Quantifying quantum computational complexity via information scrambling

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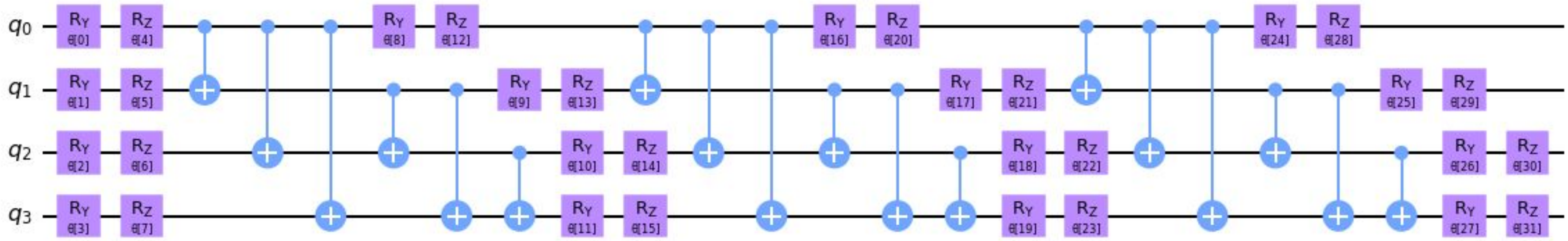
27 June 2022

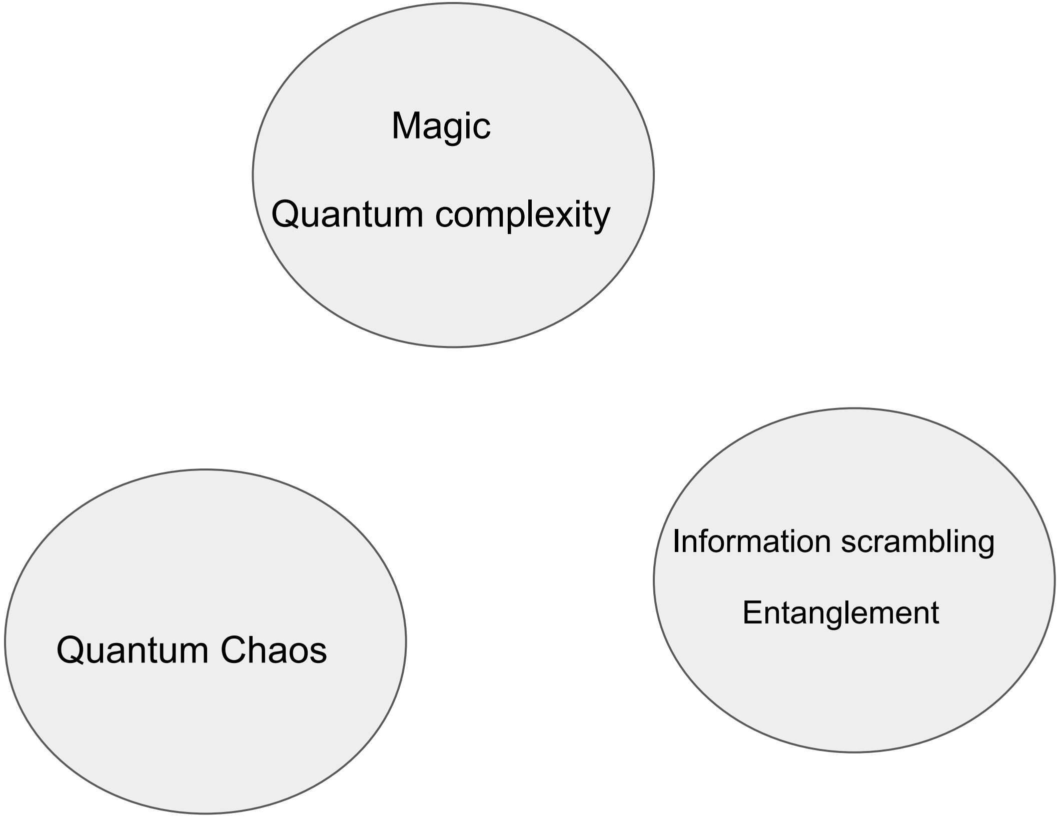
Introduction:

How much QUANTUM this quantum circuit is?

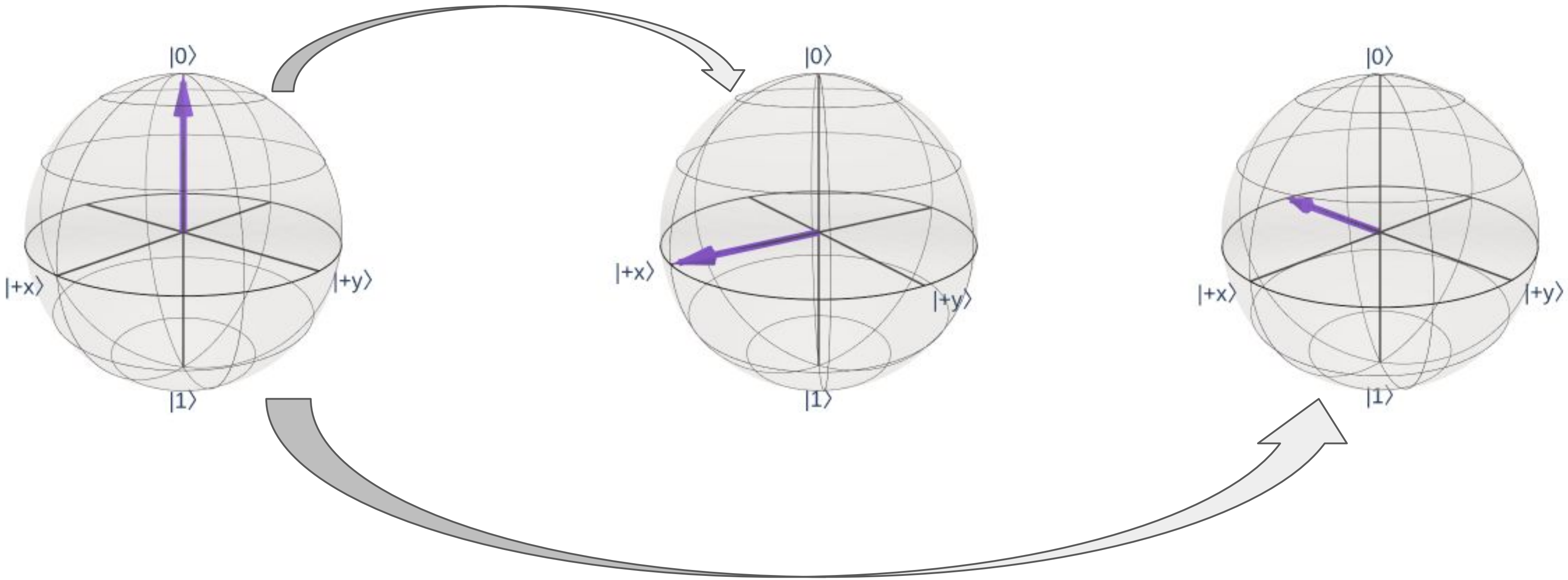


arXiv:2204.11236

Introduction:



Fault-tolerant quantum operations (in theory!)



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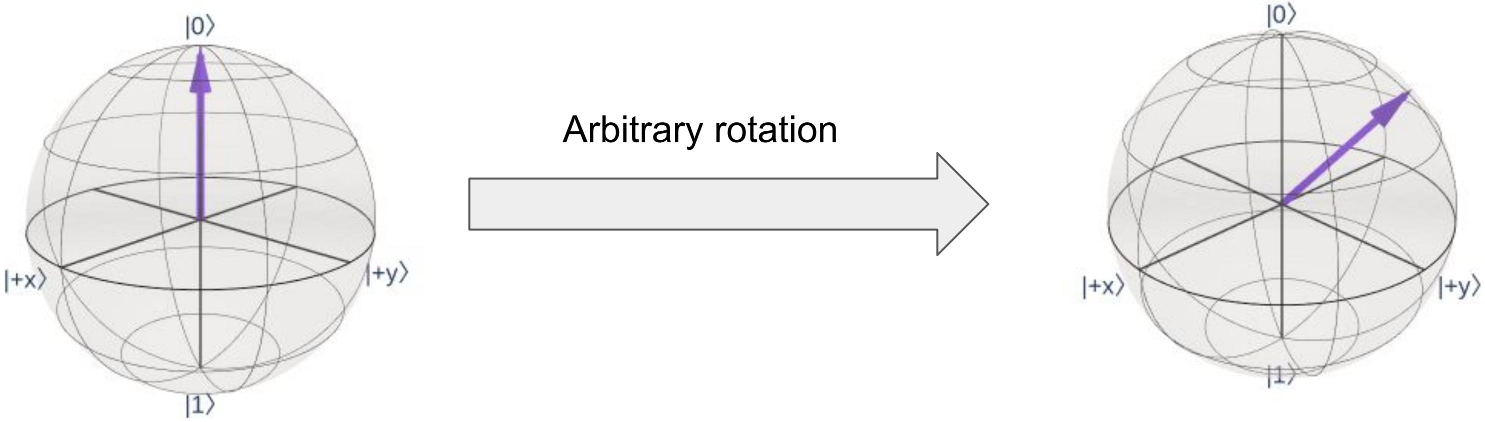
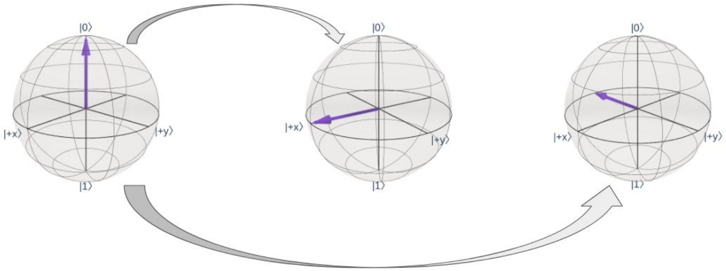
Quantum operations with errors:



Tolerance: $\left| |\psi\rangle - |\psi\rangle_T \right|^2 \leq \epsilon$

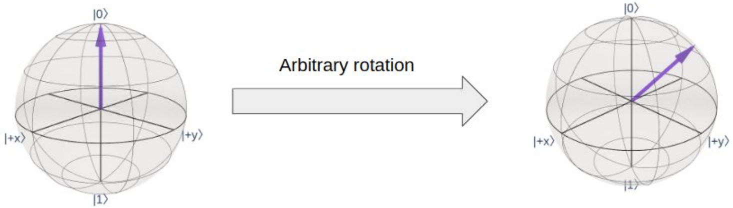
Mathematically speaking:



$$\{H, CNOT, S \,\&\, Pauli\, gates\}$$

Clifford Operations

→ Efficiently simulable on **classical** computers



$$\{R_x(\theta), R_y(\theta), R_z(\theta), \text{T-gate}, \ldots\}$$

non-Clifford Operations

→ Demands **Quantum** computers

Sergey Bravyi and Alexei Kitaev. Universal quantum computation with ideal Clifford gates and noisy ancillas. Phys. Rev. A, 71:022316, Feb 2005.
Daniel Gottesman. The heisenberg representation of quantum computers. 6 1998.

$\{H, CNOT, S \,\&\, Pauli\; gates\}$ ⇒ Stabilizer states

$\{H, CNOT, S \,\&\, Pauli\; gates\}$
$\{R_x(\theta), R_y(\theta), R_z(\theta), \text{T-gate}, \ldots\}$
⇒ Magical
(Higher Quantumness)

All non-stabilizer states are Magical but some of them are **more Magical** than others.

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Quantifying Magic:

By quantifying the magic of target state, we can have an estimate about amount of **resources** we need for such computation.

Magic Monotones:

Map: $\mathscr{H} \longrightarrow \mathbb{R} \longrightarrow$ Non-increasing under Clifford operations

Mana

- Computationally expensive.
- Only definable for odd-prime dim $\mathscr{H}$ spaces.

~~Qubits!~~

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V Veitch, S A H Mousavian, D. Gottesman, and J Emerson. The resource theory of stabilizer quantum computation. New Journal of Physics, 16(1):013009, **2014**

Information Scrambling

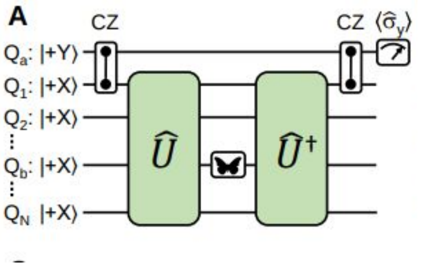
- Information spreading

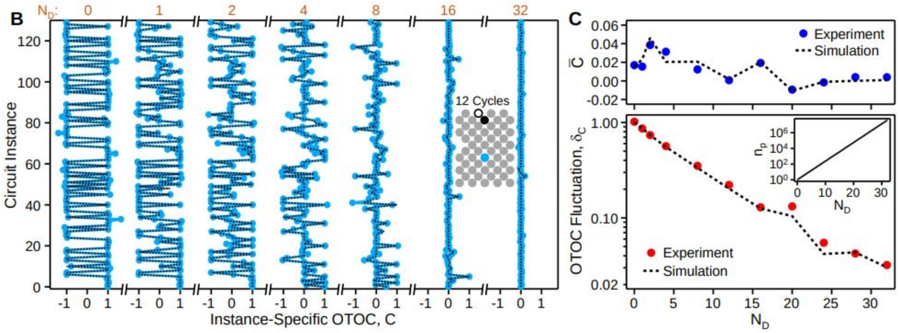
Measured with: Out-of-time ordered correlators (OTOC)

- Entanglement entropy

$$C = \langle \hat{O}^{\dagger}(t)\hat{M}^{\dagger}\hat{O}(t)\hat{M}\rangle$$

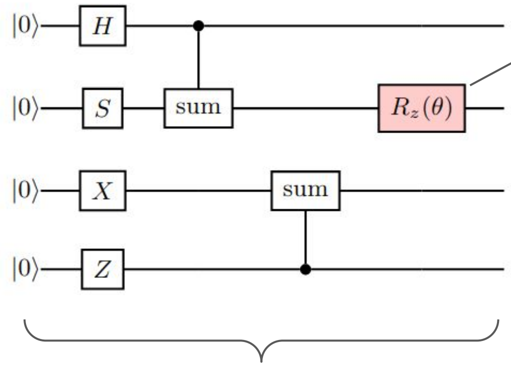
$$where\, \hat{O}(t) = \hat{U}^{\dagger}(t)\hat{B}\hat{U}(t),\ \hat{B} = \hat{\sigma}_x \,\&\, \hat{M} = \hat{\sigma}_z$$





X Mi, P Roushan et al. Information scrambling in quantum circuits. Science, 374(6574):1479–1483, Dec **2021**.

Mana vs. OTOC:



Including the non-Clifford gate in the circuit is probabilistic!

Each cycle of the circuit

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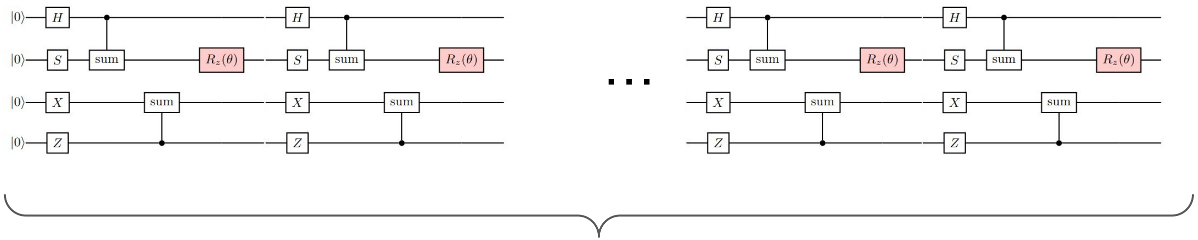
Mana vs. OTOC:



M cycles ⟹ Unitary time evolution $U(t)$

$$OTOC(t) = \Re(\langle X_{N-1}^\dagger(t) Z_1^\dagger X_{N-1}(t) Z_1 \rangle)$$

$$\delta_{OTOC} = std(OTOC(t))$$

Repeat for K instances for each θ

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Mana vs. OTOC:

$|\psi\rangle$

M cycles

Repeat for K instances for each θ

$\mathcal{M}(|\psi\rangle)$ → Mana

$\overline{\mathcal{M}}$

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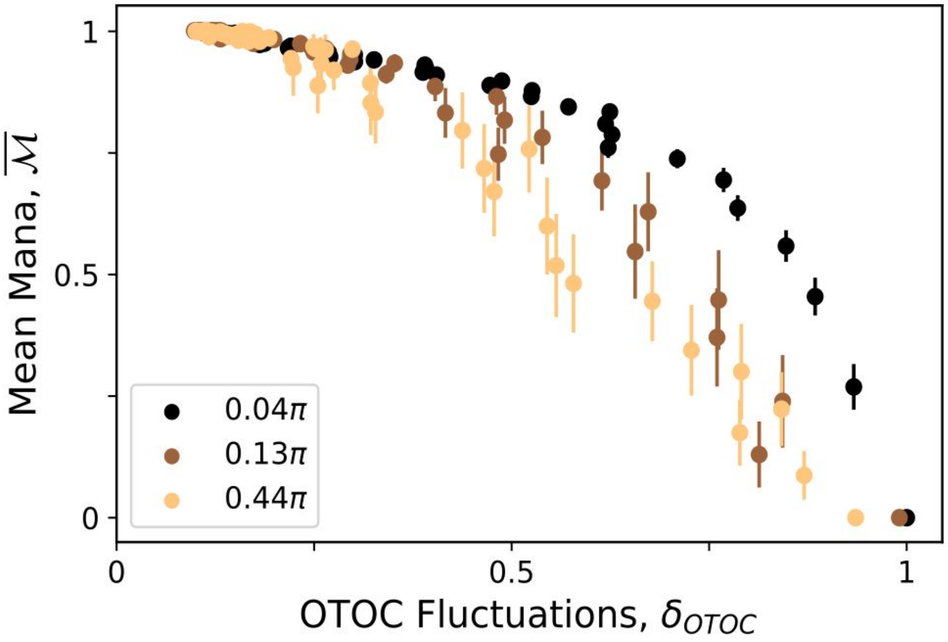
Mana vs. OTOC:



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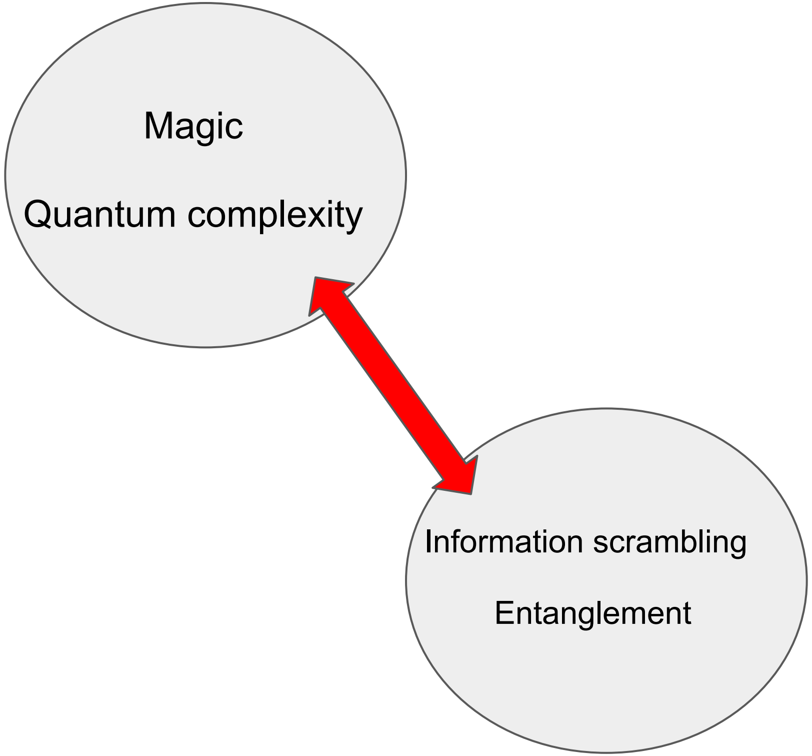

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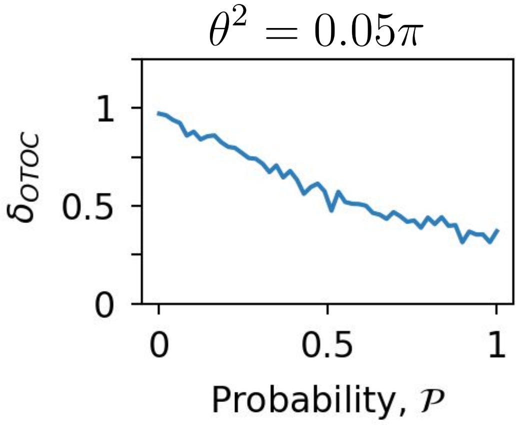
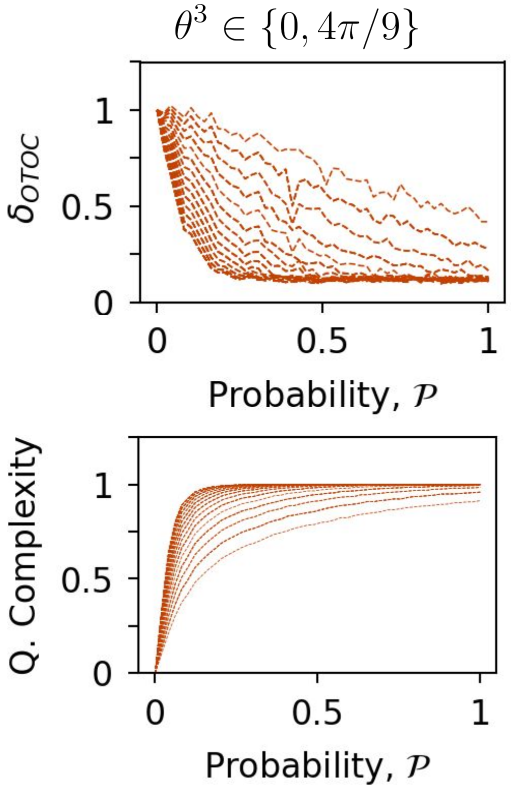
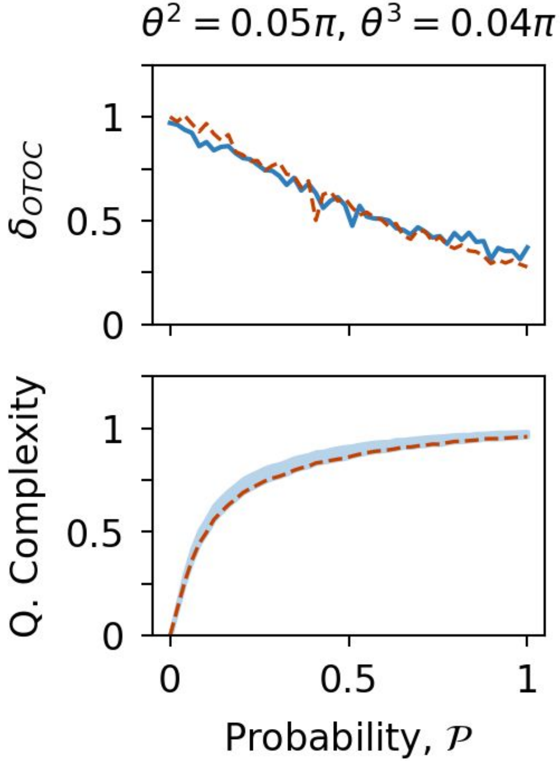
Extending Mana to other dimensions:







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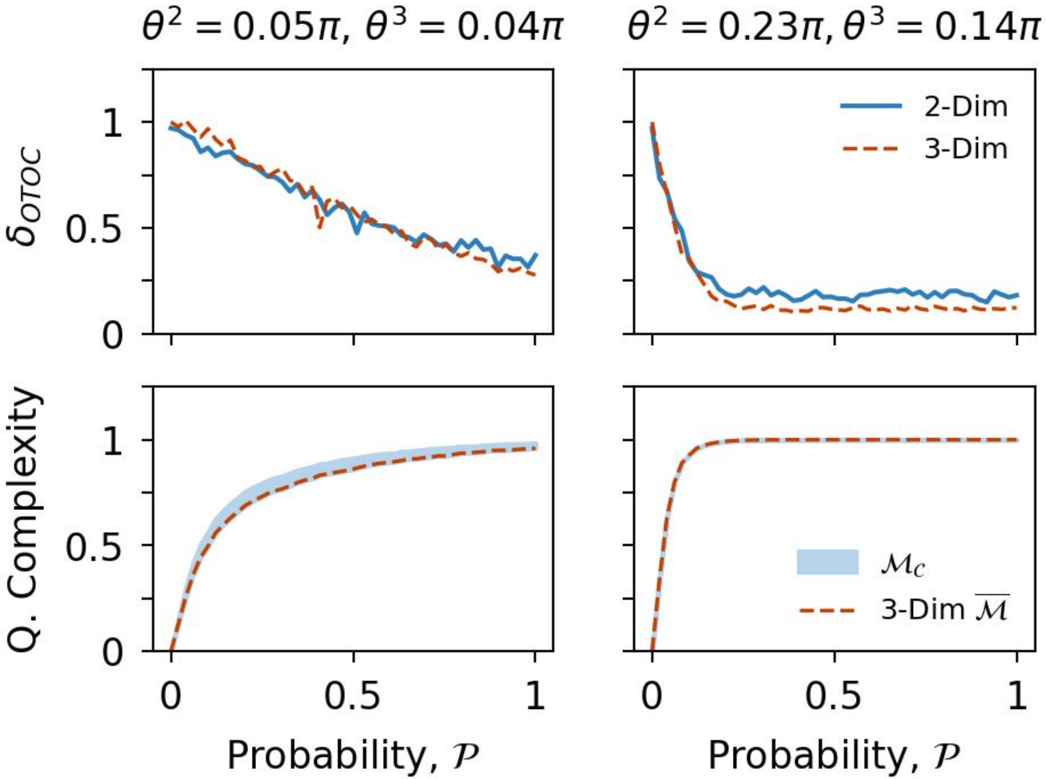
"Correlator Mana"

Extending Mana to other dimensions:



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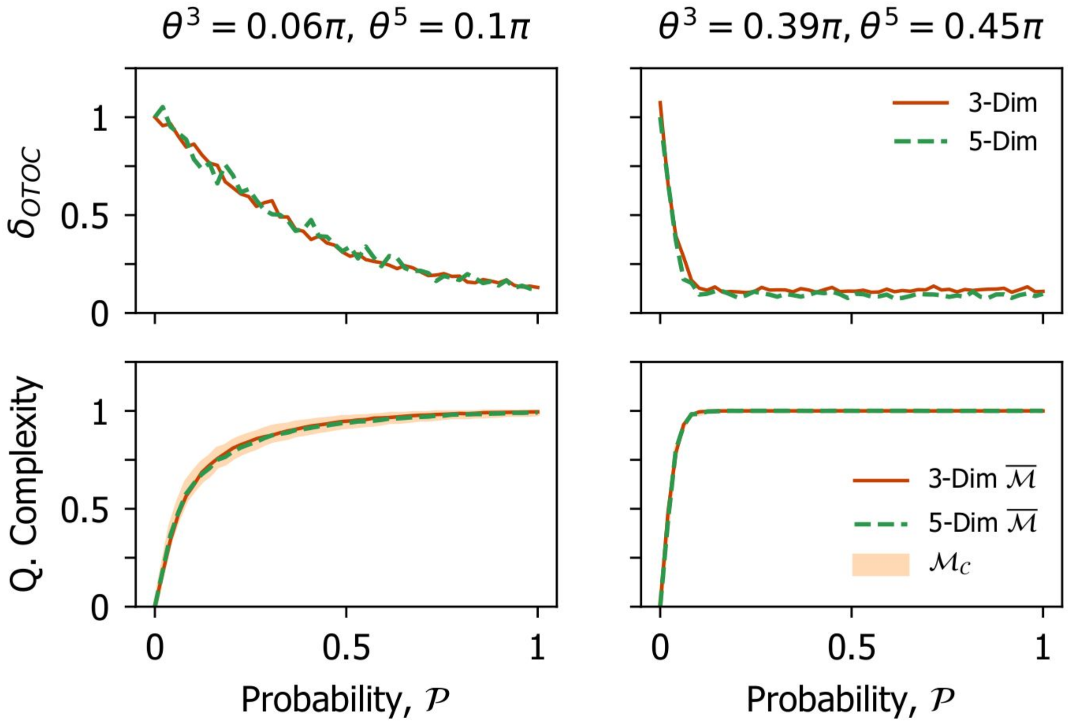
Verification:



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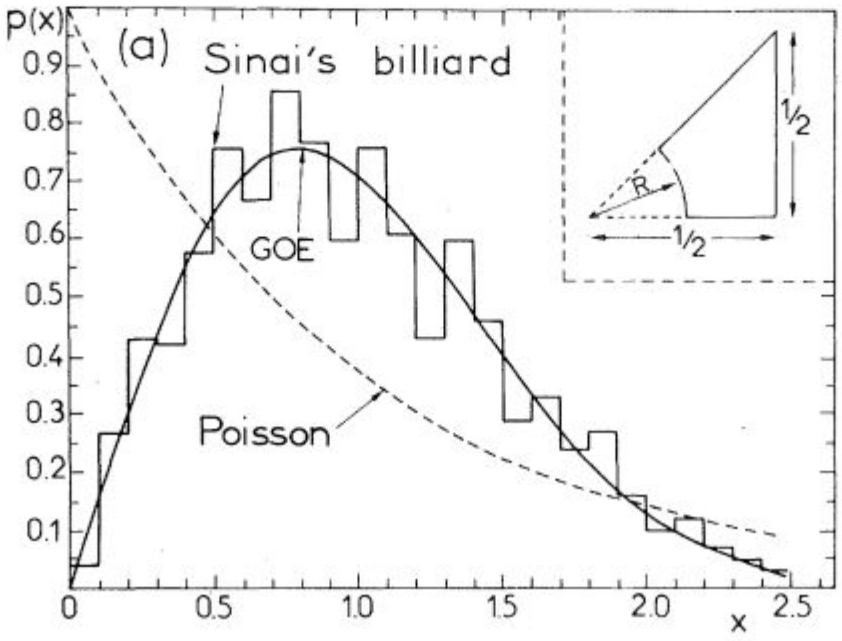
Quantum Chaos

Historically speaking:

The level spacing distribution

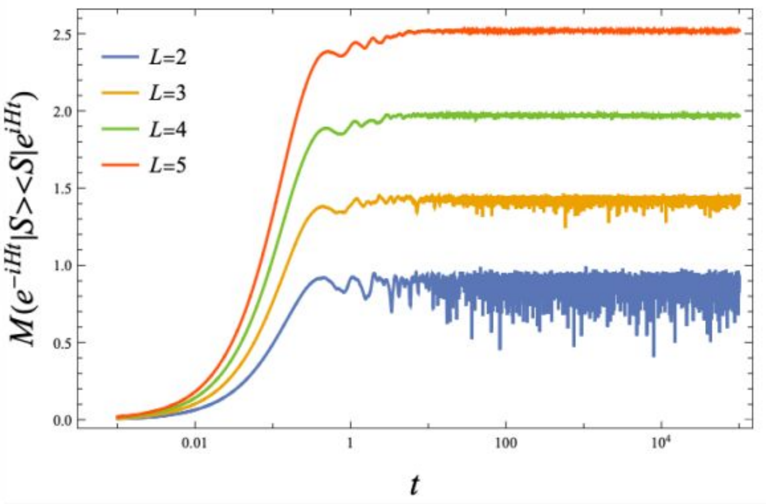


O. Bohigas, M. J. Giannoni, and C. Schmit. Characterization of chaotic quantum spectra and universality of level fluctuation laws. Phys. Rev. Lett., 52:1–4, Jan **1984**.

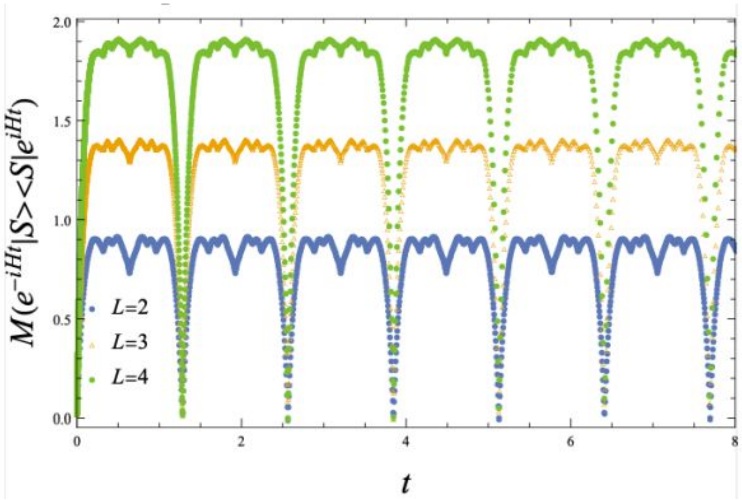
arXiv:2204.11236

Quantum Chaos

New approach based on the magic:



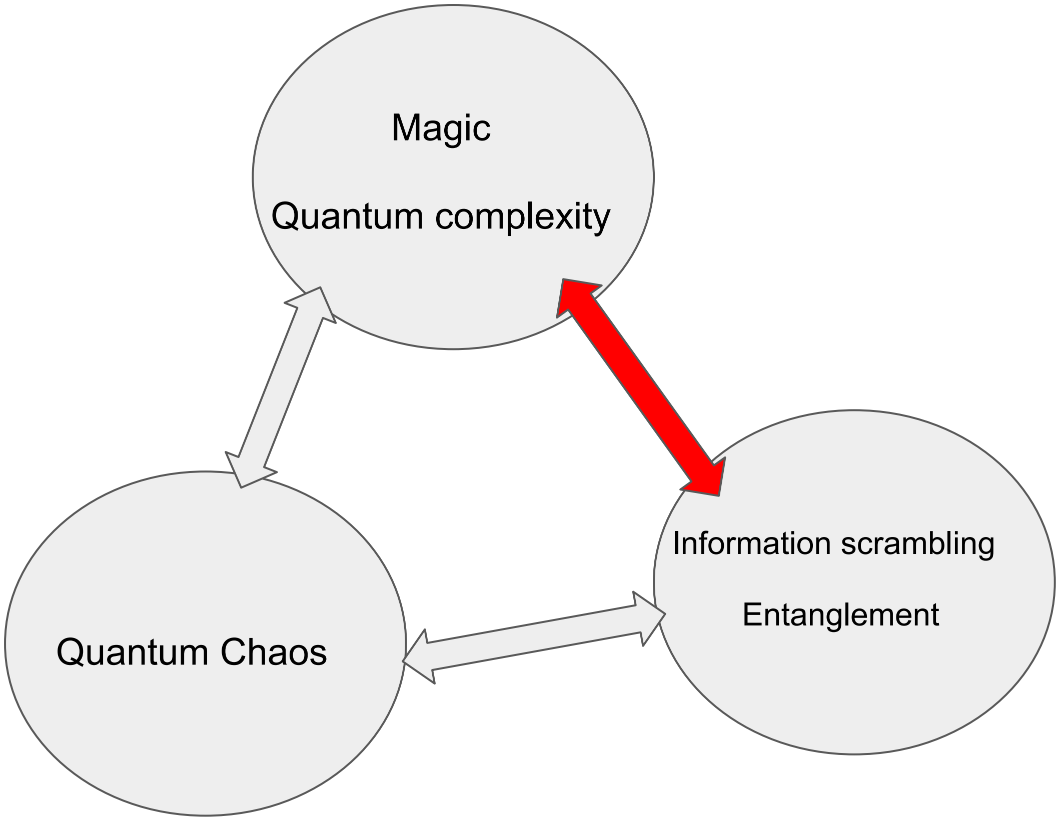
Chaotic System



Non-Chaotic System

arXiv:2204.11236

Kanato Goto, Tomoki Nosaka, and Masahiro Nozaki. Chaos by magic, 2021. arXiv:2112.14593v1



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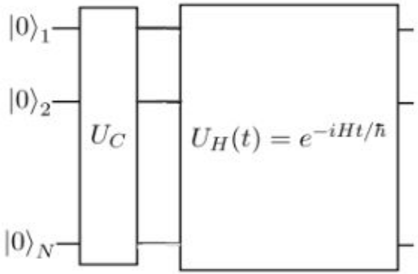
Quantum Chaos

$$H = \sum_n -S_z^n S_z^{n+1} - h_x S_x^n - h_z S_x^n$$

Chaotic for

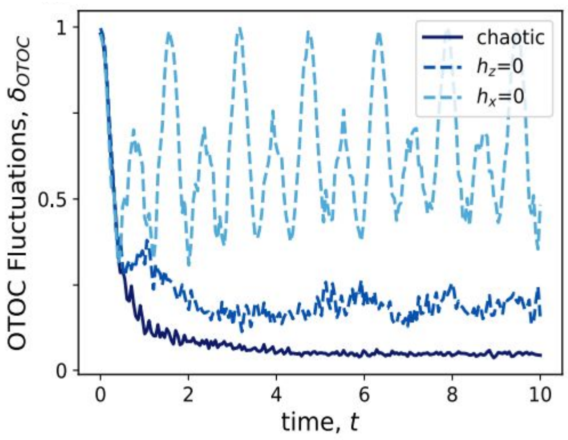
$$(h_x^*, h_z^*) = (-1.05, 0.5)$$

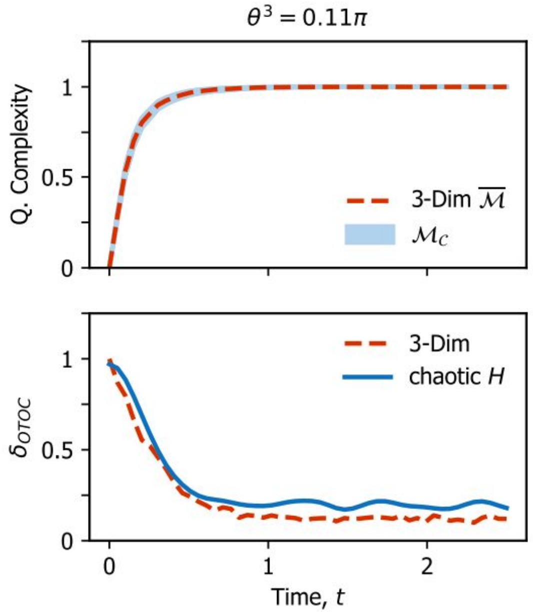
Proposed protocol:



Repeat for K instances for each time step t

Quantum Chaos





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Arash Ahmadi & Eliska Greplova

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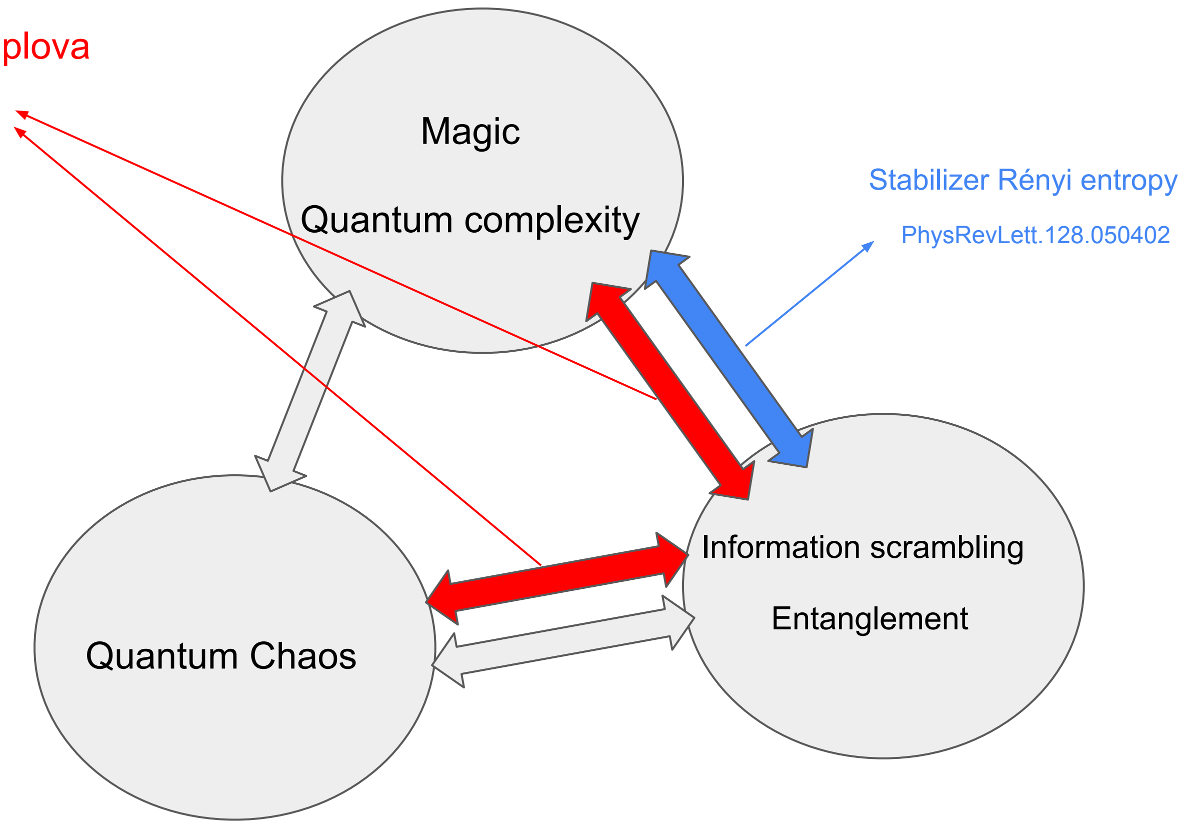

Discussion:

Can we consider OTOC as a Magic monotone?

No!

Because Magic monotone: Map: $\mathscr{H} \longrightarrow \mathbb{R}$

OTOC: Map: $(\mathscr{H}, \mathscr{L}) \longrightarrow \mathbb{R}$

Discussion:

Duality between Complexity of Random Circuit and Quantum Complexity of the State.

And another protocol to determine the chaoticity of a quantum system

Both of these results only rely on **finite(Almost constant!)** amount of measurement for exponential growth of Hilbert space which causes trouble for other methods.

50 instances for 8 qubits
and 130 for 53 qubits!

$O(10^6)$measurements for 4 qubits

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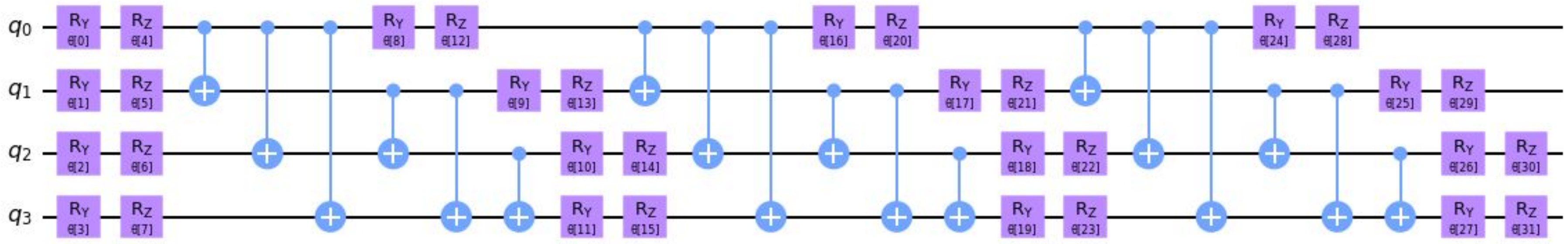
Very much QUANTUM!



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HUMMUS

Thank you for your attention!

Quantum Chaos

Nowadays: **The Information Scrambling** is dominating.

As an example:
For a system with large N degrees of freedom

$$\Re(OTOC) \sim f_1 - \frac{f_2}{N} e^{\lambda_L t}$$

More details: Daniel A. Roberts and Brian Swingle. Lieb-Robinson bound and the butterfly effect in quantum field theories. Phys. Rev. Lett., 117:091602, 2016