Emergence of classicality from information within a quantum world

Alexandre Feller

UUniversité de Lille, CNRS, Centrale Lille, UMR 9189 – CRIStAL, Lille, France

June 29, 2022

Contents

Quantum interferences, entanglement

- Superposition principle.
- Larger systems are controlled and exhibit quantum coherence.
- Composite systems exhibit entanglement (non-local correlations).
- Universality of quantum theory.

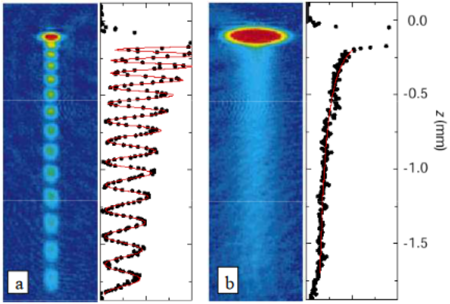


Figure: Bose-Einstein condensate coherence. [I. Bloch *et al*, Nature 403, 166 (2000)]

The problem(s)

How classical behaviors can emerge from the quantum world?

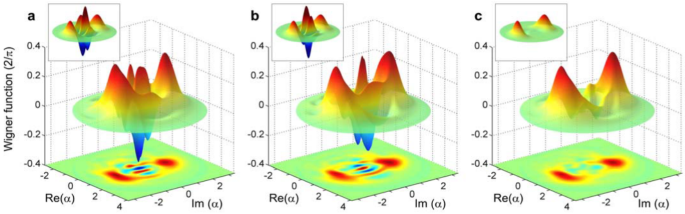
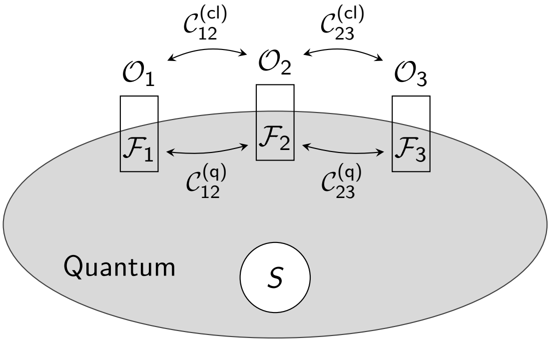


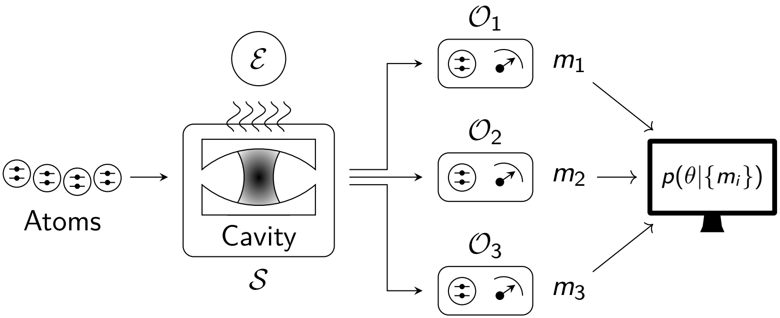
Figure: Decoherence of a coherent cat state in CQED [S. Deleglise *et al*, Nature 455, (2008)]

- Disappearance of quantum interference? Preferred states? With classical dynamics?
- How many observers agree on an outcome i.e. objectivity?
- Role of the classical sector in the interpretation? Single outcome?

A generic picture?

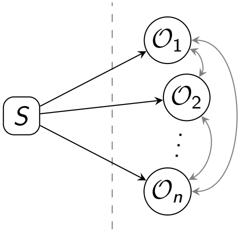


Toy example from cavity QED



Plan of the presentation

I. Sharing
quantum information?



II. Reconstruction of
classical picture?

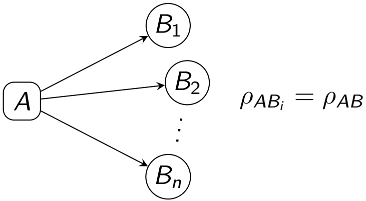
Contents

Cloning/broadcasting and estimating

- **Cloning**? $\rho_S \xrightarrow{C} \rho_S \otimes \rho_S$. No generic cloning machine.
  Approximate cloning: $\rho_S \xrightarrow{C} \tilde{\rho}_S \otimes \tilde{\rho}_S$ with $\tilde{\rho}_S \approx \rho_S$. Ex: State estimation.
- **Broadcasting**? A broadcast state of ρ_S is a state $\rho_{S_1 S_2} = \Lambda_{S \to S_1 S_2}[\rho_S]$ such that $\rho_{S_1} = \rho_{S_2} = \rho_S$. No generic broadcasting machine.
- **Local broadcasting**: A bipartite state ρ_{AB} is locally broadcastable on B if there exists a broadcast state $\rho_{A,B_1 B_2} = (\mathbb{1}_A \otimes \Lambda_{B \to B_1 B_2})[\rho_{AB}]$. Equivalently:
  1. ρ_{AB} is locally broadcastable on B,
  2. the state is quantum-classical on B:
  3. $I(A,B) = I_{\mathrm{acc}}(B,A)$ (zero discord).

Monogamy: state extension

- **Extension**: ρ_{AB} is $N-$extendable is there exists is state $\rho_{AB_1 \cdots B_N}$ such that, for all k, $\rho_{AB_k} = \rho_{AB}$.
- **Toward SEP**: An $N-$extendable state is $O(1/N)$ close to be separable. Only separable states are arbitrarily extendable.
  Proof: [Idea, Werner] Macroscopic observable $A^{(N)} = \frac{1}{N}\sum_{k=1}^{N} \mathbb{1} \otimes \cdots \otimes A \otimes \cdots \mathbb{1}$. Approximately commute $\|[A^{(N)}, B^{(N)}]\| \leq \frac{2}{N}$. Derive a de Finetti type result.



Asymptotic cloning = state estimation

- Score of the estimation task with fidelity:

$$F_e = \max_M \sum_{i,j} p_i \,\mathrm{tr}(M_j \,|\psi_i\rangle\langle\psi_i|)|\langle\phi_j|\psi_i\rangle|^2 \,. \tag{1}$$

- Score of the cloning task with fidelity:

$$F_c(N) = \max_{C_{C\to C_1\ldots C_N}} \frac{1}{N} \sum_{i,j} p_i \,\mathrm{tr}\left(C_{C\to C_j}(|\psi_i\rangle)\,|\psi_i\rangle\langle\psi_i|\,\right) . \tag{2}$$

Asymptotic cloning = state estimation

Proof:

- $F_e \leq F_c(+\infty)$ since an estimation task is a non-optimal cloning task.
- With a (symmetric) cloning map $C_{C\to C_1\ldots C_N}$ and $|\Phi^+_{AC}\rangle = \frac{1}{\sqrt{d}}\sum_i |ii\rangle$ (A ancilla), $\rho_{AC_1\ldots C_N} = (\mathbb{1}_A \otimes C_{C\to C_1\ldots C_N})(|\Phi^+_{AC}\rangle\langle\Phi^+_{AC}|)$ is an N extension of ρ_{AC}.
- Monogamy: in the limit $N \to +\infty$, ρ_{AC} must be separable. Then $C_C(\rho) = \sum_k \mathrm{tr}(M_k\rho)\,\sigma_k$ is a measure-and-prepare channel.

$$F_c(+\infty) = \max_{C_C}\sum_k p_k\langle\psi_k|C_C(|\psi_k\rangle)|\psi_k\rangle = \max_{M_j,\phi_j}\sum_{i,j} p_i\,\mathrm{tr}(M_j\,|\psi_i\rangle\langle\psi_i|)|\langle\phi_j|\psi_i\rangle|^2 = F_e\,.$$

J. Bae, & A. Acín, Physical review letters, 97(3), (2006).

Monogamy: a first de Finetti theorem

Theorem (Quantum de Finetti)

Consider $X_1, \dots, X_n$ identical quantum registers ($\mathbb{C}^d$). Consider $\rho \in \mathcal{H}_+^{\otimes n}$ a permutation invariant state and $\rho_{SEP} = \int \mathrm{tr}(M_\psi \rho) \, |\psi\rangle\langle\psi|^{\otimes n} \, \mathrm{d}\psi$ (with $M_\psi \propto |\psi\rangle\langle\psi|$). Then, for any choice of $k \in [|1, \dots, n|]$, we have

$$\|\rho^{(k)} - \rho_{SEP}^{(k)}\|_1 \underset{n \gg kd}{\leq} \frac{2(d-1)k}{n} \,. \tag{3}$$

Remark: We can remove the symmetric subspace requirement $\mathcal{H}_+$ with purification. Then $d \to d^2$ and ρ_{SEP} built from the purification.

Many observers only see classical information...

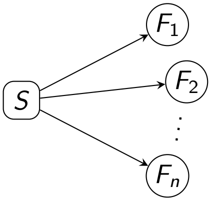
Theorem

Any symmetric distribution of information channel on N systems (with output in the symmetric subspace) can be approximated by a measure-and-prepared channel $\mathcal{M}(\rho) = \int \mathrm{tr}(M_\psi \rho)\, |\psi\rangle\langle\psi|^{\otimes N}\, \mathrm{d}\psi$ with M_ψ a POVM. In the large N limit , the accuracy of the approximation is given by:

$$\|\mathcal{C}_C^{(k)}(\rho) - \mathcal{M}^{(k)}(\rho)\|_1 \leq \frac{2(d-1)k}{N}\,, \quad N \gg kd\,. \tag{4}$$

G. Chiribella, & G. M. D'Ariano. Physical review letters, 97(25), (2006).

the same classical information!



$$S \xrightarrow{\Lambda} (F_1, \ldots, F_n)$$

$$\Lambda|_{F_i} = \Lambda_i$$

- **Objectivity of observables**: there exists a common observable X or measurement $\{M_x\}_x$ of the system shared by most observers.
- **Objectivity of outcomes**: different observers probing independent parts of the environment have full access to $\{M_x\}_x$ and agree on the outcome x.

Monogamy: a resource-dependent (1-LOCC) de Finetti theorem

More constrained bound from a resource-dependent norm evaluation:

$$\frac{1}{2}\|\rho_{AB} - \sigma_{AB}\|_{\text{1-LOCC}} = \max_{\substack{0 \leq M \leq \mathbb{1} \\ \{M, \mathbb{1} - M\} \in \text{1-LOCC}}} |\operatorname{tr}(M(\rho - \sigma))|_1 . \qquad (5)$$

Theorem (1-LOCC Quantum de Finetti)

If ρ_{AB} is a k-extendable state on $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ then:

$$\min_{\sigma_{AB} \in SEP_{AB}} \|\rho_{AB} - \sigma_{AB}\|_{1\text{-}LOCC} \leq \sqrt{\frac{2 \ln 2 \ln d_A}{k}} . \qquad (6)$$

Monogamy: a resource-dependent (1-LOCC) de Finetti theorem

Proof: [Elements] Assume the registers $B_1 \dots B_{l-1}$ classical.

1. From the chain rule of the conditional quantum mutual information, we have:

$$I(A, B_1 \dots B_k) = I(A, B_1) + I(A, B_2|B_1) + \dots + I(A, B_k|B_1 \dots B_{k-1})\,. \tag{7}$$

   The sum contains k terms, is upper bounded by $\log d_A$ (because we have classical registers), so there exists an l such that:

$$I(A, B_l|B_1 \dots B_{l-1}) \leq \frac{\log d_A}{k}\,. \tag{8}$$

2. We can write the above conditional mutual information as an average:

$$I(A, B_l|B_1 \dots B_{l-1}) = \sum_{\mathbf{x}} p_{\mathbf{x}} I(A, B_l)_{\rho^{(\mathbf{x})}_{AB_l}}\,. \tag{9}$$

Monogamy: a resource-dependent (1-LOCC) de Finetti theorem

$$
\begin{aligned}
&\frac{1}{2\ln 2}\left\|(\mathbb{1}_A\otimes M_B)(\rho_{AB}-\sum_{\mathbf{x}} p_{\mathbf{x}}\rho_A^{(\mathbf{x})}\otimes\rho_B^{(\mathbf{x})})\right\|_1^2\\
&\underset{\text{Ineq.Tri.}}{\leq}\ \frac{1}{2\ln 2}\left(\sum_{\mathbf{x}} p_{\mathbf{x}}\left\|(\mathbb{1}_A\otimes M_B)(\rho_{AB_l}^{(\mathbf{x})}-\rho_A^{(\mathbf{x})}\otimes\rho_B^{(\mathbf{x})})\right\|_1\right)^2\\
&\underset{\text{Conc.}}{\leq}\ \frac{1}{2\ln 2}\sum_{\mathbf{x}} p_{\mathbf{x}}\left\|(\mathbb{1}_A\otimes M_B)(\rho_{AB_l}^{(\mathbf{x})}-\rho_A^{(\mathbf{x})}\otimes\rho_B^{(\mathbf{x})})\right\|_1^2\\
&\underset{\text{Pinsker}}{\leq}\ \sum_{\mathbf{x}} p_{\mathbf{x}}\, I(A,B_l)_{\rho_{AB_l}^{(\mathbf{x})}} = I(A,B_l|B_1\ldots B_{l-1})\\
&\underset{(8)}{\leq}\ \frac{\log d_A}{k}\,.
\end{aligned}
$$

F. G. Brandao & A. W. Harrow. Communications in Mathematical Physics, 353(2), (2017).

What is observed?

Theorem (Observable objectivity)

For any Λ, there exists a POVM $\{M_x\}$ (objective observable) and a small set $Q \subseteq \{1, \ldots, n\}$ (excluded observers) such that for any $j \notin Q$:

$$\Lambda_j \approx_\epsilon \textit{Measurement } \{M_x\} + \textit{Post processing} \tag{10}$$

Intuition: No-cloning theorem, especially monogamy.

F. Brandao, M. Piani, and O. Horodecki (2015), Nature Communications **6**, 7908 (2015)

What is observed?

Theorem (Observable objectivity)

Let $\Lambda : D(A) \to D(F_1 \otimes \cdots \otimes F_n)$ be a quantum channel and Λ_j the reduced channel to the subsystem B_j. Given $0 < \delta < 1$, there exists a POVM measurement $\{M_x\}$) and a set P of subsystems of the environment of size $|P| \geq (1-\delta)n$ such that for all $j \in P$:

$$\|\Lambda_j - \mathcal{M}_j\|_\diamond \leq \left(27 \ln 2 \frac{d_A^6 \ln d_A}{n\delta^3}\right)^{1/3} , \tag{11}$$

with:

$$\mathcal{M}_j(\rho) = \sum_x \mathrm{tr}(M_x \rho)\sigma_{j,k} , \tag{12}$$

for states $\sigma_{j,k} \in D(B_j)$ and d_A the dimension of the space A.

Take home message

Shared information among many observers about a quantum system is essentially *classical* and about a *common* measurement.

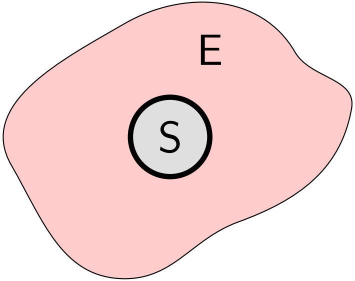
Contents

Few words about decoherence and open systems

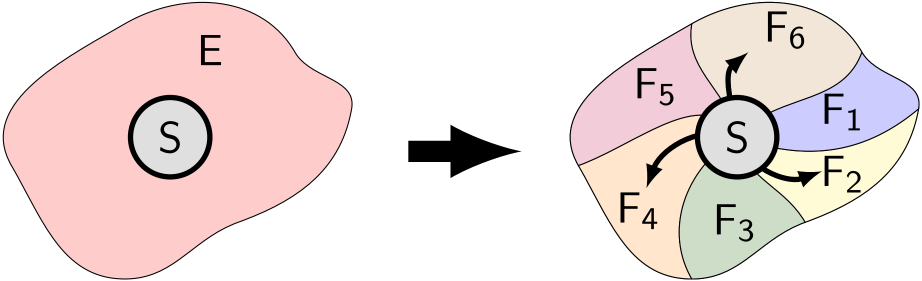
- Open quantum system: decoherence from entanglement with an unmonitored environment E.
- Einselection: selection of pointer/classical states by H_{SE}. Robust to the interaction with E:

$$|s\rangle\,|0\rangle_E \to |s(t)\rangle\,|E_s(t)\rangle \qquad \langle E_s(t)|E_{s'}(t)\rangle \underset{t\gg t_d}{\approx} 0\,.$$

- Important modeling work to do: sources of decoherence/errors in quantum systems, fundamental deviations for quantum theory?



Considering many observers



Objectivity in information theoretic terms [Quantum Darwinism, Zurek]:

$$I(S, F_f) = S[S], \quad F_f \text{ subset of fragments of relative size } f\,.$$

Typical scaling of the mutual information

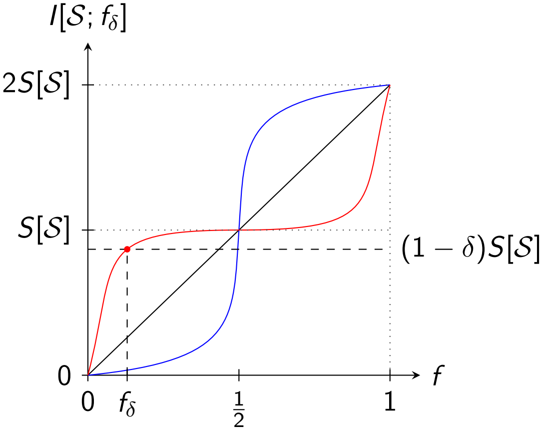


Figure: Typical scaling of the mutual information as a function of the fragment relative size (blue: random state; red: Darwinian state with plateau).

Simple examples

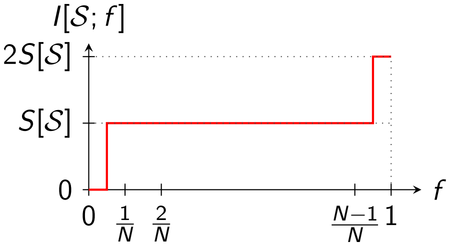


Figure: Mutual information with N qubits coupled to the qubit system with a `cnot` type interaction.

Simple examples

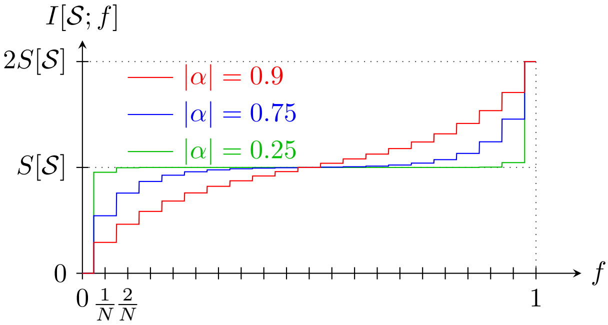


Figure: Mutual information with $20 + 1$ qubits with the "non-perfect" coupling for different values of α.

Simple examples

- Random state. Average entropy of a subsystem of size m in a space of size md [Page]:

$$S[m,d] = \sum_{k=d+1}^{md} \frac{1}{k} - \frac{m-1}{2d}.$$

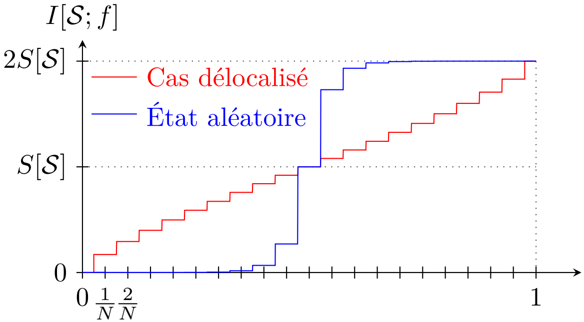
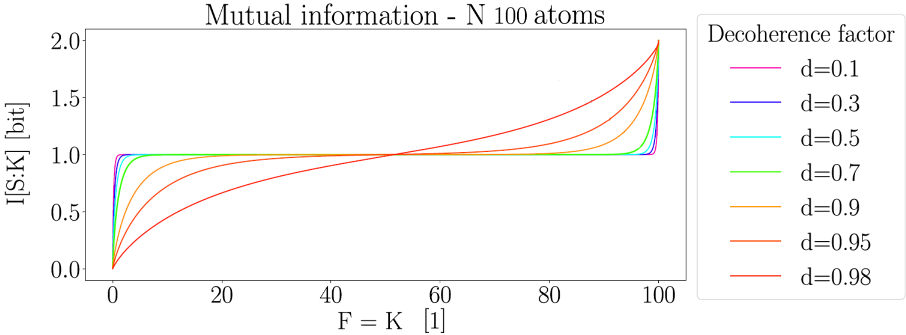


Figure: Mutual information curves for a random and "delocalized" state.

Cavity QED: atoms as fragments

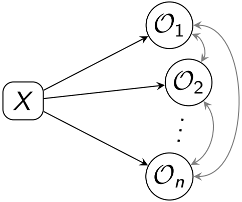


Take home message

Objectivity of outcome is achieved when many records are accessible *independently* by many observers.

An observer is a *complex* system.

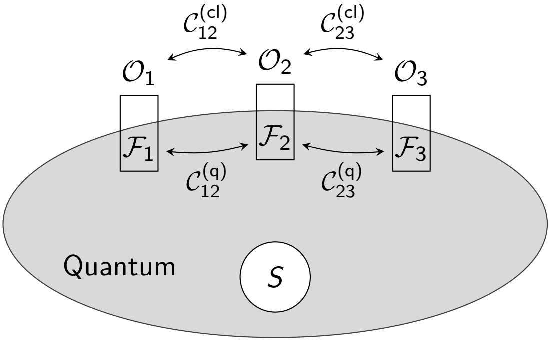
Information vs correlations vs reconstruction



II. Reconstruction of classical picture?

- Brandao's theorem does not inform us about outcomes precisely.
- Zurek's approach suggests to characterize who can be an observer.

A generic resource-based picture

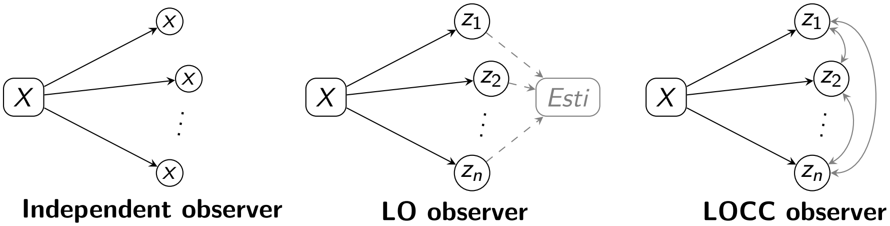


How is it observed?

- Assume collective recovery of X:

$$\rho_{XF_1\dots F_n} = \sum_x p(x)\,|x\rangle\langle x| \otimes \rho^{(x)}_{F_1\dots F_n} \tag{13}$$

- What **ressources** are needed to reconstruct a common x?



How is it observed?

Theorem (Independent observers)

Let $\rho_{XF_1\dots F_n}$ be a state as in (13). Then the following conditions are equivalent:

1. ***Information**: $I_{acc}(X, F_i) = S(X)$ for all $i \in \{1, \dots, n\}$.*
2. ***Reconstruction**: $P_{guess}(X|F_i) = 1$ for all $i \in \{1, \dots, n\}$.*
3. ***Structure**: For all $i \in \{1, \dots, n\}$, there exists an isometry W_i (i.e., $W_i^\dagger W_i = I$) that maps the space F_i to $\bar{X}_i \otimes N_i$, where $\bar{X}_i$ is isomorphic to X, such that:*

$$(\bigotimes_{i=1}^{n} W_i)\rho_{XF_1\dots F_n}(\bigotimes_{i=1}^{n} W_i^\dagger) = \sum_x p(x)\,|x\rangle\langle x|_X \otimes \left(\bigotimes_{i=1}^{n} |x\rangle\langle x|_{\bar{X}_i}\right) \otimes \rho^{(x)}_{N_1\dots N_n}$$

A hierachy of objectivity

$I_{\text{acc,LOCC}}(X) \geq$ Adaptive meas.	$I_{\text{acc,LO}}(X) \geq$ Sharing meas.	$I_{\text{acc}}(F_i)$ Indep. estimation	Example states
$S[X]$	$S[X]$	$S[X]$	F_i have a copy of X
$S[X]$	$S[X]$	0	(secret sharing)
$S[X]$	ϵ	ϵ	(information locking)
ϵ	ϵ	ϵ	(data hiding)

A. Feller, B. Roussel, I. Frérot and P. Degiovanni. Physical Review Letters **126**, 188901 (2021)
A. Feller, B. Roussel, I. Frérot, O.Fawzi and P. Degiovanni, ESA (2021)

Conclusion-Perspectives

- Concrete models, experimental tests?
- What is an observer? Role of resources?
- Classicality emerges at a meso scale?
- Single outcome? Classicality vs interpretation?