Entanglement from deformed tori

Péter Lévay

January 7, 2012

Plan of the talk

1. Motivation: the black hole-qubit correspondence

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4. Black Hole Entropy and entanglement measures

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2. Qubits from deformed tori.
3. Fermionic entanglement from deformed tori
4. Black Hole Entropy and entanglement measures
5. Application: SLOCC classes of three fermions with six single particle states
6. Conclusions

Péter Lévay: "*Qubits from extra dimensions*", Physical Review D82, 125020 (2011)

The Black Hole-Qubit Correspondence

Recently a striking correspondence has been discovered between two seemingly unrelated fields.

1. Black hole solutions in String Theory (**ST**)

The basic correspondence is between black hole entropy formulas of extremal black hole solutions in string theory and formulas for *pure state* entanglement measures of multipartite entangled systems with both distinguishable and indistinguishable constituents. Though the physical basis of the correspondence (if any) is still unknown it has repeatedly proved to be useful for obtaining interesting results on both sides. **In this talk we emphasize the QI theoretic side, with occasional references to the ST side.**

The Black Hole-Qubit Correspondence

Recently a striking correspondence has been discovered between two seemingly unrelated fields.

1. Black hole solutions in String Theory (**ST**)
2. Multipartite entanglement in Quantum Information (**QI**)

The basic correspondence is between black hole entropy formulas of extremal black hole solutions in string theory and formulas for *pure state* entanglement measures of multipartite entangled systems with both distinguishable and indistinguishable constituents. Though the physical basis of the correspondence (if any) is still unknown it has repeatedly proved to be useful for obtaining interesting results on both sides. **In this talk we emphasize the QI theoretic side, with occasional references to the ST side.**

Motivation: Entanglement from homology and cohomology

Main idea: Wrapped mem**brane**s around **homology cycles** of extra dimensions should give rise to **qubits**.

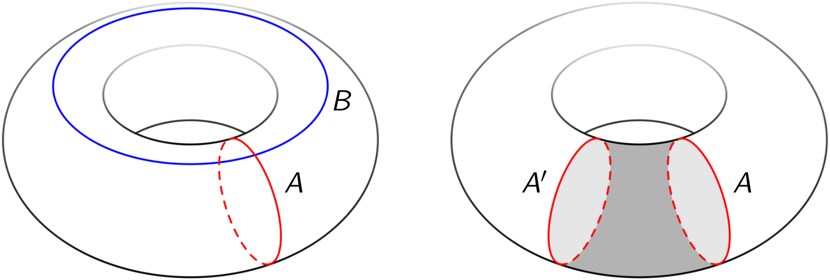
"*To wrap or not to wrap that is the qubit*" (M. J. Duff).

We would like to make this idea precise by obtaining simple entangled systems from the **cohomology** of the extra dimensions. Here we consider merely **tori**.

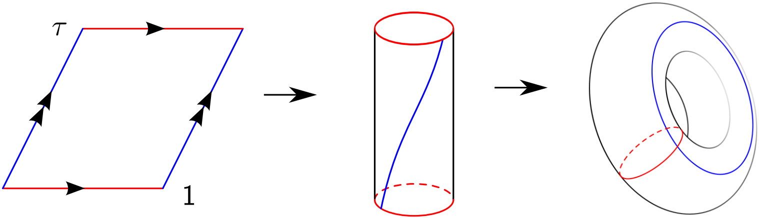
Wrapped brane configurations with different winding numbers are known to give rise to **charges** of both **electric** and **magnetic** type in our the $4D$ low energy world. Such winding configurations account for the **microstates** of certain **elementary** black hole solutions.

The **shape** and **size** of the extra dimensions is subject to quantum fluctuations. These fluctuations can be described by scalar **fields** called **moduli**. Here we show how we can construct and use **pure entangled states** depending on both the **charges** and the **moduli**.

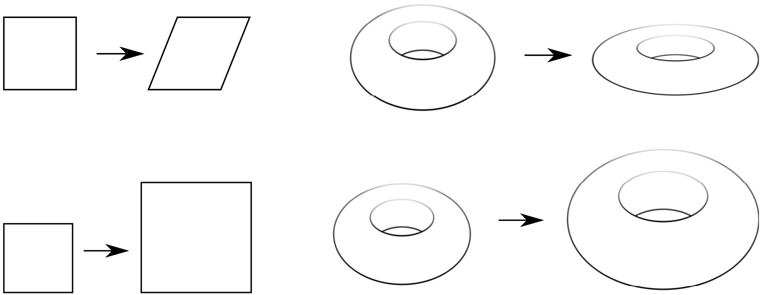
Homology base



The torus T^2 arising from a lattice of $\mathbb{C}$



Complex structure and Kähler structure deformations



One-qubit systems from deformed tori

Consider T^2 with its deformations labelled by

$$\tau \equiv x - iy \qquad y > 0$$

The complex coordinate on T^2 is $z = u + \tau v$. The space of deformations $\mathcal{M}$ has a Kähler metric

$$ds^2_{\mathcal{M}} = 2G_{\tau\overline{\tau}} d\tau d\overline{\tau} = \frac{dx^2 + dy^2}{2y^2}$$

$$G_{\tau\overline{\tau}} = \partial_\tau \partial_{\overline{\tau}} K = \frac{1}{4y^2} \qquad K = -\log(2y).$$

Define

$$\Omega_0 \equiv dz = du + \tau dv \in H^{1,0}(T^2, \mathbb{C})$$

Then by virtue of

$$\int_{T^2} du \wedge dv = 1$$

$$\int_{T^2} \Omega_0 \wedge \overline{\Omega}_0 = ie^{-K}$$

The volume form on T^2 is

$$\omega = i d\overline{z} \wedge dz.$$

The Hodge star $*$ is acting as

$$*dz = idz, \qquad *d\overline{z} = -id\overline{z}$$

In order to reinterpret one-forms on T^2 as **qubits** we use the hermitian inner product

$$\langle \xi | \eta \rangle \equiv \int_{T^2} \xi \wedge *\overline{\eta}, \qquad \xi, \eta \in H^1(T^2, \mathbb{C})$$

Define the one-form Ω as

$$\Omega \equiv e^{K/2} \Omega_0$$

then the correspondence

$$i\Omega \leftrightarrow |0\rangle, \qquad i\overline{\Omega} \leftrightarrow |1\rangle$$

defines the orthonormal computational base.

Remark: Notice that the symbols $|0\rangle$ and $|1\rangle$ would rather define a **family of basis states** labelled by the complex deformation parameter τ. Hence the notation $|0,\tau\rangle$, $|1,\tau\rangle$ would be more appropriate.
Define the flat covariant derivative as

$$D_{\hat{\tau}}\Omega \equiv (\overline{\tau}-\tau)D_\tau\Omega \equiv (\overline{\tau}-\tau)\left(\partial_\tau + \frac{1}{2}\partial_\tau K\right)\Omega = \overline{\Omega}$$

$$D_{\hat{\tau}}\overline{\Omega} \equiv (\overline{\tau}-\tau)\left(\partial_\tau - \frac{1}{2}\partial_\tau K\right)\overline{\Omega} = 0$$

Then

$$D_{\hat{\tau}} \leftrightarrow \sigma_+, \qquad D_{\hat{\overline{\tau}}} \leftrightarrow \sigma_-, \qquad * \leftrightarrow -\sigma_3$$

Hence the covariant derivatives act as **projective bit flip errors**, and the Hodge star as the negative of the **parity check** operator.

In the case of wrapped branes as qubits the homology classes are **real** hence $\Gamma \in H^1(T^2, \mathbb{R})$

$$\Gamma = p\alpha - q\beta, \qquad \alpha = du, \quad \beta = dv.$$

One can express this in the **Hodge diagonal basis** as

$$\Gamma = -e^{K/2}(p\overline{\tau} + q)i\Omega + e^{K/2}(p\tau + q)i\overline{\Omega}.$$

According to our correspondence between one-forms and qubits we can represent this as a state in the computational base satisfying an extra reality condition

$$|\Gamma\rangle = \Gamma_0|0\rangle + \Gamma_1|1\rangle, \qquad \Gamma_1 = -\overline{\Gamma}_0 = e^{K/2}(p\tau + q). \qquad (1)$$

The state $|\Gamma\rangle$ is **unnormalized** with norm squared satisfying

$$||\Gamma||^2 = \langle\Gamma|\Gamma\rangle = 2e^K|p\tau + q|^2 = \frac{1}{y}|p\tau + q|^2$$

It is a unitary and a symplectic i.e. $SL(2,\mathbb{R})$ invariant at the same time. The latter means that under the set of combined transformations

$$\tau \mapsto \frac{a\tau + b}{c\tau + d}, \qquad \begin{pmatrix} -p \\ q \end{pmatrix} \mapsto \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} -p \\ q \end{pmatrix} \tag{2}$$

$||\Gamma||^2$ remains invariant.

Notice also that in matrix representation the state $|\Gamma\rangle$ can be given the form

$$\begin{pmatrix} \Gamma_0 \\ \Gamma_1 \end{pmatrix} = \frac{1}{\sqrt{2y}} \begin{pmatrix} \overline{\tau} & -1 \\ -\tau & 1 \end{pmatrix} \begin{pmatrix} -p \\ q \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & -1 \\ i & 1 \end{pmatrix} \frac{1}{\sqrt{y}} \begin{pmatrix} y & 0 \\ -x & 1 \end{pmatrix} \begin{pmatrix} -p \\ q \end{pmatrix}$$

With new notational conventions our state can be given the deceptively simple appearance

$$|\Gamma\rangle = \mathcal{S}|\gamma\rangle = US|\gamma\rangle, \qquad |\gamma\rangle = -p|0\rangle + q|1\rangle$$

Coordinates

$$z^a = u^a + \tau^a v^a, \qquad \tau^a = x^a - iy^a \qquad y^a > 0, \qquad a = 1, 2, 3$$

Holomorphic three-form

$$\Omega_0 = dz^1 \wedge dz^2 \wedge dz^3.$$

We have as usual

$$\int_{T^6} \Omega_0 \wedge \overline{\Omega}_0 = i(8y^1y^2y^3) = ie^{-K}$$

$$G_{a\overline{b}} = \partial_a \partial_{\overline{b}} K$$

is a Kähler metric on the manifold $\mathcal{M} \simeq [SL(2,\mathbb{R})/SO(2)]^{\times 3}$ The flat covariant derivatives are

$$D_{\hat{a}}\Omega = (\overline{\tau}^a - \tau^a) D_a \Omega = (\overline{\tau}^a - \tau^a) \left(\partial_a + \frac{1}{2} \partial_a K \right) \Omega$$

$T^2 \times T^2 \times T^2$ and three qubits

$$\Omega = e^{K/2} dz^1 \wedge dz^2 \wedge dz^3, \qquad \overline{\Omega} = e^{K/2} d\overline{z}^1 \wedge d\overline{z}^2 \wedge d\overline{z}^3,$$

$$D_{\hat{1}}\Omega = e^{K/2} d\overline{z}^1 \wedge dz^2 \wedge dz^3, \qquad \overline{D}_{\hat{\overline{1}}}\overline{\Omega} = e^{K/2} dz^1 \wedge d\overline{z}^2 \wedge d\overline{z}^3,$$

$$D_{\hat{2}}\Omega = e^{K/2} dz^1 \wedge d\overline{z}^2 \wedge dz^3, \qquad \overline{D}_{\hat{\overline{2}}}\overline{\Omega} = e^{K/2} d\overline{z}^1 \wedge dz^2 \wedge d\overline{z}^3,$$

$$D_{\hat{3}}\Omega = e^{K/2} dz^1 \wedge dz^2 \wedge d\overline{z}^3, \qquad \overline{D}_{\hat{\overline{3}}}\overline{\Omega} = e^{K/2} d\overline{z}^1 \wedge d\overline{z}^2 \wedge dz^3$$

Now we regard the 8 complex dimensional untwisted primitive part of the 20 dimensional space $H^3(T^6, \mathbb{C}) \equiv H^{3,0} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{0,3}$ equipped with the Hermitian inner product

$$\langle \varphi | \eta \rangle \equiv \int_{T^6} \varphi \wedge * \overline{\eta}$$

as a Hilbert space isomorphic to $\mathcal{H} \equiv (\mathbb{C}^2)^{\times 3} \simeq \mathbb{C}^8$ of three qubits.

$T^2 \times T^2 \times T^2$ and three qubits

We define the basis states of our computational base to be given by the correspondence

$$-i\Omega \leftrightarrow |000\rangle \quad -iD_{\hat{1}}\Omega \leftrightarrow |001\rangle \quad -iD_{\hat{2}}\Omega \leftrightarrow |010\rangle \quad -iD_{\hat{3}}\Omega \leftrightarrow |100\rangle$$

$$-i\overline{\Omega} \leftrightarrow |111\rangle \quad -i\overline{D}_{\hat{\hat{1}}}\Omega \leftrightarrow |110\rangle \quad -i\overline{D}_{\hat{\hat{2}}}\Omega \leftrightarrow |101\rangle \quad -i\overline{D}_{\hat{\hat{3}}}\Omega \leftrightarrow |011\rangle$$

$$(D_{\hat{1}}, D_{\hat{2}}, D_{\hat{3}}) \leftrightarrow (I \otimes I \otimes \sigma_+, I \otimes \sigma_+ \otimes I, \sigma_+ \otimes I \otimes I)$$

$$(D_{\hat{\hat{1}}}, D_{\hat{\hat{2}}}, D_{\hat{\hat{3}}}) \leftrightarrow (I \otimes I \otimes \sigma_-, I \otimes \sigma_- \otimes I, \sigma_- \otimes I \otimes I)$$

$$* \leftrightarrow -\sigma_3 \otimes \sigma_3 \otimes \sigma_3$$

Wrapped three-branes on $T^2 \times T^2 \times T^2$

Now for a real three-form representing the cohomology class of a wrapped $D3$ brane configuration we take

$$\Gamma = p^I \alpha_I - q_I \beta^I \in H^3(T^6, \mathbb{Z}),$$

with summation on $I = 0, 1, 2, 3$ and

$$\alpha_0 = du^1 \wedge du^2 \wedge du^3, \qquad \beta^0 = -dv^1 \wedge dv^2 \wedge dv^3$$

$$\alpha_1 = dv^1 \wedge du^2 \wedge du^3, \qquad \beta^1 = du^1 \wedge dv^2 \wedge dv^3$$

with the remaining ones obtained via cyclic permutation. With the choice of orientation

$$\int_{T^6} (du^1 \wedge dv^1) \wedge (du^2 \wedge dv^2) \wedge (du^3 \wedge dv^3) = 1$$

we have

$$\int_{T^6} \alpha_I \wedge \beta^J = \delta_I^J$$

.

A charge and moduli dependent three-qubit state

Define

$$W = q_0 + q_1\tau^1 + q_2\tau^2 + q_3\tau^3 + p^1\tau^2\tau^3 + p^2\tau^1\tau^3 + p^3\tau^1\tau^2 - p^0\tau^1\tau^2\tau^3$$

Using the correspondence between three-forms and three-qubit states we can write $\Gamma \leftrightarrow |\Gamma\rangle$ where

$$|\Gamma\rangle = \Gamma_{000}|000\rangle + \Gamma_{001}|001\rangle + \cdots + \Gamma_{110}|110\rangle + \Gamma_{111}|111\rangle,$$

where

$$\Gamma_{111} = -e^{K/2}W(\tau^3, \tau^2, \tau^1) = \overline{\Gamma}_{000},$$
$$\Gamma_{001} = -e^{K/2}W(\overline{\tau}^3, \overline{\tau}^2, \tau^1) = \overline{\Gamma}_{110}$$

and the remaining amplitudes are given by cyclic permutation. Let us put the 8 charges p^I and q_I with $I = 0, 1, 2, 3$ to a $2 \times 2 \times 2$ array γ_{kji} $k, j, i = 0, 1$ as follows

$$\begin{pmatrix} \gamma_{000} & \gamma_{001} & \gamma_{010} & \gamma_{100} \\ \gamma_{111} & \gamma_{110} & \gamma_{101} & \gamma_{011} \end{pmatrix} = \begin{pmatrix} -p^0 & -p^1 & -p^2 & -p^3 \\ -q_0 & q_1 & q_2 & q_3 \end{pmatrix}$$

An alternative form

The three-qubit state $|\Gamma\rangle$ can alternatively be written in the following form

$$|\Gamma\rangle = \mathcal{S}_3 \otimes \mathcal{S}_2 \otimes \mathcal{S}_1 |\gamma\rangle$$

$$|\gamma\rangle = \gamma_{000}|000\rangle + \gamma_{001}|001\rangle + \cdots + \gamma_{110}|110\rangle + \gamma_{111}|111\rangle$$

and the matrix representative of the operator $\mathcal{S}_3 \otimes \mathcal{S}_2 \otimes \mathcal{S}_1$ is

$$\frac{1}{\sqrt{8y^3y^2y^1}} \begin{pmatrix} \overline{\tau}^3 & -1 \\ -\tau^3 & 1 \end{pmatrix} \otimes \begin{pmatrix} \overline{\tau}^2 & -1 \\ -\tau^2 & 1 \end{pmatrix} \otimes \begin{pmatrix} \overline{\tau}^1 & -1 \\ -\tau^1 & 1 \end{pmatrix}$$

One-half the norm $\frac{1}{2}||\Gamma||^2$ is

$$e^K(|W(\tau^3,\tau^3,\tau^1)|^2 + |W(\overline{\tau}^3,\tau^2,\tau^1)|^2 + W(\tau^3,\overline{\tau}^2,\tau^1)|^2 + |W(\tau^3,\tau^2,\overline{\tau}^1)|^2$$

This quantity in the string theoretical context can be reinterpreted as the charge and moduli dependent **Black Hole Potential** V_{BH}.

Fermionic entanglement and T^6

We choose analytic coordinates for the complex torus such that the holomorphic one-forms are defined as

$$dz^a = du^a + \tau^{ab} dv^b$$

where now $\tau^{ab}, \quad 0 \leq a, b \leq 3$ is the period matrix of the torus with the convention

$$\tau^{ab} = x^{ab} - iy^{ab}. \tag{3}$$

For principally polarized Abelian verieties we have the additional constraints

$$\tau^{ab} = \tau^{ba}, \qquad y^{ab} > 0.$$

We choose as usual $\Omega_0 = dz^1 \wedge dz^2 \wedge dz^3$, and the orientation

$$\int_{T^6} du^1 \wedge dv^1 \wedge du^2 \wedge dv^2 \wedge du^3 \wedge dv^3 = 1$$

.

Now we exploit the full 20 dimensional space of $H^3(T^6, \mathbb{C})$. We expand $\Gamma \in H^3(T^6, \mathbb{C})$ in the basis

$$\alpha_0 = du^1 \wedge du^2 \wedge du^3, \qquad \alpha_{ab} = \frac{1}{2}\varepsilon_{aa'b'} du^{a'} \wedge du^{b'} \wedge dv^b$$

$$\beta^0 = -dv^1 \wedge dv^2 \wedge dv^3, \qquad \beta^{ab} = \frac{1}{2}\varepsilon_{ba'b'} du^a \wedge dv^{a'} \wedge dv^{b'}.$$

Now

$$\Omega_0 = \alpha_0 + \tau^{ab}\alpha_{ab} + \tau^{\sharp}{}_{ab}\beta^{ba} - (\mathrm{Det}\tau)\beta^0,$$

where $\tau\tau^{\sharp} = \mathrm{Det}(\tau)\mathbf{1}$.
An element Γ of $H^2(T^6, \mathbb{C})$ can be expanded as

$$\Gamma = p^0\alpha_0 + P^{ab}\alpha_{ab} - Q_{ab}\beta^{ab} - q_0\beta^0. \qquad (4)$$

We can rewrite this as

$$\Gamma = \sum_{1 \leq A < B < C \leq 6} \gamma_{ABC} f^A \wedge f^B \wedge f^C$$

Here

$$(f^1, f^2, f^3, f^4, f^5, f^6) = (du^1, du^2, du^3, dv^1, dv^2, dv^3).$$

and γ_{ABC} is a completely antisymmetric tensor of rank three with 20 independent components. Using

$$(1, 2, 3, 4, 5, 6) \leftrightarrow (1, 2, 3, \overline{1}, \overline{2}, \overline{3})$$

the identification between the components of γ_{ABC} and the charges is

$$p^0 = \gamma_{123}, \qquad \begin{pmatrix} P^{11} & P^{12} & P^{13} \\ P^{21} & P^{22} & P^{23} \\ P^{31} & P^{32} & P^{33} \end{pmatrix} = \begin{pmatrix} \gamma_{23\overline{1}} & \gamma_{23\overline{2}} & \gamma_{23\overline{3}} \\ \gamma_{31\overline{1}} & \gamma_{31\overline{2}} & \gamma_{31\overline{3}} \\ \gamma_{12\overline{1}} & \gamma_{12\overline{2}} & \gamma_{12\overline{3}} \end{pmatrix},$$

$$q^0 = -\gamma_{\overline{123}}, \qquad \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{pmatrix} = \begin{pmatrix} \gamma_{1\overline{23}} & \gamma_{1\overline{31}} & \gamma_{1\overline{12}} \\ \gamma_{2\overline{23}} & \gamma_{2\overline{31}} & \gamma_{2\overline{12}} \\ \gamma_{3\overline{23}} & \gamma_{3\overline{31}} & \gamma_{3\overline{12}} \end{pmatrix}.$$

Γ can also be regarded as an unnormalzed three fermion state with six single particle states.

Fermionic entanglement and T^6

Using new moduli dependent basis vectors

$$e^A = f^{A'} S_{A'}{}^A, \qquad S_{A'}{}^A = \begin{pmatrix} I & I \\ \tau & \overline{\tau} \end{pmatrix}$$

one can write

$$\Gamma = \frac{1}{6!} \Gamma_{A'B'C'} \left(-ie^{K/2} e^{A'} \wedge e^{B'} \wedge e^{C'} \right),$$

where

$$\Gamma_{A'B'C'} = \mathcal{S}_{A'}{}^A \mathcal{S}_{B'}{}^B \mathcal{S}_{C'}{}^C \gamma_{ABC},$$

and

$$\mathcal{S} \equiv -ie^{-K/6} S^{-1} = -ie^{-K/6} (\tau - \overline{\tau})^{-1} \begin{pmatrix} -\overline{\tau} & I \\ \tau & -I \end{pmatrix}.$$

Fermionic entanglement and T^6

Notice also that the basis states

$$E^A \wedge E^B \wedge E^C \equiv -ie^{K/2} e^A \wedge e^B \wedge e^C \qquad 1 \leq A < B < C \leq 6,$$

now form an orthonormal basis with respect to the usual Hermitian inner product

$$\langle \varphi | \eta \rangle \equiv \int_{T^6} \varphi \wedge * \overline{\eta}, \quad \varphi, \eta \in H^3(T^6, \mathbb{C})$$

It is also important to realize that now we have the *same* matrix $\mathcal{S} \in GL(6, \mathbb{C})$ acting on all indices of γ_{ABC}.
This reflects the fact known from the theory of quantum entanglement that the SLOCC group for a quantum system consisting of *indistinguishable* subsystems is represented by the same $GL(6, \mathbb{C})$ matrix acting on each entry of a tensor representing the set of amplitudes of the system.

$T^2 \times T^2 \times T^2$ alias three qubits embedded in T^6 as three fermions

How do we recover the case of three qubits? In order to see this just notice that in the three qubit case we merely have γ_{ABC} with labels $123, 12\overline{3}, \dots, \overline{12}3, \overline{123}$. Moreover, the 3×3 matrix τ is now diagonal, hence the form of $\mathcal{S}$ is

$$\mathcal{S} = \frac{1}{2} e^{-K/6} \begin{pmatrix} -\overline{\tau}^1/y^1 & 0 & 0 & 1/y^1 & 0 & 0 \\ 0 & -\overline{\tau}^2/y^2 & 0 & 0 & 1/y^2 & 0 \\ 0 & 0 & -\overline{\tau}^3/y^3 & 0 & 0 & 1/y^3 \\ \tau^1/y^1 & 0 & 0 & -1/y^1 & 0 & 0 \\ 0 & \tau^2/y^2 & 0 & 0 & -1/y^2 & 0 \\ 0 & 0 & \tau^3/y^3 & 0 & 0 & -1/y^3 \end{pmatrix}$$

Let us use the correspondence $321 \leftrightarrow 000, 32\overline{1} \leftrightarrow 001$ etc. Looking at the structure of the tensor product $\mathcal{S} \otimes \mathcal{S} \otimes \mathcal{S}$ and recalling that $e^{-K/2} = \sqrt{8y^1y^2y^3}$ we quickly recover the structure of the matrices $\mathcal{S}_3 \otimes \mathcal{S}_2 \otimes \mathcal{S}_1$ known from the three qubit case.

Effective low energy $4D$ field content in the IIB picture

Geometry: $M \times K$ now : $K = T^2 \times T^2 \times T^2$ or T^6

1. $g_{\mu\nu}(x) \leftrightarrow$ describing the $4D$ spacetime geometry of M

$$h^{2,1} \equiv \mathrm{dim} H^{2,1}(K, \mathbb{C})$$

For $T^2 \times T^2 \times T^2$ and T^6 respectively we have

$$h^{2,1} = 3, \qquad h^{2,1} = 9$$

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1. $g_{\mu\nu}(x) \leftrightarrow$ describing the $4D$ spacetime geometry of M
2. $\tau^a(x), a = 1, \ldots h^{2,1}(K) \leftrightarrow$ **volume preserving** fluctuations of K

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4. $\mathcal{F}^I_{\mu\nu}(x), I = 0, 1, \dots h^{2,1}(K) \leftrightarrow$ Maxwell-type fields

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Effective low energy $4D$ field content in the IIB picture

Geometry: $M \times K$ now : $K = T^2 \times T^2 \times T^2$ or T^6

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3. $G_{ab}(x) \leftrightarrow$ metric on the space of deformations $\mathcal{M}$ of K.
4. $\mathcal{F}^I_{\mu\nu}(x), I = 0, 1, \dots h^{2,1}(K) \leftrightarrow$ Maxwell-type fields
5. $\mathcal{N}_{IJ}(\tau(x)) \leftrightarrow$ coupling depending on the deformation fields

$$h^{2,1} \equiv \dim H^{2,1}(K, \mathbb{C})$$

For $T^2 \times T^2 \times T^2$ and T^6 respectively we have

$$h^{2,1} = 3, \qquad h^{2,1} = 9$$

The bosonic part of the $4D$ Effective Action

$$\begin{aligned}\mathcal{S} &= \frac{1}{8\pi G_N}\int d^4x\sqrt{|g|}\{-\frac{R}{2}+G_{a\overline{b}}\partial_\mu\tau^a\partial_\nu\overline{\tau}^{\overline{b}}g^{\mu\nu} \\ &+ (\mathrm{Im}\mathcal{N}_{IJ}\mathcal{F}^I\mathcal{F}^J+\mathrm{Re}\mathcal{N}_{IJ}\mathcal{F}^{I*}\mathcal{F}^J)\}+\dots\end{aligned}$$

Our aim is to solve the Euler-Lagrange equations arising from this action under special conditions.$\mapsto$**Black Hole Solutions**

Extremal Black Hole Solutions

The solutions we are searching for are

1. Static

The solutions we find are of extremal Reissner-Nordström type.
Supersymmetric solitons.

Extremal Black Hole Solutions

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Supersymmetric solitons.

The solutions we are searching for are

1. Static
2. Spherically symmetric
3. Asymptotically Minkowski

The solutions we find are of extremal Reissner-Nordström type.
Supersymmetric solitons.

Extremal Black Hole Solutions

The solutions we are searching for are

1. Static
2. Spherically symmetric
3. Asymptotically Minkowski
4. Extremal

The solutions we find are of extremal Reissner-Nordström type.
Supersymmetric solitons.

The solutions we are searching for are

1. Static
2. Spherically symmetric
3. Asymptotically Minkowski
4. Extremal
5. Supersymmetric (BPS)

The solutions we find are of extremal Reissner-Nordström type.
Supersymmetric solitons.

If we take the ansatz for the space-time metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^{2U(r)} dt^2 + e^{-2U(r)} \left(dr^2 + r^2 d\Omega^2 \right)$$

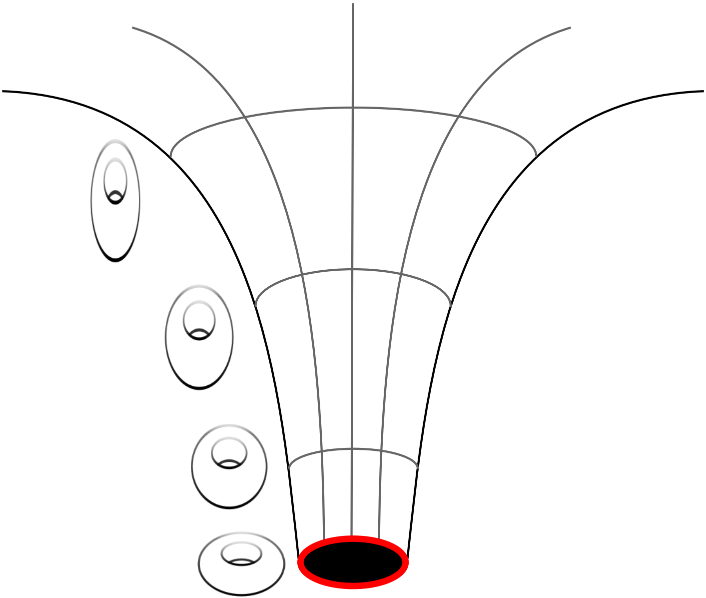
and introduce a spherically symmetric ansatz also for the gauge-fields $\mathcal{F}^I$ with electric and magnetic charges q_I and p^I and employing the new variable

$$\varrho \equiv \frac{1}{r}$$

we get the action ($T \equiv \int dt$ is the elapsed time and dot denotes $\frac{d}{d\varrho}$)

$$S_{4D}/T = \frac{1}{2G_N} \int_0^\infty d\varrho \left(\dot{U}^2 + G_{a\overline{b}} \dot{\tau}^a \dot{\overline{\tau}}^{\overline{b}} + G_N \frac{1}{2} ||\Gamma||^2 \right)$$

The equations derived from this effective action describe the RADIAL dynamics of the space-time warp factor and the fluctuating extra dimensions in the near horizon $\varrho \to \infty$ limit.



Near horizon geometry

The near horizon geometry of the black hole turns out to be $AdS_2 \times S^2$

$$ds^2 = \left(-\frac{4r^2}{G_N||\Gamma||^2_\infty}dt^2 + \frac{G_N||\Gamma||^2_\infty}{4r^2}dr^2\right) + \frac{G_N}{4}||\Gamma||^2_\infty \left(d\theta^2 + \sin^2\theta d\Phi^2\right)$$

The horizon area is

$$A = \pi G_N||\Gamma||^2_\infty$$

hence the thermodynamic Bekenstein-Hawking entropy is

$$S_{BH} = \frac{A}{4G_N} = \frac{\pi}{4}||\Gamma||^2_\infty$$

Solving the equations of motion yields the stabilized values $\tau^a(\infty)$ in terms of the charges p^I and q_I, $I = 0, 1, 2, 3$ which gives the **three-qubit state** on the event horizon

$$|\Gamma\rangle_\infty = (-D)^{1/4}\left(e^{i\alpha}|000\rangle_\infty - e^{-i\alpha}|111\rangle_\infty\right)$$

$$\tan\alpha = \sqrt{-D}\frac{p^0}{2p^1p^2p^3 + p^0(p^0q_0 + p^1q_1 + p^2q_2 + p^3q_3)}$$

$$\begin{aligned} D &= (p^0q_0 + p^1q_1 + p^2q_2 + p^3q_3)^2 \\ &- 4((p^1q_1)(p^2q_2) + (p^2q_2)(p^3q_3) + (p^3q_3)(p^1q_1)) \\ &+ 4p^0q_1q_2q_3 - 4q_op^1p^2p^3 \end{aligned}$$

Now the **entropy** is

$$S_{BH} = \pi\sqrt{-D(|\gamma\rangle)}$$

Note that in the theory of three-qubit entanglement the quantity $\tau_3 = 4|D(|\gamma\rangle)|$ is a genuine entanglement measure of the **charge** state $|\gamma\rangle$.

Black Hole Entropy and the generalization of the three-tangle for Three-Fermions on $\mathbb{C}^6$. T^6

Solving the equations of motion yields the stabilized values $\tau_{ab}(\infty)$ in terms of the 20 charges p^0, q_0, P^{ab} and Q_{ab} $a, b = 1, 2, 3$ which gives the **three-fermion state** on the event horizon

$$\Gamma_\infty = (-\mathcal{D})^{1/4}\left(e^{i\alpha}E^1 \wedge E^2 \wedge E^3 - e^{-i\alpha}E^{\overline{1}} \wedge E^{\overline{2}} \wedge E^{\overline{3}}\right)$$

$$\tan\alpha = \sqrt{-\mathcal{D}}\frac{p^0}{2\mathrm{Det}P + p^0(\mathrm{Tr}(PQ) + p^0 q_0)}$$

$$\mathcal{D} = (p^0 q_0 + \mathrm{Tr}(PQ))^2 - 4\mathrm{Tr}(P^\sharp Q^\sharp) + 4p^0\mathrm{Det}(Q) - 4q_0\mathrm{Det}(P)$$

Now the **entropy** is

$$S_{BH} = \pi\sqrt{-\mathcal{D}(\gamma)}$$

Application: Classification of three fermions with six single particle states

Let $V = \mathbb{C}^6$ and V^* its dual. Let $\{e^A\}$ with $A = 1, \ldots 6$ an orthonormal basis for V^*. Then a three-fermion state is represented by the three-form

$$\gamma = \frac{1}{3!}\gamma_{ABC} e^A \wedge e^B \wedge e^C \in \wedge^3 V^*$$

The SLOCC equivalence classes are defined as

$$\gamma' \sim \gamma \quad \text{iff} \quad \gamma' = (S \otimes S \otimes S)\gamma, \quad S \in GL(6, \mathbb{C})$$

How to obtain the **different** SLOCC orbits?

Idea: use $\mathcal{D}(\gamma)$ occurring in the entropy formula as a genuine tripartite measure similar to the three-tangle in the three-qubit case. Motivation : $\mathcal{D}$ is an $SL(6, \mathbb{C})$ invariant.

Classification of three fermions with six single particle states

Notice that $\mathcal{D}(\gamma)$ can be written in the alternative form

$$\mathcal{D}(\gamma) = -\frac{1}{3}\epsilon^{ABCDEF}\gamma_{ABC}\tilde{\gamma}_{DEF}$$

where $\tilde{\gamma}$ is called **the dual state** of γ

$$\tilde{\gamma}_{ABC} = \frac{1}{72}\epsilon^{KLMK'L'M'}\gamma_{AKL}\gamma_{MBC}\gamma_{K'L'M'}$$

It can be shown that $\tilde{\gamma}$ is a $GL(6,\mathbb{C})$ **covariant** i.e.

$$\tilde{\gamma} \mapsto (S \otimes S \otimes S)\tilde{\gamma}$$

hence $\mathcal{D}(\gamma)$ is an $SL(6,\mathbb{C})$ **invariant** as claimed.

For **normalized states** the analogue of the three-tangle is

$$0 \leq \mathcal{T} \equiv 4|\mathcal{D}(\gamma)| \leq 1$$

Classification of three fermions with six single particle states

P. Lévay and P. Vrana ,Phys. Rev. **A78**, *022329 (2008)*
G. B. Gurevich, Trudy Sem. Vektor. Tenzor. Anal. **6**, *28 (1948)*
There are **four** disjoint SLOCC classes with representatives

$$\gamma = \frac{1}{2}(e^1\wedge e^2\wedge e^3+e^1\wedge e^{\overline{2}}\wedge e^{\overline{3}}+e^{\overline{1}}\wedge e^2\wedge e^{\overline{3}}+e^{\overline{1}}\wedge e^{\overline{2}}\wedge e^3), \quad \mathcal{T}(\gamma)\neq 0$$

$$\gamma = \frac{1}{\sqrt{3}}(e^1\wedge e^2\wedge e^{\overline{3}}+e^1\wedge e^{\overline{2}}\wedge e^3+e^{\overline{1}}\wedge e^2\wedge e^3), \qquad \mathcal{T}(\gamma)=0, \quad \tilde{\gamma}\neq 0$$

$$\gamma = \frac{1}{\sqrt{2}}(e^1\wedge e^2\wedge e^3+e^1\wedge e^{\overline{2}}\wedge e^{\overline{3}}), \qquad \mathcal{T}(\gamma)=0, \quad \tilde{\gamma}=0, \quad \mathcal{P}_{\mathcal{A},\mathcal{B}}(\gamma)\neq 0$$

$$\gamma = e^1\wedge e^2\wedge e^3, \qquad \mathcal{T}(\gamma)=0, \quad \tilde{\gamma}=0, \quad \mathcal{P}_{\mathcal{A},\mathcal{B}}(\gamma)=0$$

$$\mathcal{P}_{\mathcal{A},\mathcal{B}}(\gamma)\equiv\sum_{i=1}^{4}(-1)^{i-1}\gamma_{A_1A_2B_i}\gamma_{B_1B_2B_3B_4\hat{B}_i}$$

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1. We have shown how qubits are emerging from the geometry of tori serving as extra dimensions in IIB compactifications.
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3. Our results provide a framework for understanding the attractor mechanism as a "distillation" procedure.
4. The idea **"Qubits from extra dimensions"** also turn out to be very useful for generalizing our results to flux attractors.
5. For toroidal models one can also show that the natural arena where qubits show up is the realm of fermionic entanglement of indistinguishable constituents. The notion **fermionic** is associated with the structure of p -forms related to p-branes.
6. The idea can also be generalized for Calabi-Yau compactifications. In this case two and three partite systems like a qubit and a qu**d**it, and two qubits and a qu**d**it arise. $d = h^{2,1} + 1$.