Simulation of the pile-soil interface in a pull out test

Civil Engineering: steel reinforced concrete (pull out test)



Sliding of a glass indentor on a polymer block (recoverable adhesion)



A unified model for adhesive interfaces

(joint work with Gianpietro Del Piero, University of Ferrara)

The idea is to give a general thermomechanical framework from which the various interface laws could be deducted.

The concepts presented in this work are similar of the ones of Generalized Standart Material introduced by Halphen and Nguyen. They are used here for interfaces

References :

G. Del Piero, M. Raous, ***A unified model for adhesive interfaces with damage, viscosity and friction****, European Journal of Mechanics - A/Solids, 29(4), 2010, pp. 496-507.*

B. Halphen, Q.S. Nguyen, ***Sur les matériaux standard généralisés****, Journal de Mécanique, 14, 1975, 39-63*.

About some key ideas

- Contact problems relate **non smooth mechanics** and this **non smooth character is fundamental** (instabilities, etc …)
  - ➢ *Regularization treats another problem*

- **Numerical methods** solving the non smooth problem **do exist**

- **Mathematical analysis** is still **opened** (specially on dynamics problems)

- When **uniqueness** is proved, it is **only for small values of the friction** coefficient …

Just remember that

- ➢ when μ goes to zero we effectively tend to a **frictionless boundary condition,**
- ➢ but when μ goes to infinite we do not tend to clamped boundary conditions but towards **many difficulties !....**

Thank for your attention