Program of the workshop final version Jan. 5, 2012

GEOMETRY OF QUANTUM ENTANGLEMENT CIRM, Marseille, January 2012

**Jan 9 Monday:**

9.00 – 9.35 Marius Junge
9.40 – 10.15 Janusz Grabowski
*Coffee*
10.45 – 11.40 Ion Nechita I
11.45 – 12.20 Adam Sawicki
*LUNCH*
15.00 – 15.35 Remy Mosseri
15.40 – 16.15 Nilanjana Datta
*Coffee*
16.50 – 17.45 Thomas Vidick I
17.50 – 18.25 Andris Ambainis

**Jan 10 Tuesday:**

9.00 – 9.55 Gilles Pisier I
10.00 – 10.35 Peter Levay
*Coffee*
11.00 – 11.55 Michał Horodecki I
12.00 – 12.35 Mary-Beth Ruskai
*LUNCH*
15.00 – 15.55 Thomas Vidick II
16.00 – 16.35 Debbie Leung
*Coffee*
17.05 – 18.00 Ion Nechita II
18.05 – 18.40 Dariusz Chruscinski

**Jan 11 Wednesday**:

9.00 – 9.55 Michał Horodecki II
10.00 – 10.35 Toby Cubitt
*Coffee*
11.00 – 11.55 Gilles Pisier II
12.00 – 12.35 Ashley Montanaro
LUNCH
& **free afternoon !**

**Jan 12 Thursday** :

9.00 – 9.35 Christopher King
9.40 – 10.15 Gniewomir Sarbicki
*Coffee*
10.50 – 11.25 Łukasz Skowronek
11.30– 12.05 Seung-Hyeok Kye
12.10 - 12.30 OPEN PROBLEMS I
*LUNCH*
15.00 – 15.35 Andreas Winter
15.40 – 16.15 Graeme Smith
*Coffee*
16.50 – 17.25 Michał Studziński
17.30 – 18.40 OPEN PROBLEMS II

**Jan 13 Friday**

9.00 – 9.35 Arram Harrow
9.40 – 10.15 Michael Walter
*Coffee*
10.50 – 11.25 Final (surprise) talk
Early LUNCH + departure

# ABSTRACTS

**HORODECKI Michal**, Univ. of Gdansk

Selected results and open problems of quantum entanglement theory

The purpose of this talk is to present selected basic challenges of entanglement theory. The latter is devoted to analysis of properties of quantum states of composite systems (i.e. positive operators of unit trace acting on a tensor product of two or more Hilbert spaces). The basic challenges are of the following sort: given a quantum state of composite system, how to verify whether it is entangled? Given a state, can it be transformed into another by some class of transformations whose definition bases on tensor structure?

The latter question involves some particular problems that have been extensively studied and are still, in general, open. Namely, we can distinguish a so-called "maximally entangled state", and ask whether a given state can be transformed into such state, for various choices of classes of operations. We can also distinguish a subset of states (e.g., private states, motivated by quantum cryptography). In both cases the problem is to characterize sets of states that can be transformed into the distinguished state(s) by means of some given operations.

We shall concentrate on an important variant of these questions assumes that arbitrarily large number of copies of an input states is available, and small errors are tolerated, provided they vanish with growing number of copies (this is motivated by classical Shannon information theory). We shall present the present state-of-art concerning those questions, mainly for bipartite systems.

Another question is relation of entanglement to the so-called local hidden variable models. The latter are made in order to mimic quantum phenomena by classical theory. Entanglement, being exclusively quantum feature, should intuitively lead to a contradiction with such models, by violating the so-called Bell inequalities, which have to be satisfied by any local hidden variable model. It turns out, however, that the relations are more complicated. We shall present some known results and open problems regarding this issue.

**NECHITA Ion**, Univ. de Toulouse

Statistical properties of random quantum channels

We study ensembles of quantum channels from a random matrix theory point of view, with emphasis on the spectral statistics of output states. The entanglement of a single or of an entire subspace of pure states belonging to a tensor product is computed and shown to be relevant to additivity violations for the minimal output entropy. Finally, problems beyond Hastings's counterexample to the additivity conjecture will be discussed.

**PISIER Gilles**, Texas A&M University

Topics on non-commutative Grothendieck theorems

Probably the most famous of Grothendieck's contributions to Banach space theory is the result that he himself described as "the fundamental theorem in the metric theory of tensor products". That is now commonly referred to as "Grothendieck's theorem" (GT in short), or sometimes as "Grothendieck's inequality". This had a major impact first in Banach space theory (roughly after 1968), then, later on, in C*-algebra theory (roughly after 1978). More recently, in this millennium, a new version of GT has been successfully developed in the framework of "operator spaces" or non-commutative Banach spaces. In addition, GT independently surfaced in several quite unrelated fields: in connection with Bell's inequality in quantum mechanics, in graph theory where the Grothendieck constant of a graph has been introduced and in computer science where the Grothendieck inequality is invoked to replace certain NP hard problems by others that can be treated by "semidefinite programming" and hence solved in polynomial time.

We will review the recent connections between Grothendieck's inequality and Bell's, the trilinear Bell type deviation of Junge et al, as well as the operator space versions of Grothendieck's theorem.

**VIDICK Thomas**, MIT CSAIL

Coping with high-dimensional entanglement in multi-player games

A multi-player game is given by an interaction between a (trusted) referee and cooperating players. The only restriction placed on the players is that they are not allowed to communicate for the duration of the game. Multi-player games are an ideal framework in which to study the properties of entanglement, and there are many examples of games known in which players using entanglement can have dramatically higher odds of winning over those of the best classical players.
Finding optimal entangled strategies, or even computing the optimal success probability of entangled players, in such games is made difficult by the absence of upper bounds on the amount of entanglement that may be useful to them. In this talk I will survey some of the specific tools and techniques that have been developed to cope with arbitrarily high dimensional strategies in multiplayer games.

**Andris Ambainis**

On random nonlocal games

We initiate a study of random instances of nonlocal games. We show that quantum strategies are better than classical strategies for random instances of XOR games. Namely, if we consider a random 2-player XOR game with n questions to each player (for large n), the quantum values is at least 1.21... times the classical value, with probability 1-o(1). Several interesting random matrix questions emerge from our work.

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**Dariusz Chruściński**

Non-Markovian dynamics vs. quantum entanglement

We analyze two recently proposed measures of non-Markovianity: one based on the concept of divisibility of the corresponding dynamical map and the other one based on distinguishability of quantum states. It is shown that there is an intricate relation between non-Markovian evolution and quantum entanglement.

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**Toby Cubitt**

The Algebraic Geometry of Entanglement

Abstract: In [1], we completely determined the (algebraic) geometry of subspaces with bounded Schmidt rank. For every pair of local dimensions dA and dB, and every r, we determined the largest dimension of a subspace consisting only of entangled states of Schmidt rank r or larger. We not completely characterised the geometry of such subspaces, but also gave an explicit method for constructing examples, for any dA, dB and r. This result is a simple application of classical algebraic geometry. But it has found a surprising number of applications, from proving additivity violation of minimum output p-Renyi entropies for p close to 0 [2], to proving superactivation of zero-error capacities of quantum channels [3], and even taking a first step towards the final frontier of additivity research: additivity of the classical capacity of a quantum channel, the biggest unresolved additivity question in quantum Shannon theory. I will talk about this simple result, and some of its striking applications.

[1] J. Math. Phys. 49, 022107 (2008)
[2] Comm. Math. Phys. 284, 281-290 (2008)
[3] arXiv:0906.2547[quant-ph] (to appear in IEEE Trans. Inform. Theory)

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**Nilanjana Datta**

How many singlets are needed to create a single copy of a bipartite state using LOCC?

The answer is seen to be given in terms of the Schmidt number of the density matrix, a quantity introduced by Barbara Terhal and Pawel Horodecki. They had shown that k-positive maps witness the Schmidt number, just as positive maps witness entanglement, and established that the Schmidt number is non-increasing under LOCC. We complete the picture by imparting an operational significance to the logarithm of the Schmidt number in the context of entanglement manipulation.

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**Janusz Grabowski**

Segre maps and entanglement for multipartite systems of indistinguishable particles

Abstract: We elaborate the concept of entanglement for multipartite system with bosonic and fermionic constituents and its generalization to systems with arbitrary parastatistics. We use the representation theory of symmetry groups to formulate a unified approach to this problem in terms of simple tensors with appropriate symmetry. For an arbitrary parastatistics, we define the S-rank generalizing the notion of the Schmidt rank. The S-rank, defined for all types of tensors, serves for distinguishing the entanglement of pure states.
The entanglement is characterized also in terms of generalized Segre maps, supplementing thus an algebraic approach to the problem by a more geometric point of view.

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**Aram Harrow**

Different permutations are almost orthogonal

Consider the n! different unitaries that permute n d-dimensional quantum systems. If d>=n, then these are linearly independent. In this talk, I'll explain a sense in which they are approximately orthogonal if d >> n, or if d >> n^2. This simple fact has several implications, which I'll discuss:
1. There is no efficient product test (in the sense of my previous work with Ashley Montanaro) that uses only LOCC measurements between the different copies of the state to be tested.
2. Random maximally entangled states have similar moments to fully random states.
3. Random quantum circuits on n qubits with poly(n) gates are approximate poly(n)-designs. (Joint work with Fernando Brandao and Michal Horodecki).
4. An alternate proof of the Hastings result that random unitaries give quantum expanders.
5. The N-party data-hiding scheme of Eggeling and Werner can be achieved with only poly(N) local dimension.

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**Marius Junge** (a joint work with Carlos Palazuelos)

Additivity and restricted capacity

We introduce a notion of restricted capacity. Here restricted refers to the dimension of a maximal entangled state which is added to increase the (classical) capacity of a channel. We discuss the connection to Gorothendieck's notion of absolutely summing maps and how to find non-additivity results with tools from Compressed Sensing.

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**Chris King**

Properties of typical output states from channels in high dimensions.

Abstract: The concentration of measure phenomenon has been used to analyze typical behavior of high-dimensional random channels, leading to the proof of existence of counterexamples to the additivity conjectures. In this talk another application will be described, concerning the behavior of typical output states from multiple products of a fixed channel, when the number of factors in the product increases. Concentration of measure implies that the output entropy per channel use for typical states will converge to the average output entropy per channel use. Explicit calculations are shown for some classes of examples, and the relation to entanglement of typical output states is also discussed.

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**Seung-Hyeok Kye**

Classification of bi-qutrit PPT entangled edge states by their ranks

Abstract:
We exhibit examples of bi-qutrit PPT (positive partial transpose) entangled edge states with maximal ranks. The ranks of the states and their partial transposes are $8$ and $6$, respectively. This completes the classification of bi-qutrit PPT entangled edge states by their types. Recall that an edge state is said to be of type $(p,q)$ if the ranks of the state itself and its partial transpose are $p$ and $q$, respectively. It turns out that possible types for bi-qutrit PPT edge states are (4,4), (5,5), (6,5), (7,5), (8,5), (6,6), (7,6) and (8,6), if we list up the cases with $p \ge q$ by symmetry. This talk is based on the cowork with Young-Hoon Kiem and Joongseob Lee, and the another cowork with Hiroyuki Osaka.

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**Peter Levay**

Entanglement from deformed tori

We show that for toroidal compactification of type IIB string theory simple qubit systems arise naturally from the geometrical data of deformed tori. We also demonstrate that the natural arena for such qubits to show up is rather an embedded one within the realm of fermionic entanglement. Interestingly for such embedded systems extremal black hole solutions can be described using the language of tripartite fermionic entanglement, with the formula for the black hole entropy providing a fermionic generalization of the three-tangle well-known from three-qubit systems.

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**Debbie Leung**

Joint work with Frederic Dupuis, Jan Florjanczyk, and Patrick Hayden

Locking classical information

Locking is the quantum phenomenon in which removal of a small part of a quantum system induces a large suppression in a correlation. In this talk, we consider locking of classical information in the following scenario. A classical message M is encoded into a quantum system N, which is then subject to random unitary transformation followed by the removal of a small subsystem D. We consider how the remaining system is correlated with M, and show that there is a critical size for discarded D at which the correlation changes rapidly from being near-maximum to negligible. This critical size is logarithmic in the message size, plus the amount of redundancy in the encoding (from M to N), the entropy deficit in the original message M, and the amount of quantum side information available.

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**Ashley Montanaro**

Weak multiplicativity for random quantum channels

For many years, an important open problem in quantum information theory was whether the minimum output entropy of quantum channels was additive; this problem was recently solved (with a negative answer) by Hastings. A closely related family of conjectures concerns the maximum output p-norms of quantum channels. For all p>1, it is known that there exist channels N such that, given two uses of N, one can achieve a significantly higher output p-norm using an entangled input (across the two uses of N) than would be possible with product state inputs. However, the situation is far from clear in the setting where one is allowed n>2 uses of N; indeed, it is not even clear whether the maximum output p-norms necessarily go to 0 in the limit of large n.

In this talk, I will discuss the possibility of proving a weak variant of multiplicativity for random quantum channels. This would be the statement that, as n increases, the maximum output p-norms necessarily decrease exponentially with n.

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**Rémy Mosseri**

Entanglement in the symmetric sector of n qubits

I will discuss the entanglement properties of symmetric states of n qubits. The Majorana representation maps a generic such state into a system of n points on a sphere. Entanglement invariants, either under local unitaries (LU) or stochastic local operations and classical communication (SLOCC), can then be addressed in terms of the relative positions of the Majorana points. In the LU case, an over complete set of invariants can be built from the inner product of the radial vectors pointing to these points; this is detailed for the well documented three-qubits case. In the SLOCC case, cross ratio of related Möbius transformations are shown to play a central role, examplified here for four qubits. Finally, as a side result, we also analyze the manifold of maximally entangled 3 qubit state, both in the symmetric and generic case.

Ref : P. Ribeiro et R. Mosseri, "Entanglement in the Symmetric Sector of N qubits, Phys. Rev. Lett. **106**, 180502 (2011)

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**Mary Beth Ruskai**,

Some old and new results on quantum marginals and reduced density matrices

It is well known that the set of m-body reduced density matrices of N-body states (possibly satisfying a symmetry constraint) is convex. This also holds for vectors of m-body quantum marginals.

Recent work in quantum information theory has regenerated interest in the role of reduced density matrices in many-body quantum theory and led to the resolution of some long-standing questions. One of these is the realization that an extreme point can have a non-unique pre-image, and that such situations are related to quantum error correction codes.

This talk will provide a brief survey of old and new work, and describe some of the open questions which remain.

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**Gniewomir Sarbicki**

Exposed positive maps and entanglement witnesses - a sufficient condition.

Abstract: The knowledge about exposed points of the set of positive maps are important when one investigates the geometry of the boundary of this set. Exposedness of a map implies its extremality. More, the set of exposed points is dense in the set of extremal points, so any entangled state can be detected by an exposed map.

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**Adam Sawicki**

Symplectic geometry of quantum correlations

I will present a description of entanglement in composite quantum systems in terms of symplectic geometry. In particular, using the Kostant-Sternberg theorem, I will show that separable states form a unique symplectic orbit, whereas orbits of entangled states are characterized by different degrees of degeneracy of the canonical symplectic form on the complex projective space. The degree of degeneracy may be thus used as a new geometric measure of entanglement. The above statements remain true for systems with an arbitrary number of components, moreover the presented method is general and can be applied also under different additional symmetry conditions stemming, e.g., from the indistinguishability of particles. I will show how to calculate the degeneracy for various multiparticle systems providing also simple criteria of separability.

This is a joint work with Marek Kus and Alan Huckleberry.

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**Łukasz Skowronek**

Bound entangled states of lowest rank and general unextendible product bases

We shall discuss the subject of unextendible product bases with the orthogonality condition dropped and prove that the lowest rank non-separable positive-partial-transpose states, i.e., states of rank 4 in 3 × 3 systems are always SLOCC equivalent to a projection onto the orthogonal complement of a linear subspace spanned by an orthogonal unextendible product basis. The product vectors in the kernels of the states belong to a non-zero measure subset of all general unextendible product bases, nevertheless, they can always be locally transformed to the orthogonal form. This fully confirms the surprising numerical results recently reported by Leinaas et al. Parts of the talk rely heavily on the use of Bezout's theorem from algebraic geometry.

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**Graeme Smith**

Fun with Gaussian States and Channels

Gaussian states form a natural subclass of all states on a bosonic system. They are relatively easy to generate in a lab (at least for some parameter values) and have nice mathematical properties. For example, while there are PPT bound entangled Gaussian states, there is no NPT bound entangled Gaussian state. Gaussian operations are a nice subclass of all operations that are both easy to describe and relatively easy to implement. While in some ways information theory with Gaussian states and channels is simpler than the usual finite-dimensional setting, there is still substantial complexity, and I'll point out what we know about privacy, superactivation, and other exotic phenomena in the Gaussian regime.

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**Michał Studziński**

We consider distillation of entanglement from two qubit states which are mixtures of three mutually orthogonal states: two pure entangled states and one pure product state. We distill entanglement from such states by projecting $n$ copies of the state on permutationally invariant subspace and then applying one-way hashing protocol. We find analytical expressions for the rate of the protocol. We also generalize this method to higher dimensional systems. To get analytical expression for two qubit case, we faced a mathematical problem of diagonalizing a family of matrices enjoying some symmetries w.r.t. the symmetric group. We have solved this problem in two ways: (i) directly, by use of {\bf Schur-Weyl decomposition} and {\bf Young symmetrizers} (ii) showing that the problem is equivalent to a problem of diagonalizing adjacency matrices in a particular instance of a so called algebraic association scheme.

(a joint work with Mikołaj Czechlewski, Andrzej Grudka, Michał Horodecki and Marek Mozrzymas)

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**Michael Walter**

Probability Measures for the Quantum Marginal Problem

Given local density matrices of Alice, Bob and Charlie, can they arise from a global pure state? (This is the quantum marginal problem.) And if so, how likely is this when choosing the pure state at random (according to the Fubini--Study measure)? In this talk, I will explain how to answer this question using the tools of symplectic geometry, and illustrate our results in the three-qubit example. As time permits, I will also indicate how it is related to the asymptotic representation theory of the unitary and symmetric groups.

This is joint work with my supervisor Matthias Christandl and with Stavros Kousidis and Brent Doran from ETH Zurich.

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**Andreas Winter**

Operator norms in quantum information: Distinguishability of states under restricted families of measurements

We consider the problem of ambiguous discrimination of two quantum states when we are only allowed to perform a restricted set of measurements. Let the bias of a POVM be defined as the total variational distance between the outcome distributions for the two states to be distinguished. The performance of a set of measurements can then be defined as the ratio of the bias of this POVM and the largest bias achievable by any measurements. We first provide lower bounds on the performance of various POVMs acting on a single system such as the isotropic POVM, and spherical 2 and 4-designs, and show how these bounds can lead to certainty relations. Furthermore, we prove lower bounds for several interesting POVMs acting on multipartite systems, such as the set of local POVMS, POVMs which can be implemented using local operations and classical communication (LOCC), separable POVMs, and finally POVMs for which every bipartition results in a measurement having positive partial transpose (PPT). In particular, our results show that a scheme of Terhal et al. for hiding data against local operations and classical communication has the best possible dimensional dependence. I would like to use the presentation to highlight open questions following from our work: These include the possibility of extending the results to more than two parties, and the clarification of the relation between the norms corresponding to LOCC and separable measurements.

[Largely based on a joint paper with Will Matthews and Stephanie Wehner, Comm. Math. Phys. 291:813–843 (2009).]