Quantum generalisation of Einstein's Equivalence Principle can be verified with entangled clocks as quantum reference frames

Based on

C. Cepollaro, F. Giacomini, Quantum generalisation of Einstein's Equivalence Principle can be verified with entangled clocks as quantum reference frames, arXiv:2112.03303

Workshop: Quantum Information and the Frontiers of Quantum Theory, Lyon, 28/06/2022

Carlo Cepollaro,
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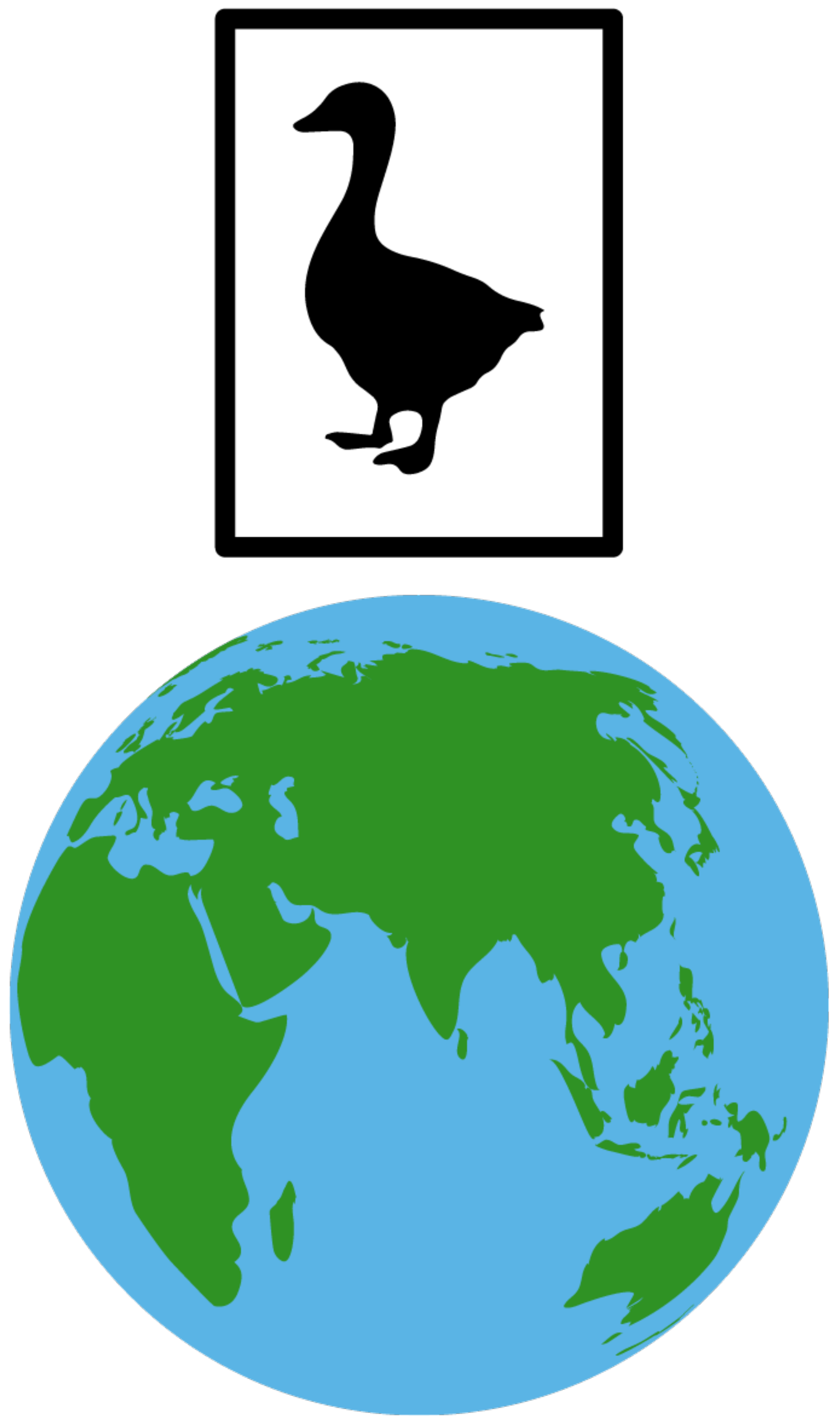
Motivation

- Different proposals of a quantum extension of the equivalence principle have been suggested:
  - Y. Aharonov and G. Carmi, Quantum aspects of the equivalence principle, Found. Phys. 3, 493 (1973).
  - R. Penrose, On gravity's role in quantum state reduction, Gen. Rel. Grav. 28, 581 (1996).
  - C. Lämmerzahl, On the equivalence principle in quantum theory, Gen. Relativ. Gravit. 28, 1043 (1996).
  - M. Zych and C. Brukner, Quantum formulation of the Einstein equivalence principle, Nature Physics 14, 1027 (2018).
  - L. Hardy, Implementation of the Quantum Equivalence Principle, in Progress and Visions in Quantum Theory in View of Gravity: Bridging foundations of physics and mathematics (2019).
  - **F. Giacomini and C. Brukner, Einstein's equivalence principle for superpositions of gravitational fields and quantum reference frames, arXiv:2012.13754 [quant-ph] (2021).**
  - C. Marletto and V. Vedral, Sagnac interferometer and the quantum nature of gravity, Journal of Physics Communications 5, 051001 (2021).
- It is fundamental to test these ideas.

The Einstein’s Equivalence Principle (EEP)

- In any and every locally inertial frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.



C. W. Misner, K. S. Thorne, J. A. Wheeler, Gravitation (W. H. Freeman, 1973).

The three aspects of the principle

- Weak Equivalence Principle (WEP): The local effects of motion in a curved spacetime (gravitation) are indistinguishable from those of an accelerated observer in flat spacetime.

C.M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 17 (2014).

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- Weak Equivalence Principle (WEP): The local effects of motion in a curved spacetime (gravitation) are indistinguishable from those of an accelerated observer in flat spacetime.

- Local Lorentz Invariance (LLI): The outcome of any local nongravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.

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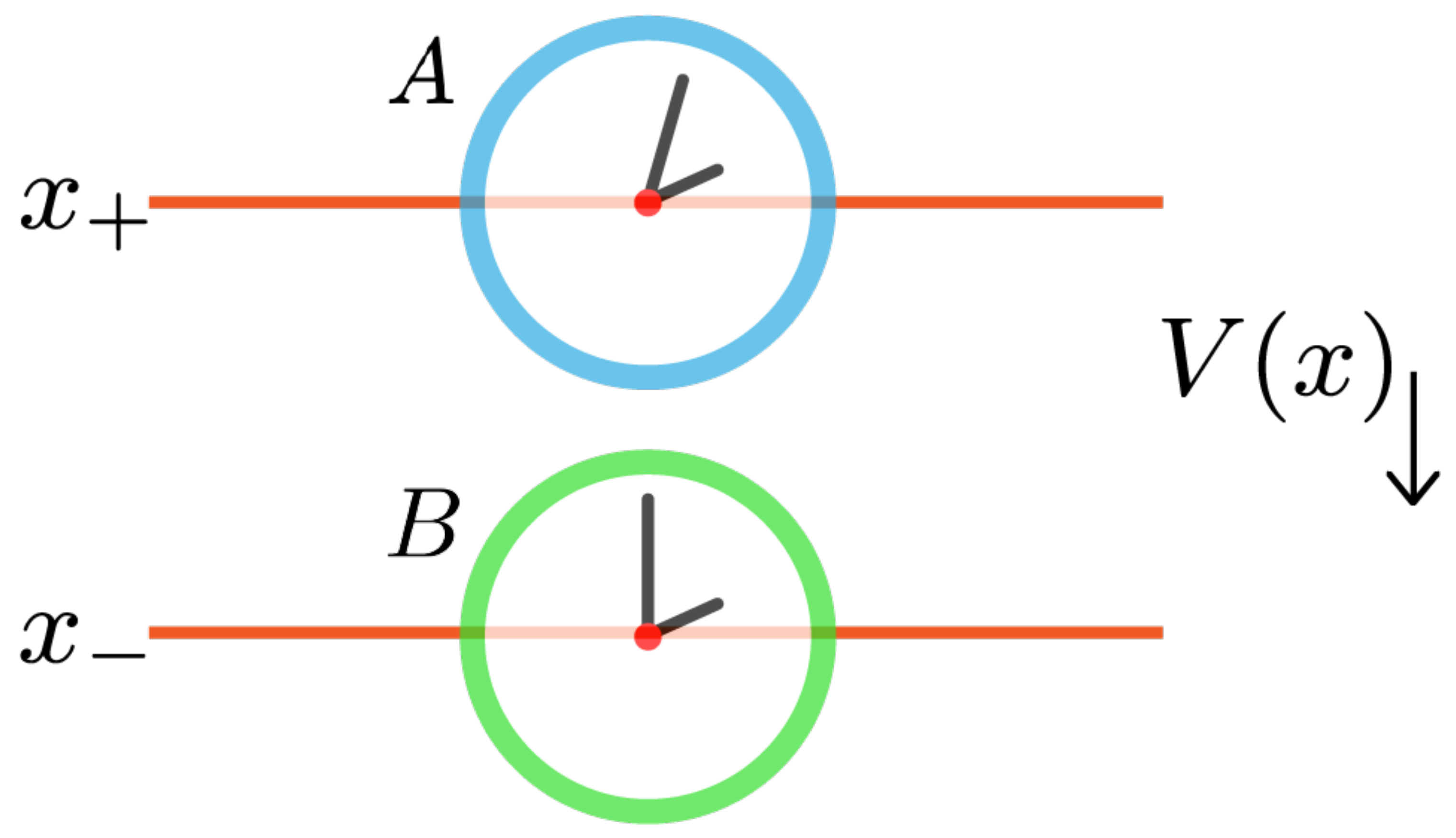
- Local Lorentz Invariance (LLI): The outcome of any local nongravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.

- Local Position Invariance (LPI): The outcome of any local nongravitational experiment is independent of where and when in the universe it is performed.

C.M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 17 (2014).

Classical tests of the principle

- WEP tests: Comparison between accelerations of two bodies of different composition in an external gravitational field.
- LLI tests: Tests of special relativity (e.g., Michelson-Morley experiment).
- LPI tests: Gravitational redshift experiment.



$$\frac{\Delta \nu}{\nu} = \frac{\Delta V}{c^2}$$

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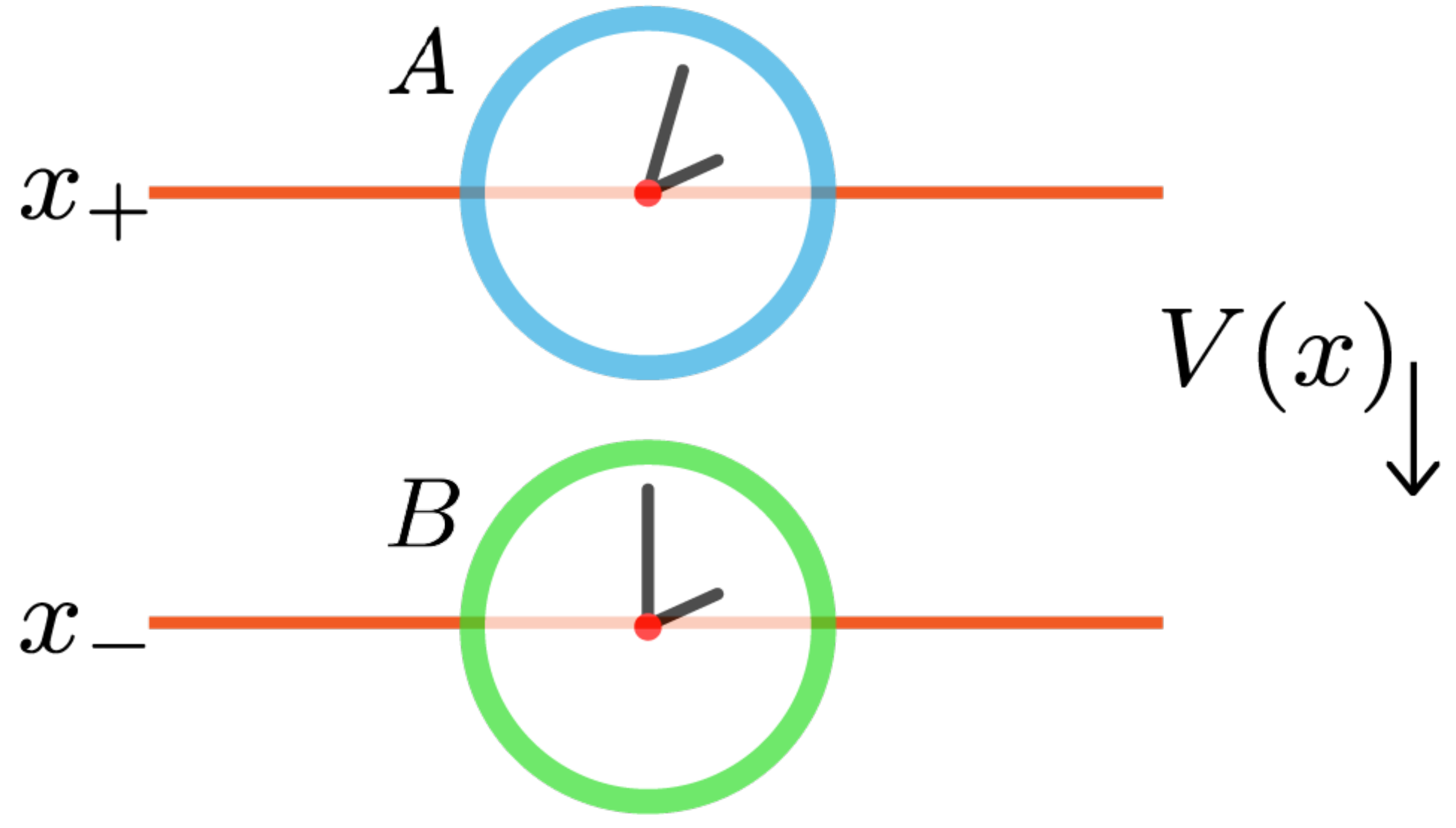
- LPI tests: Gravitational redshift experiment.
    - What happens when clocks are in a quantum superposition of different heights?



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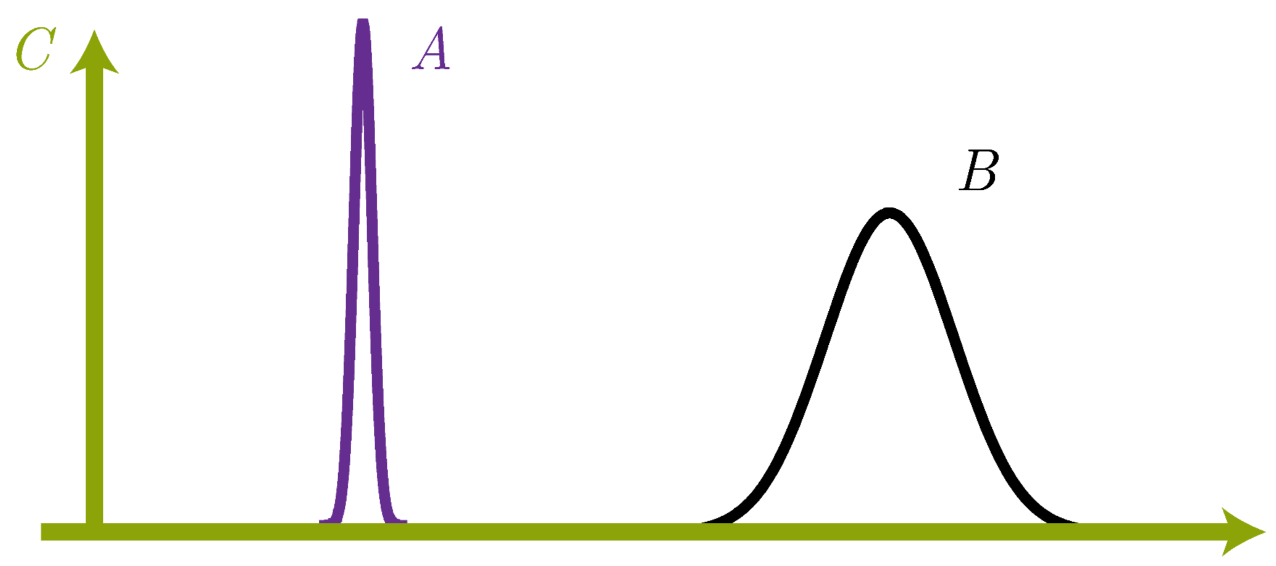
Quantum Reference Frames (QRFs)

- Quantum Reference Frames are reference frames associated to quantum particles that can be delocalized.
- The classical translation:



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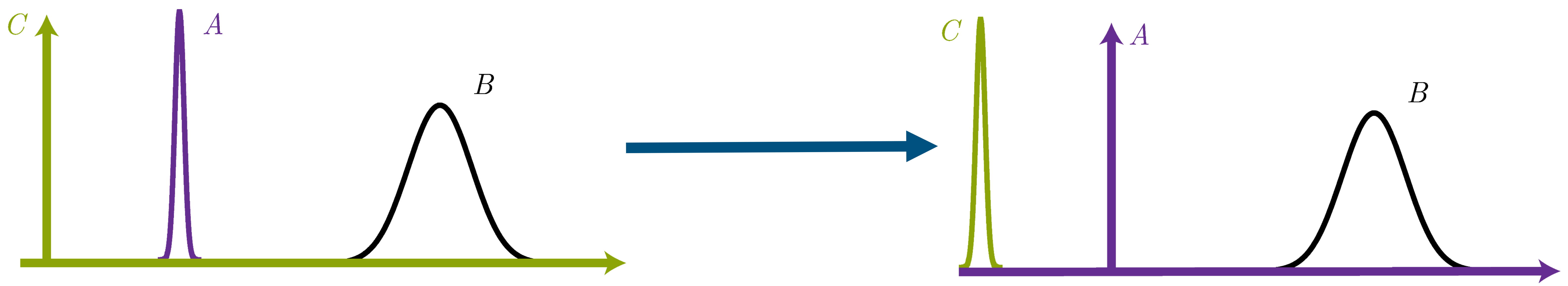

$$\boxed{\hat{T}_{x_A} = e^{i x_A \hat{p}_B}}$$

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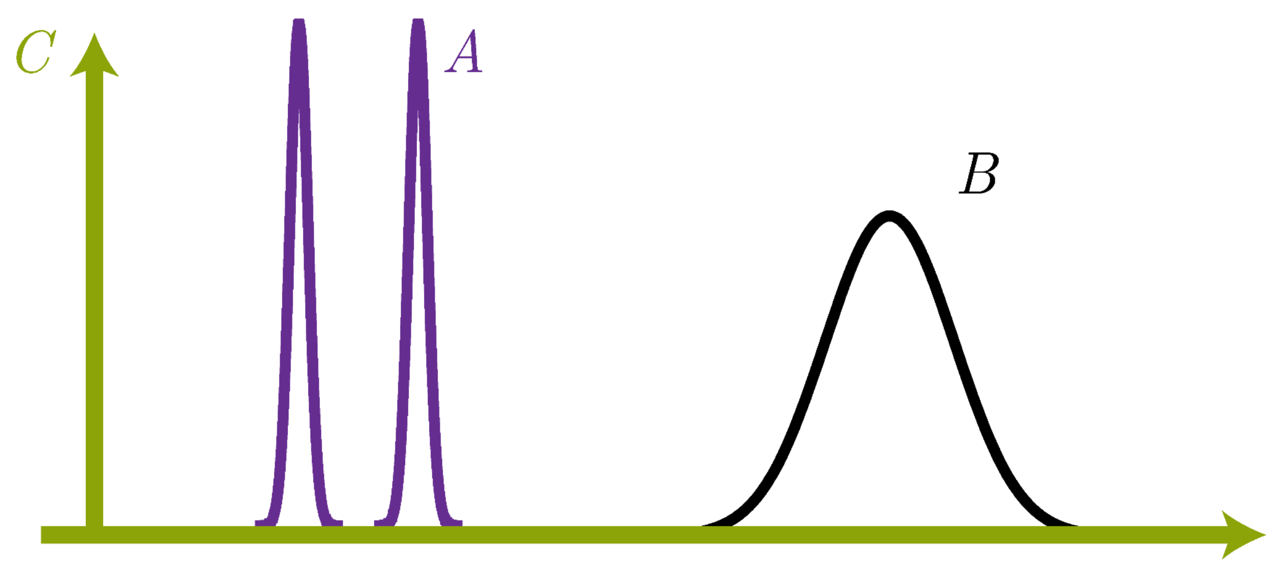

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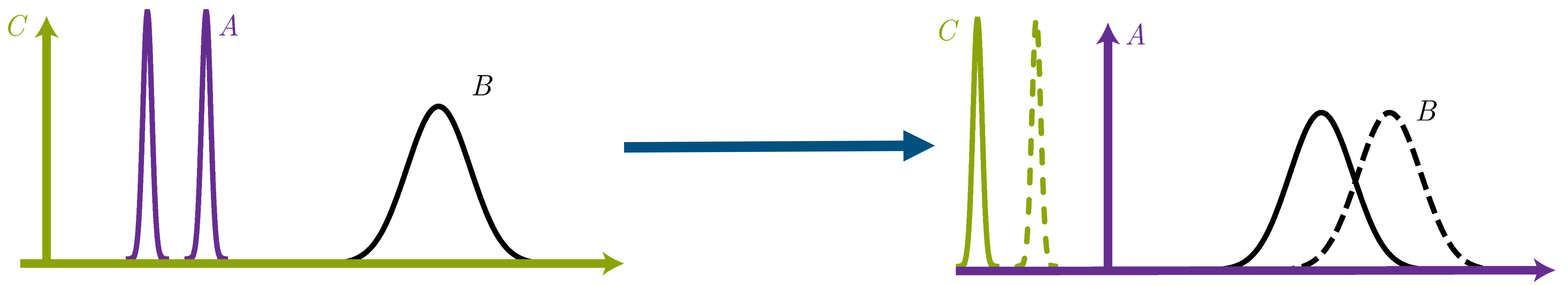

$$\boxed{\hat{T}_{x_A} = e^{ix_A\hat{p}_B} \rightarrow \hat{T}_{\hat{x}_A} = e^{i\hat{x}_A\hat{p}_B}}$$

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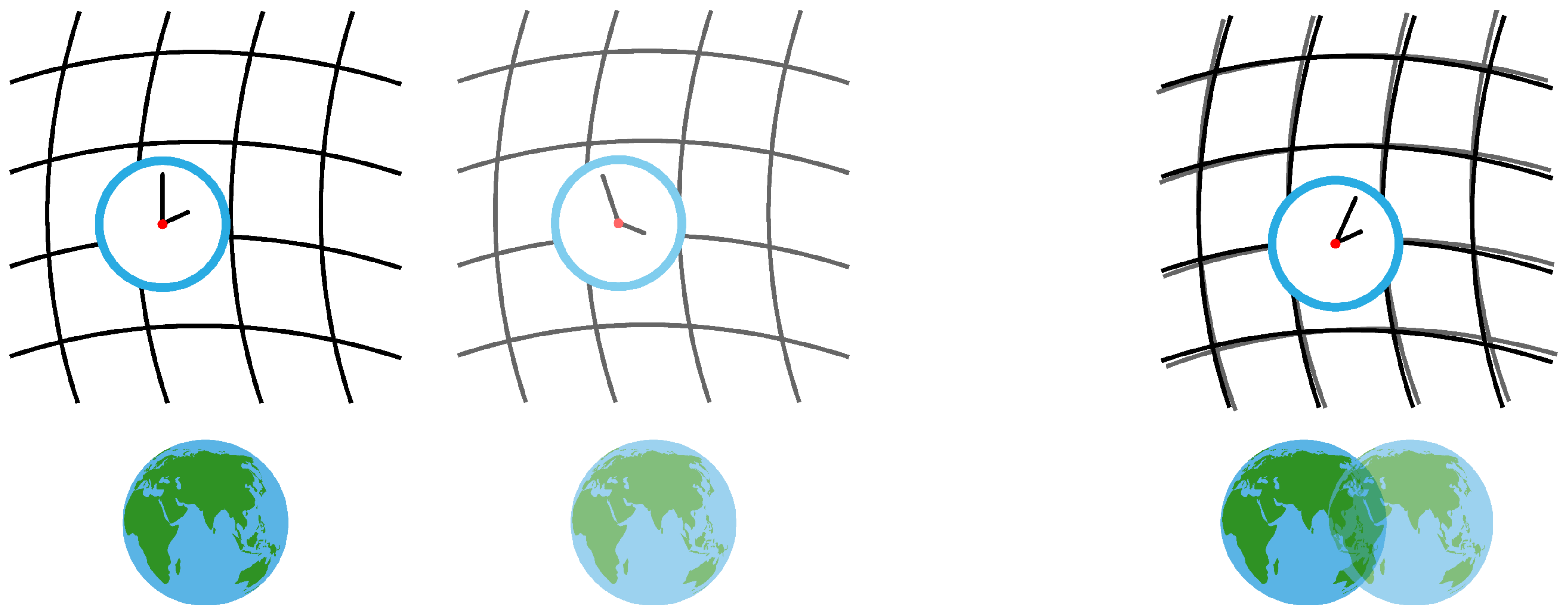
Quantum locally inertial frame

- A mass in spatial superposition generates a superposition of gravitational fields. A clock that evolves in this scenario will get entangled with the gravitational field.



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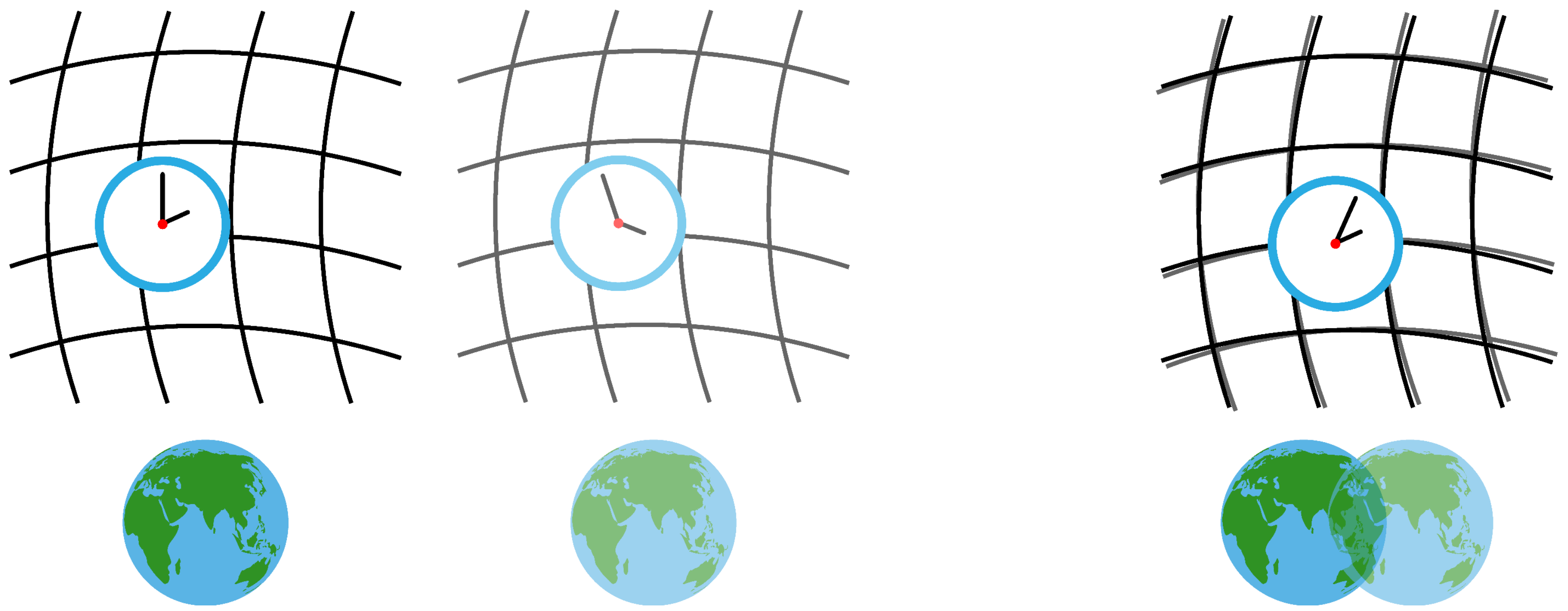
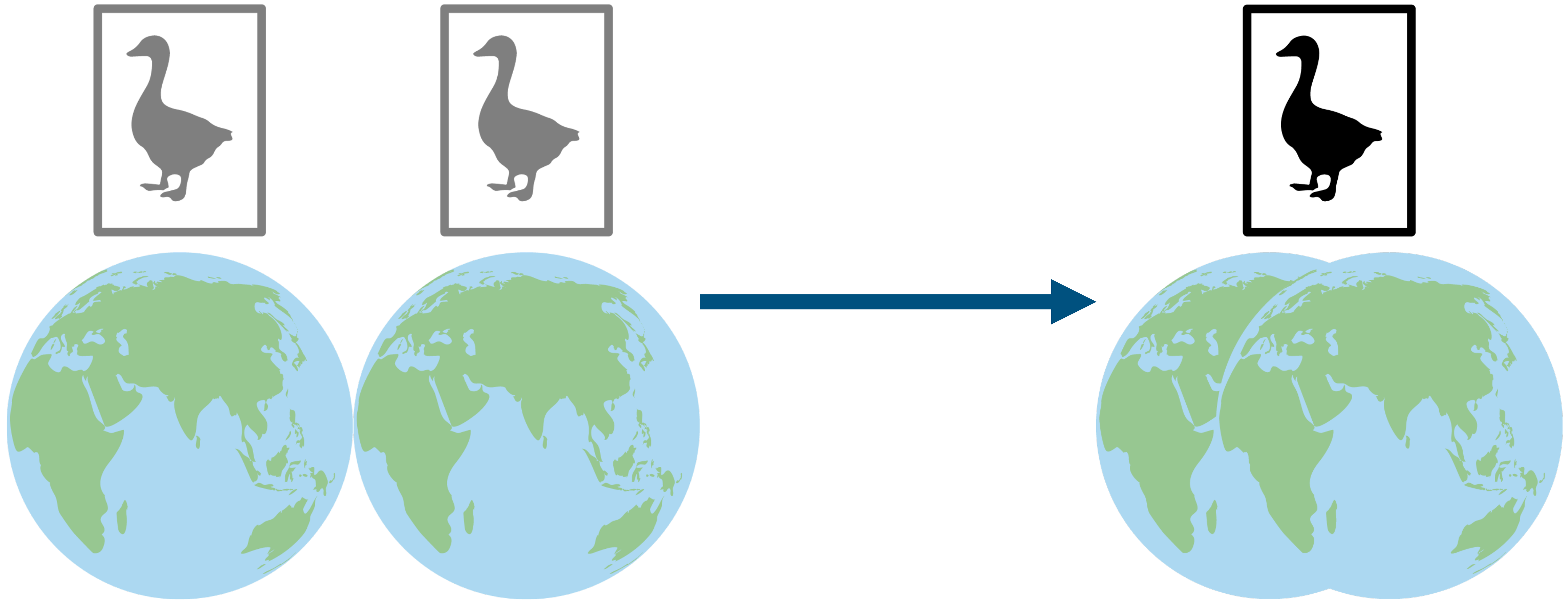


- The **Quantum locally inertial frame** (QLIF) of the clock is a frame associated to a quantum particle where the metric is locally flat and well defined. This can be reached through a QRF transformation.

F. Giacomini, Č. Brukner, Einstein's Equivalence principle for superpositions of gravitational fields and quantum reference frames, arXiv:2012.137544.

The EEP for QRFs

- In any and every **Quantum** Locally Inertial Frame (QLIF), anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.



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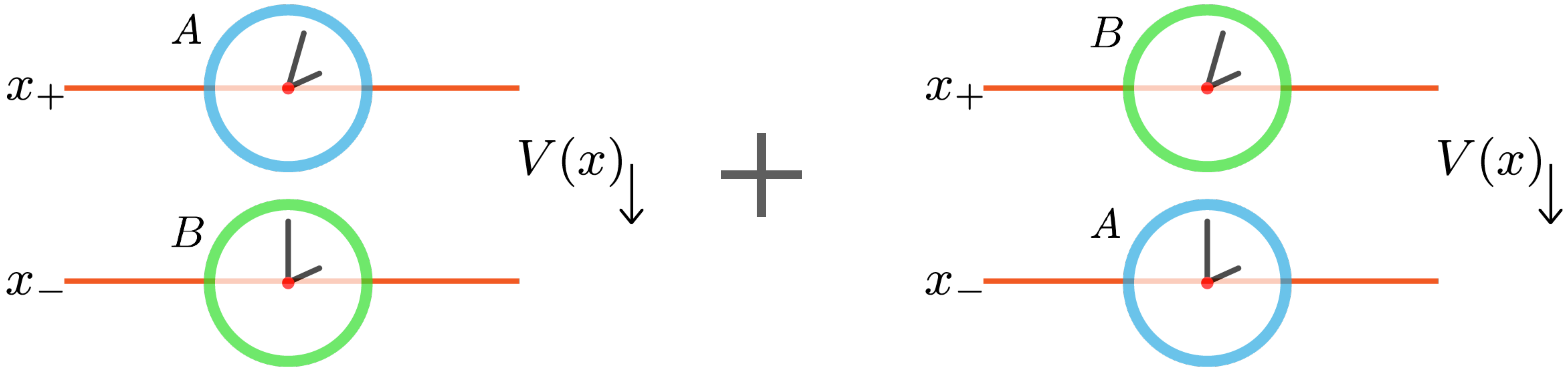
The three aspects of the principle revisited

- **(Q-WEP)** The local effects of (quantum) motion in a superposition of uniform gravitational fields are indistinguishable from those of an observer in flat spacetime that undergoes a quantum superposition of accelerations. [1]
- **(Q-LLI)** The outcome of any local nongravitational experiment is independent of the velocity of the freely falling quantum reference frame in which it is performed.
- **(Q-LPI)** The outcome of any local nongravitational experiment is independent of the position of the quantum reference frame in which it is performed.

1. F. Giacomini, E. Castro-Ruiz, Č. Brukner, Quantum mechanics and the covariance of physical laws in quantum reference frames, Nat. Commun. 10, 494 (2019).

Quantum Universality of Gravitational Redshift (Q-UGR)

- In order to test Q-LPI, one has to test Q-UGR.
- For example: Given this quantum superposition:

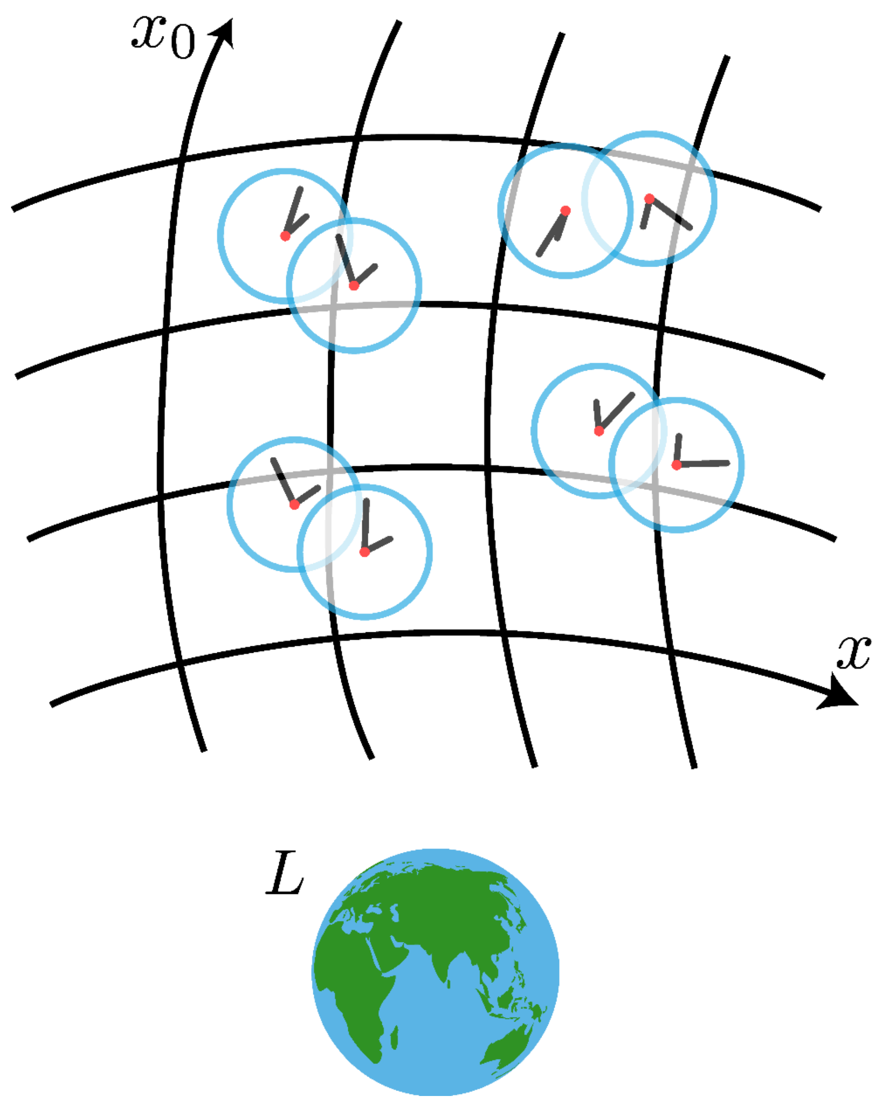


- What is the time of the second clock according to the first one?

Spacetime Quantum Reference Frames (SQRFs)

- SQRFs are reference frames associated to non-interacting quantum particles that move in a gravitational field generated by a mass L, given by a Newtonian metric.

$$
\begin{aligned}
g_{00} &= 1 + 2\frac{V(\mathbf{x}-\mathbf{x_L})}{c^2},\\
g_{01} &= g_{10} = 0,\\
g_{11} &= -1.
\end{aligned}
$$



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Spacetime Quantum Reference Frames (SQRFs)

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- External DOF: fix the quantum reference frame.
- Internal DOF: clocks, as in the Page-Wootters mechanism.

$$g_{00} = 1 + 2\frac{V(\mathbf{x} - \mathbf{x_L})}{c^2},$$
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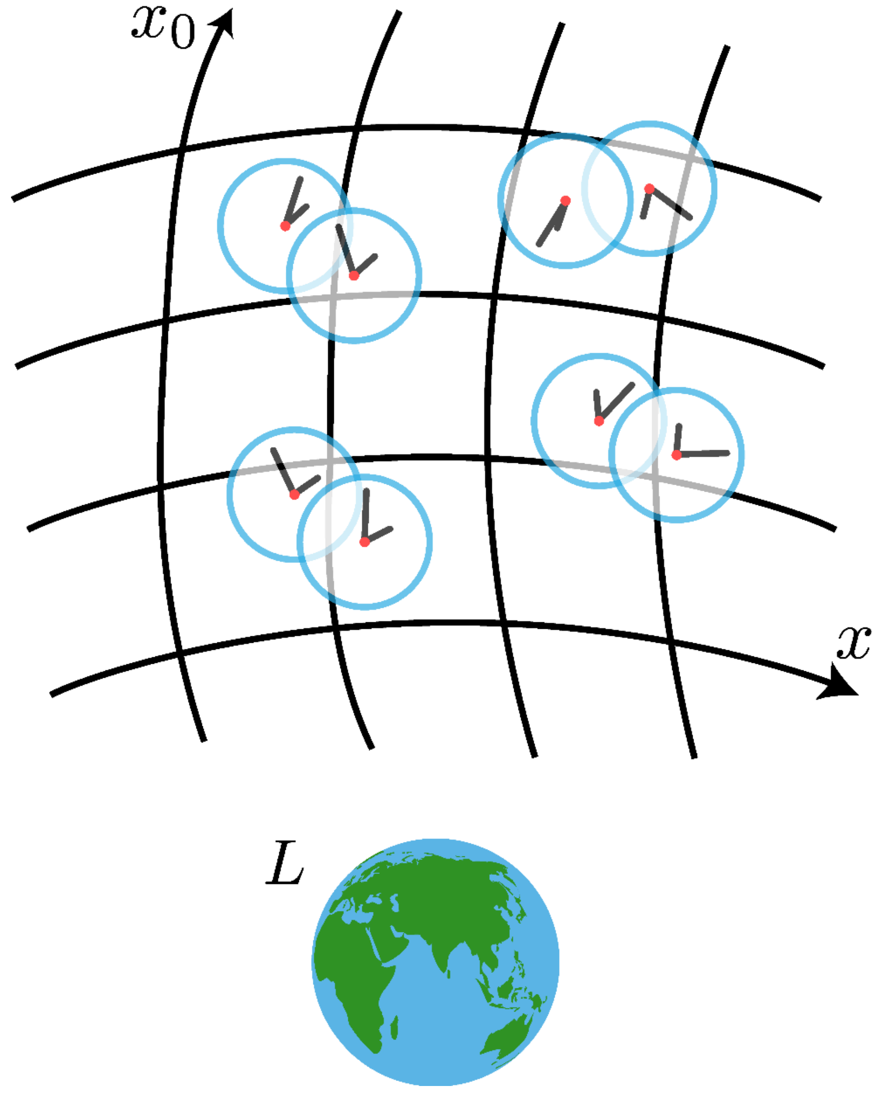

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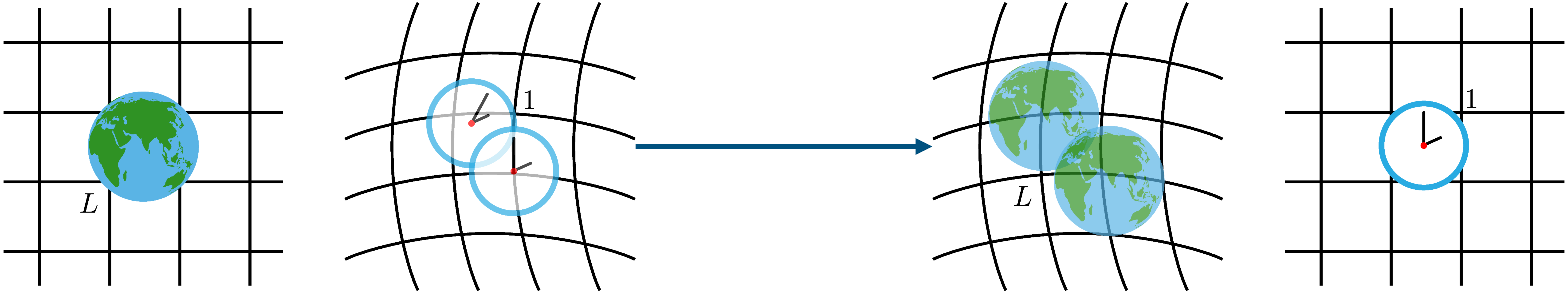
Spacetime Quantum Reference Frames (SQRFs)

- The formalism allows to describe the QLIF of a particle, and to change from one QLIF to another.

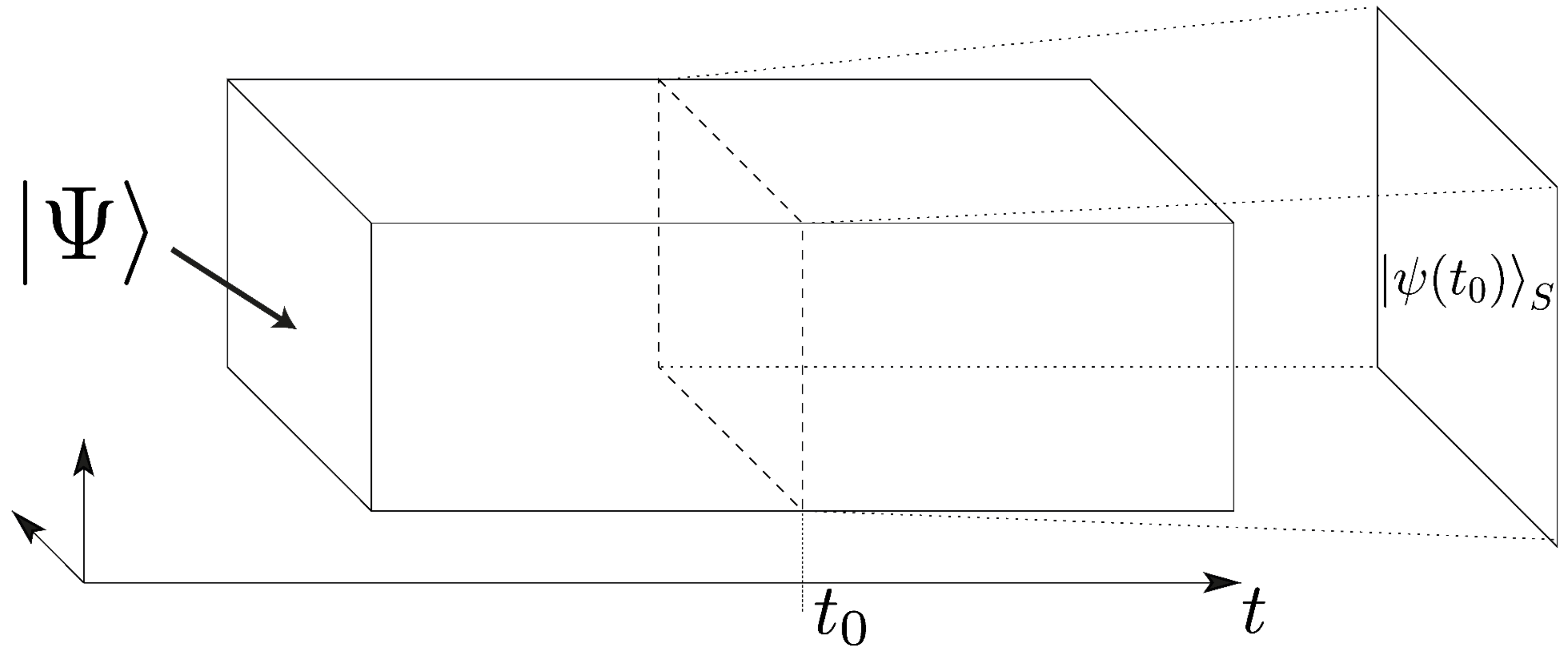


Spacetime Quantum Reference Frames (SQRFs)

- The state from the point of view of a given particle is a history state, as in the Page-Wootters mechanism.

$$|\psi\rangle^{(L)} = \int d\tau_L \, |\psi(\tau_L)\rangle^{(L)} \, |\tau_L\rangle$$

$$|\psi(\tau_L)\rangle^{(L)} = e^{-\frac{i}{\hbar}\hat{H}^{(L)}\tau_L} \, |\psi_0^{(L)}\rangle$$



V. Giovannetti, S. Lloyd, L. Maccone, Quantum time, Phys. Rev. D 92, 045033 (2015).

The Hamiltonian in the laboratory frame

$$\hat{H}^{(L)} = mc^2 + \frac{\hat{\mathbf{p}}^2}{2m} + mV(\hat{\mathbf{x}}) + \hat{H}_I \left(1 + \frac{V(\hat{\mathbf{x}})}{c^2} - \frac{\hat{\mathbf{p}}^2}{2m^2c^2}\right)$$

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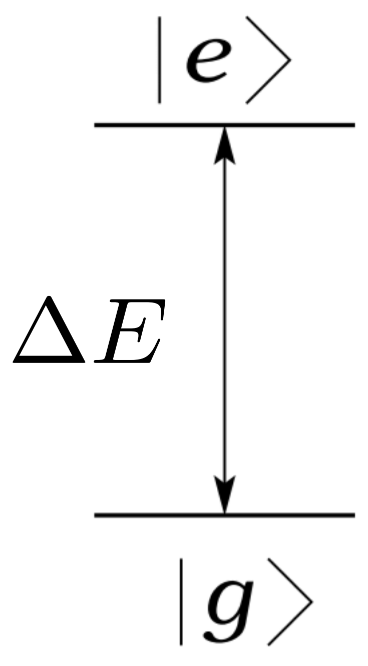
$$\omega' = \omega \left(1 + \frac{V(x)}{c^2} - \frac{p^2}{2m^2c^2}\right) = \omega \frac{d\tau}{dt} \qquad \omega = \frac{\Delta E}{\hbar}$$



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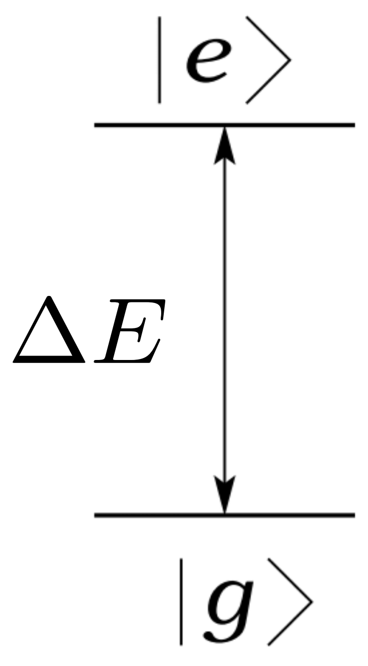

The same Hamiltonian can be found starting from the Newtonian Hamiltonian and performing the substitution[1,2]:

$$m \rightarrow \hat{M} = m + \frac{\hat{H}_I}{c^2}$$

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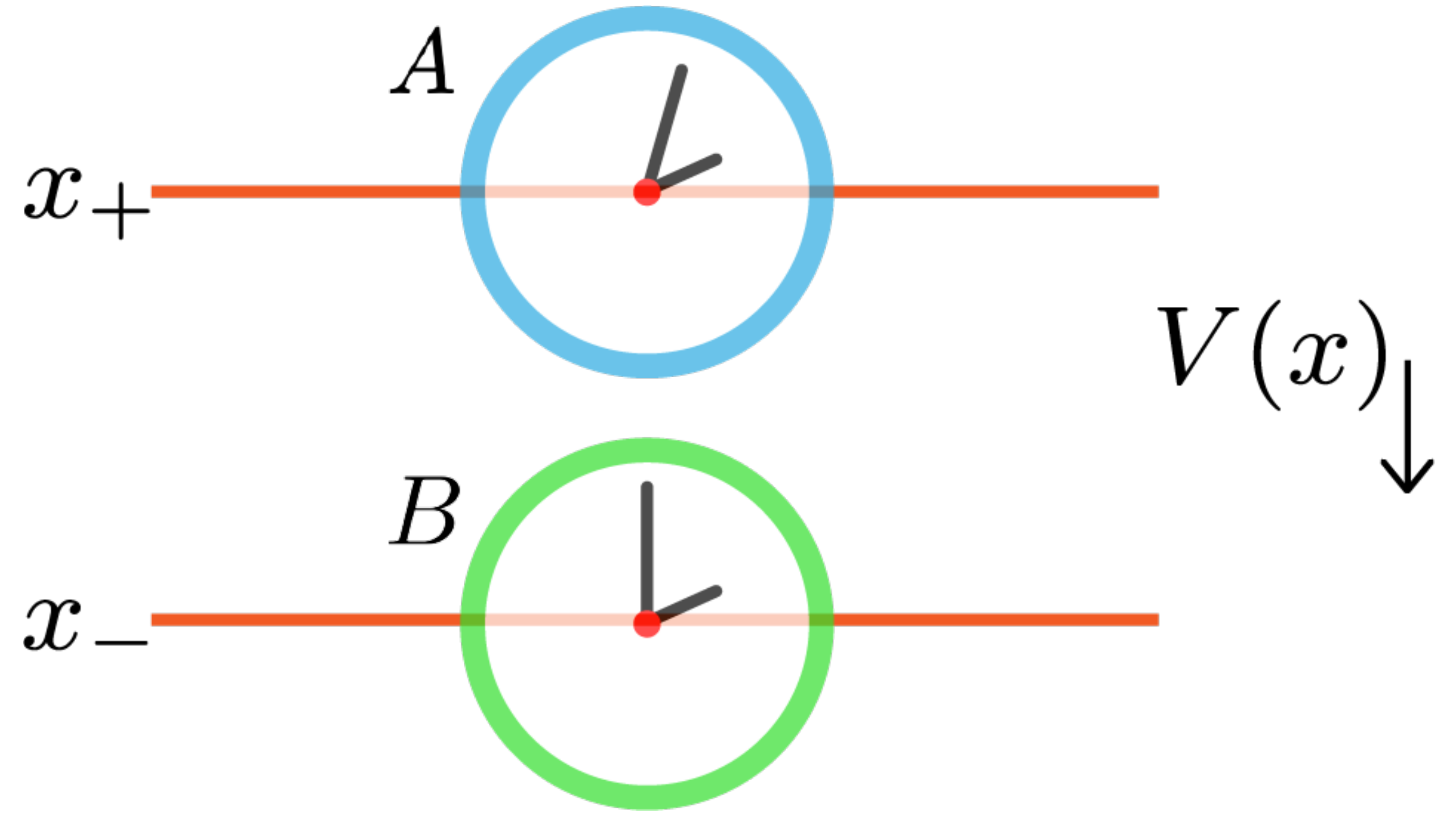
UGR and SQRFs

- Using SQRFs we can analyze classical UGR:

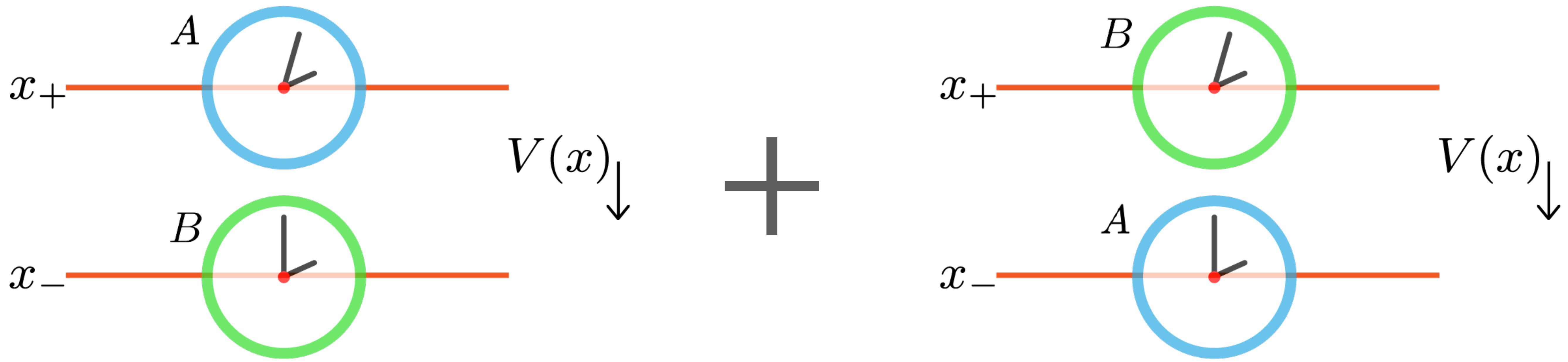
$$|\psi_0\rangle_{AB}^{(L)} = |x_+\rangle_{\mathbf{A}} \, |x_-\rangle_{\mathbf{B}} \, |\tau_{in} = 0\rangle_{C_A} \, |\tau_{in} = 0\rangle_{C_B}$$

$$|\psi(t_A)\rangle_{BL}^{(A)} = |\Psi(t_A, x_+, x_-)\rangle_{\mathbf{BL}} \left| \left(1 + \frac{V(\mathbf{x}_-) - V(\mathbf{x}_+)}{c^2}\right) t_A \right\rangle_{C_B}$$



Examples of Q-UGR

$$|\psi_0\rangle^{(L)}_{AB} = \frac{|x_+\rangle_{\mathbf{A}}\,|x_-\rangle_{\mathbf{B}} + |x_-\rangle_{\mathbf{A}}\,|x_+\rangle_{\mathbf{B}}}{\sqrt{2}}\,|\tau_{in}\rangle_{C_A}\,|\tau_{in}\rangle_{C_B}$$



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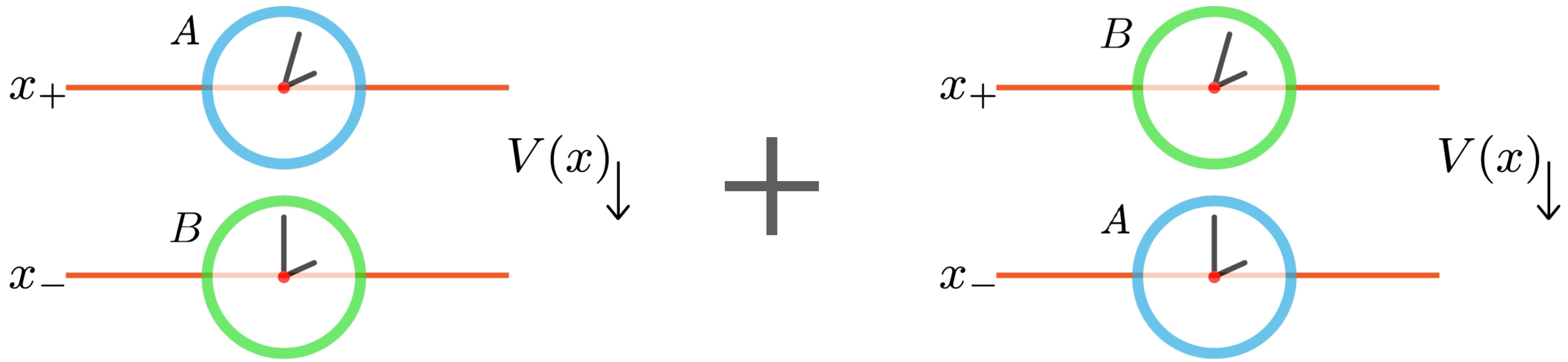

$$|\psi^{(A)}(\tau_A)\rangle_{BL} = \frac{1}{\sqrt{2}}\left(|\Psi(\tau_A, x_+, x_-)\rangle_{\mathbf{B}L} |\tau_A + \Delta\tau\rangle_{C_B} + |\Psi(\tau_A, x_-, x_+)\rangle_{\mathbf{B}L} |\tau_A - \Delta\tau\rangle_{C_B}\right)$$

- In the QRF of the clock there is a superposition of gravitational time dilations, which is due to the relative delocalisation of the clocks: there is a redshift factor for each possible position of particle B as seen from A.

Model for violations

- Let’s assume that there are three different types of masses: $\hat{M} = m + \frac{\hat{H}_I}{c^2}$
- Inertial mass, coupled with the momentum. $\hat{M}^{(i)}(\hat{\mathbf{p}})$
- Gravitational mass, coupled with the gravitational field. $\hat{M}^{(g)}(\hat{\mathbf{x}})$
- Rest mass, uncoupled. $\hat{M}^{(r)}$

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$$\hat{M}^{(i)}(\hat{\mathbf{p}}) \neq \hat{M}^{(g)}(\hat{\mathbf{x}}) \qquad \text{(Q-WEP)} \qquad \hat{\eta}(\hat{\mathbf{x}}, \hat{\mathbf{p}}) = \hat{\mathbb{1}} - \hat{M}^{(g)}(\hat{\mathbf{x}})\, \hat{M}^{(i)-1}(\hat{\mathbf{p}})$$

$$\hat{M}^{(r)} \neq \hat{M}^{(i)}(\hat{\mathbf{p}}) \qquad \text{(Q-LLI)} \qquad \hat{\beta}(\hat{\mathbf{p}}) = \hat{\mathbb{1}} - \hat{H}^{(i)}(\hat{\mathbf{p}})\, \hat{H}^{(r)-1}$$

$$\hat{M}^{(r)} \neq \hat{M}^{(g)}(\hat{\mathbf{x}}) \qquad \text{(Q-LPI)} \qquad \hat{\alpha}(\hat{\mathbf{x}}) = \hat{\mathbb{1}} - \hat{H}^{(g)}(\hat{\mathbf{x}})\, \hat{H}^{(r)-1}$$

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Non-existence of QLIFs when violations are introduced

- When Q-LPI is violated, one finds: $\cancel{|\psi\rangle^{(A)} = \int d\tau_A \, |\psi(\tau_A)\rangle^{(A)} \, |\tau_A\rangle}$

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$$|\psi\rangle^{(A)} \sim \int d\tau_A dq_L dt_A \, |\chi(\tau_A, \mathbf{q}_L, t_A)\rangle \, |\varphi(\tau_A, \mathbf{q}_L, t_A)\rangle$$

- It is not a history state anymore: the state of the clock in its reference frame depends explicitly on the position of the clock.

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$\boxed{\text{Violation of Q-LPI}}$

Violations in the laboratory QLIF

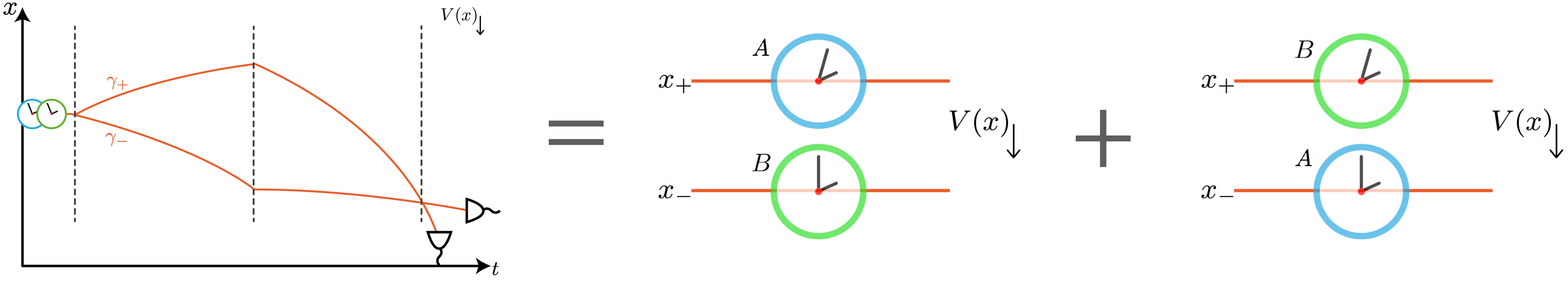
- It is still possible to describe the point of view of the laboratory. This allows us to make predictions about experiments, even if the EEP for QRFs is violated. The new Hamiltonian, e.g. when only Q-LPI is violated, is

$$\hat{H}^{Q-LPI} = \frac{\mathbf{p}^2}{2m} + mV(\mathbf{x}) + \hat{H}_I^{(r)} + \hat{H}_I^{(g)}(\mathbf{x})\frac{V(\mathbf{x})}{c^2} - \hat{H}_I^{(r)}\frac{\mathbf{p}^2}{2m^2c^2}.$$

- How can we measure the difference with the standard Hamiltonian?

Atomic clock interferometers

- A natural choice are atomic clock interferometers:



$$|\psi_0\rangle^{(L)}_{AB} = \frac{|x_+\rangle_{\mathbf{A}}|x_-\rangle_{\mathbf{B}} + |x_-\rangle_{\mathbf{A}}|x_+\rangle_{\mathbf{B}}}{\sqrt{2}}|\tau_{in}\rangle_{C_A}|\tau_{in}\rangle_{C_B} \qquad |D_\pm\rangle_{\mathbf{AB}} = \frac{|x_+\rangle_{\mathbf{A}}|x_+\rangle_{\mathbf{B}} \pm |x_-\rangle_{\mathbf{A}}|x_-\rangle_{\mathbf{B}}}{\sqrt{2}}$$

The probabilities in the presence of violations

- The measurement probabilities depend on the violation parameters for the three different aspects:

$$P_{\pm}^{(Q-LPI)} = f_1(\alpha_{ij}(x))$$

$$P_{\pm}^{(Q-LLI)} = f_2(\beta_{ij}(p))$$

$$P_{\pm}^{(Q-WEP)} = f_3(\eta_{ij}(x,p))$$

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$$P_{\pm}^{(Q-WEP)} = f_3(\eta_{ij}(x,p))$$

It is a test for the EEP for QRFs

Take-home messages

- We introduced a model for violations of the Einstein’s Equivalence Principle for Quantum Reference Frames.
- We showed that entangled atomic clocks in an atomic clock interferometer can be used to test the Einstein’s Equivalence Principle for Quantum Reference Frames.
- We found a link between different proposals of quantum extensions of the equivalence principle.

Thank you!