A beginner's guide to the landscape of GPTs

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Outline

- Motivation
- Operational Theory
- Generalized Probabilistic Theories (GPTs)
- Further reading

Motivation

To better understand the *probabilistic* features of quantum theory within a broad landscape of probabilistic theories

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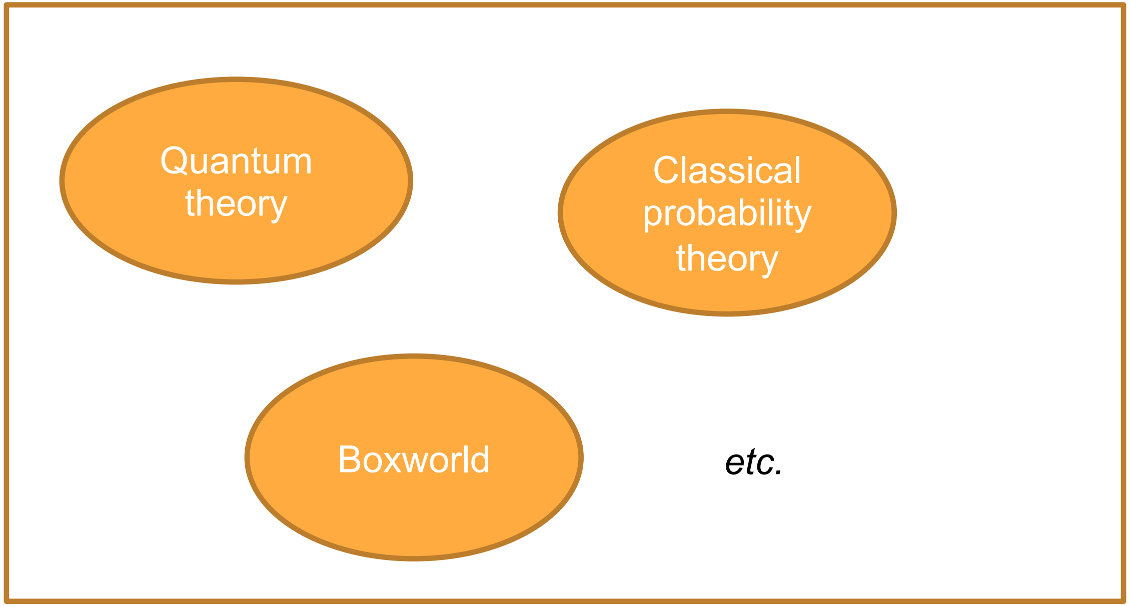
- Probabilities, however, do not *require* quantum theory, *cf.* classical probabilistic theories, but also more generally
- A general framework for probabilistic theories? GPTs!

Motivation


Quantum theory
Classical probability theory
Boxworld
etc.

Motivation

What sorts of questions can GPTs answer?

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- quantum foundations sans interpretational baggage (mostly); a form of operationalism
- information-theoretic axiomatizations of quantum theory, *e.g.,* Hardy 2001 (quant-ph/0101012), Masanes-Müller 2011 (arXiv:1004.1483)
- quantum-like phenomena beyond quantum theory, *e.g.,* Barrett 2007 (quant-ph/0508211)

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- how much ‘quantumness’ do quantum protocols really require?

Motivation

Not an exhaustive list!

Physical / mathematical as another axis of motivations

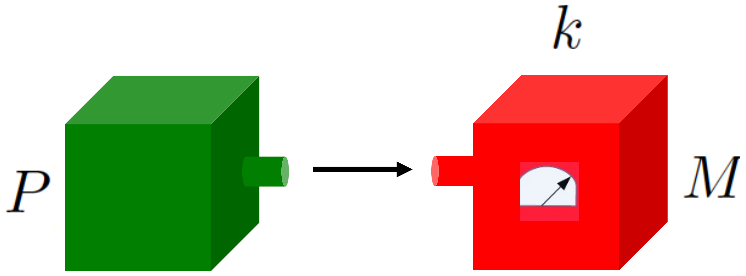
Motivation

This tutorial:

Bottom-up, physical perspective, motivated by quantum foundations, not targeting specific results (too many of them!)

Operational Theory

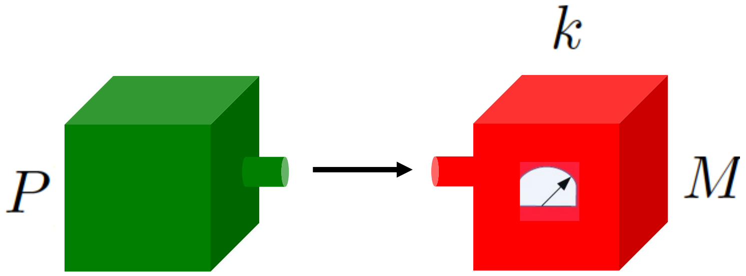
Operational primitives, blackbox view, care about observed data



$$p(k|M,P)$$

Generalized Probabilistic Theory (GPT)

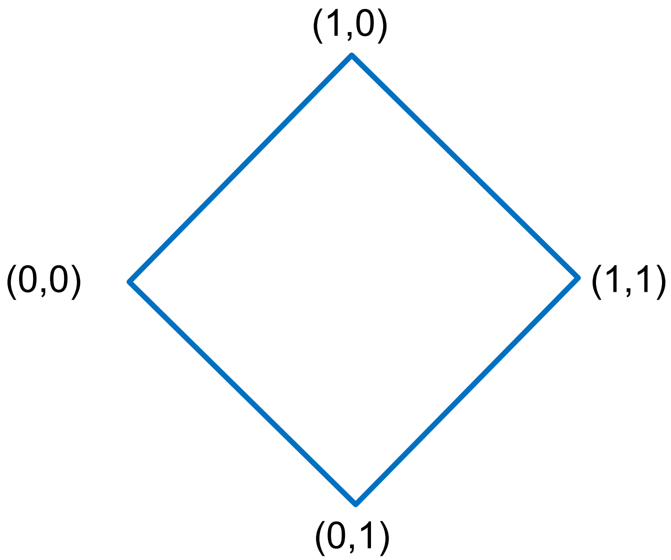
obtained by quotienting the operational theory w.r.t. its operational equivalences



$$p(k|M,P) = \langle \vec{e}_{[k|M]}, \vec{s}_P \rangle$$

Continue on the board

Examples of GPTs: simplicial (bit)



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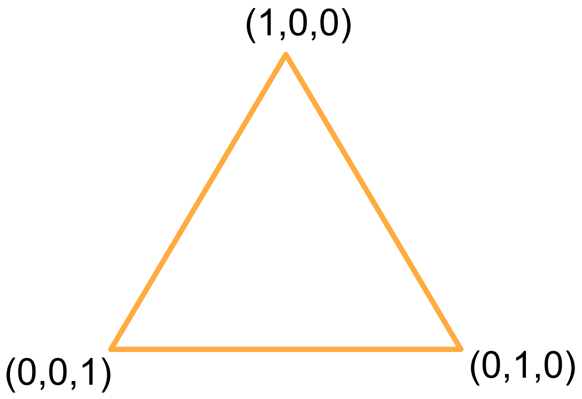
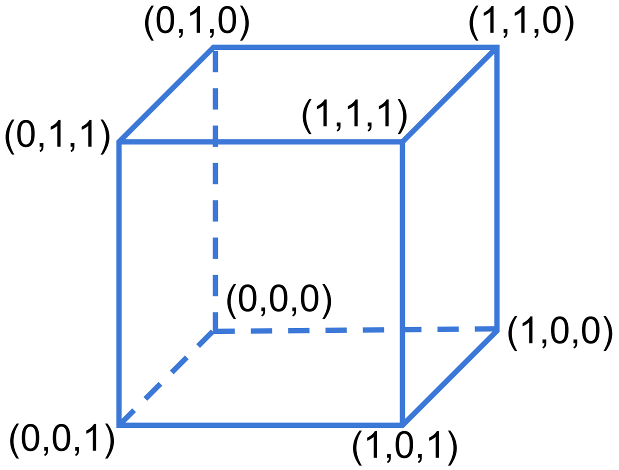
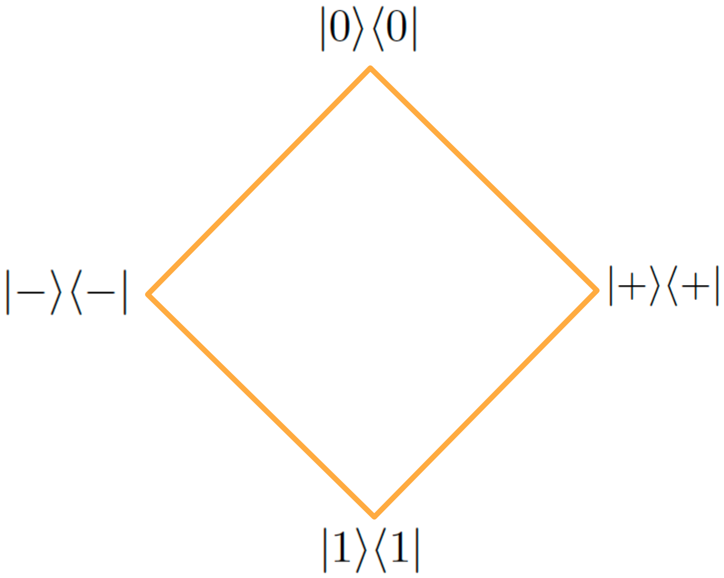
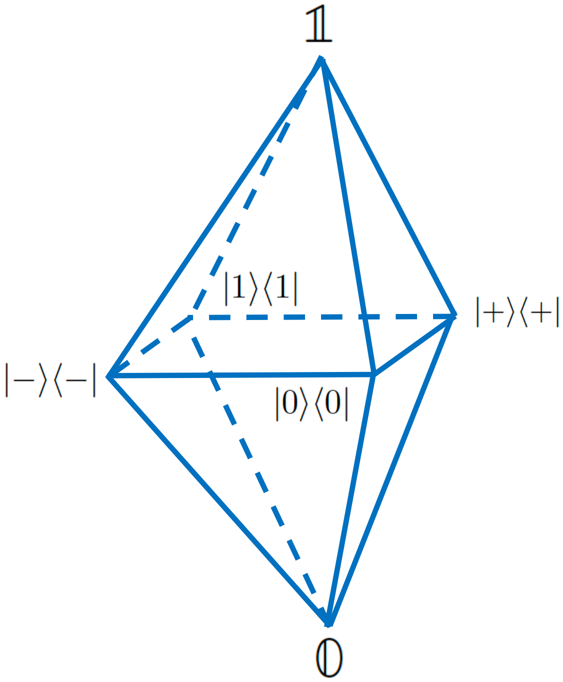




Examples of GPTs: non-simplicial (stabilizer rebit)





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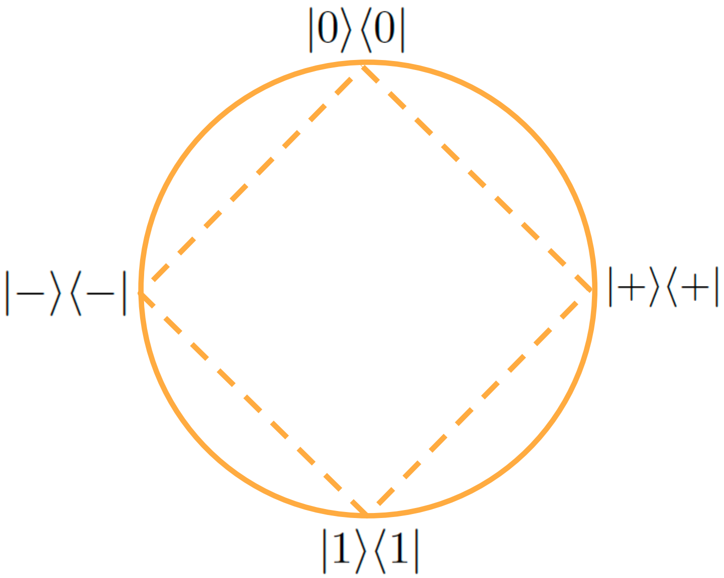

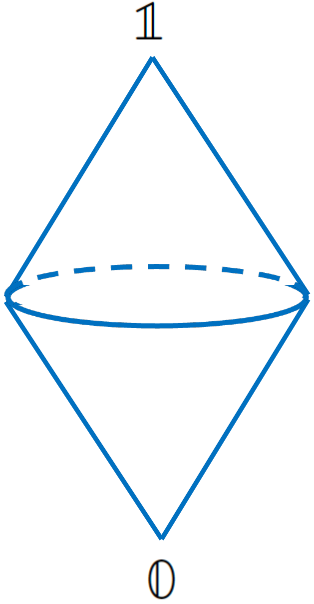

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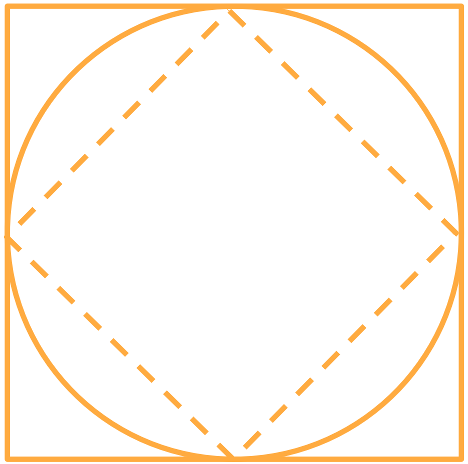
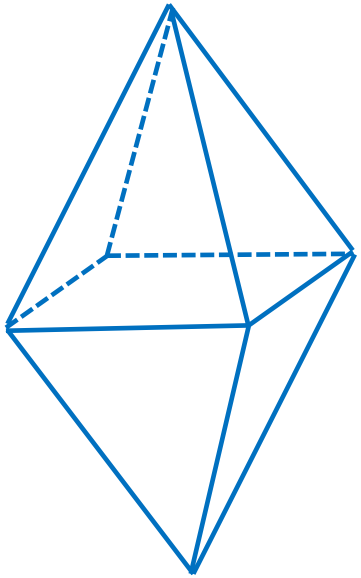


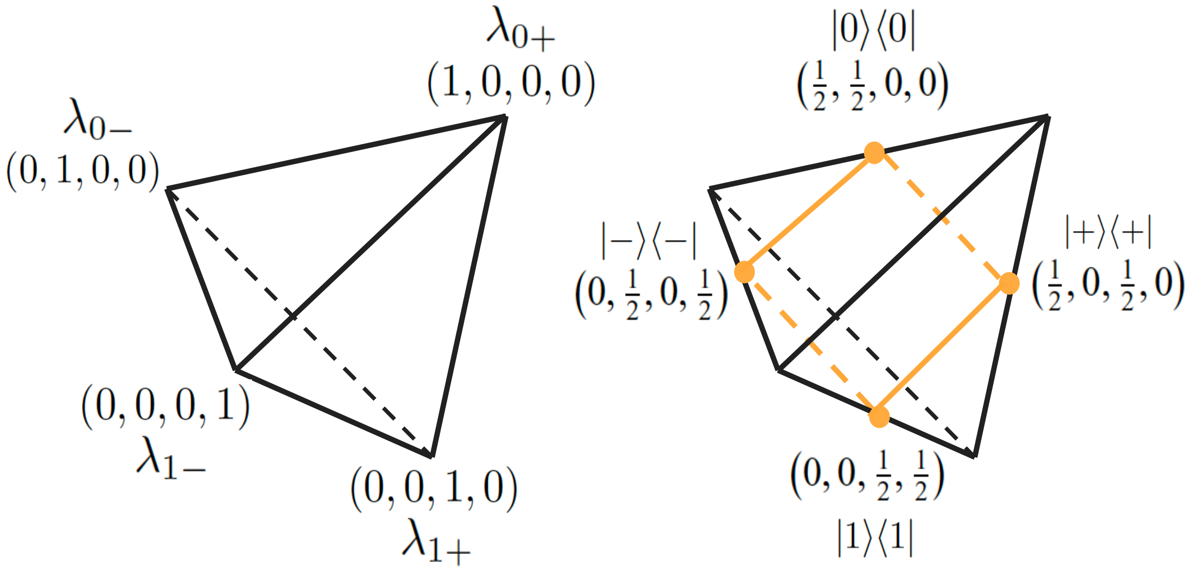


A classification of GPTs according to their (non)classicality

Simplicial ("strictly classical")	Simplex-embeddable ("weakly nonclassical")	Not simplex-embeddable ("strongly nonclassical")
Simplex + Dual	Simplex + Dual + Restriction	All other GPTs
Unique convex decomposition All measurements compatible	Non-unique convex decomposition Incompatible measurements, *etc.* But no contextuality!	Generalized contextuality!

[Ref: Schmid *et al.*, arXiv:1911.10386]

Simplex-embeddability of stabilizer rebit as a GPT



Effect space similarly embedded within a 4-dimensional hypercube with 2^4 vertices

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- The framework assumes an implicit 'Heisenberg cut', not obvious it can do justice to extended Wigner Friend scenarios of the Frauchiger-Renner type (see, however, arXiv: 1904.06247)
- Operationalism as a pragmatic tool vs. operationalism as a philosophy of physics
- Doesn't resolve interpretational issues but does provide a useful 'outside' perspective

(arXiv:1401.7254)

OPINION

Quantum Theory Needs No 'Interpretation' besides Ours

Christopher A. Fuchs and Asher Peres

Recently there has been a spate of articles, reviews, and letters in PHYSICS TODAY promoting various "interpretations" of quantum theory of our experimental activity, then we must be prepared for that, too.

The thread common to all the nonstandard "interpretations" is the carry an umbrella. Probability theory is simply the quantitative formulation of how to make rational decisions in the face of uncertainty.

Further reading

- Quantum theory: informational foundations and foils, Chiribella and Spekkens, introductory chapter (arXiv:1805.11483), and chapters 4,5,6,8
- Some negative remarks on operational approaches to quantum theory, Fuchs and Stacey (Ch. 8 above), (arXiv:1401.7254)
- Probabilistic theories and reconstructions of quantum theory, Müller (arXiv:2011.01286)
- Generalized Probabilistic Theories without the no-restriction hypothesis, Janotta and Lal (arXiv:1302.2632)
- Experimentally bounding deviations from quantum theory in the landscape of generalized probabilistic theories, Mazurek *et al.* (arXiv:1710.05948)

Further reading

- Quantum mechanics as quantum information (and only a little more), Fuchs (arXiv:quant-ph/0205039)
- Information processing in generalized probabilistic theories, Barrett (arXiv:quant-ph/0508211)
- The computational landscape of general physical theories, Barrett *et al.* (arXiv:1702.08483)
- Characterization of noncontextuality in the framework of generalized probabilistic theories (arXiv:1911.10386)
- Multi-agent paradoxes beyond quantum theory (arXiv: 1904.06247)

Merci!