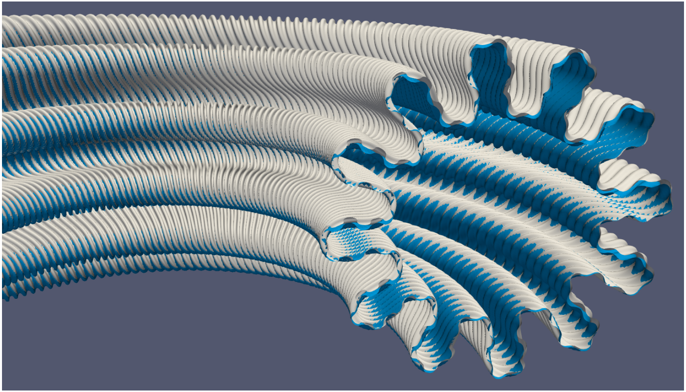
H-Principle and Convex Integration Theory

Vincent Borrelli

Université Lyon 1



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A tool to detect *h*-principles

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Problem ($\sim$ 1930 until 1960).– Given two surfaces of $\mathbb{R}^3$, does there exist a *regular* homotopy joining them ?

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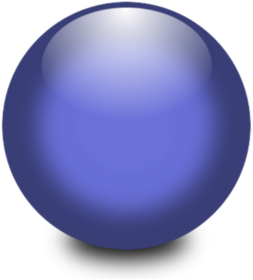
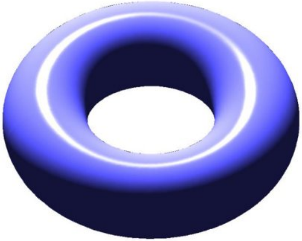
Problem ($\sim$ 1930 until 1960).– Given two surfaces of $\mathbb{R}^3$, does there exist a *regular* homotopy joining them ?

Regular means that at each time the surface remains smooth (no sharp bend, no crease).

Self-intersections are allowed.

An example





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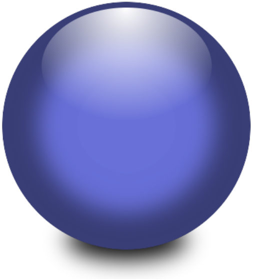
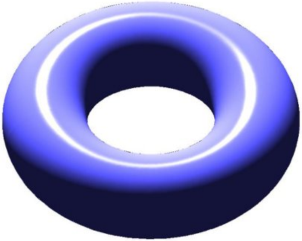
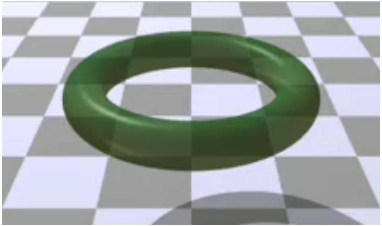
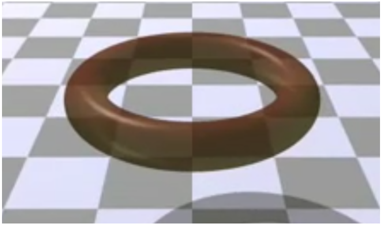




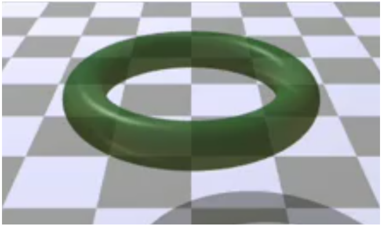
Obviously impossible.– There is a topological obstruction !

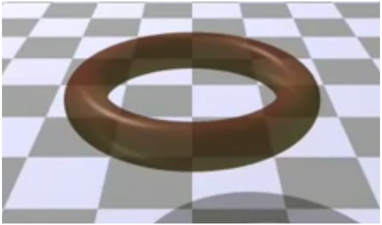
Another example





Another example

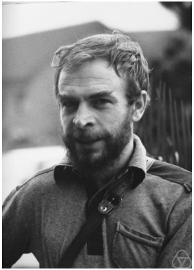




VIDEO

No obstruction !

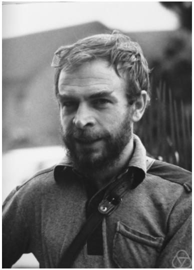
What is the *h*-principle ?



Mikhail Gromov

• Informal definition : We say that the *h***-principe** holds on a differential problem if the obstructions to find solutions come from algebraic topology.

What is the *h*-principle ?



Mikhail Gromov

• Informal definition : We say that the *h*-**principe** holds on a differential problem if the obstructions to find solutions come from algebraic topology.

• For instance, the *h*-principle holds on the problem of finding regular homotopies.

What is the *h*-principle ?

• The *h*-principle holds for numerous problems : in Symplectic/Contact/Riemannian Geometry, in Foliation Theory...

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• The *h*-principle holds for numerous problems : in Symplectic/Contact/Riemannian Geometry, in Foliation Theory...

• In these lectures, we shall focus on one of the techniques to detect the presence of the *h*-principle : Convex Integration Theory.

General Philosophy of these lectures

• To present Convex Integration Theory in a historical perspective in order to emphasize the key ideas behind its development.

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Sometimes, you do not need tons of abstract and conceptual theories to get a breakthrough.

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• To present Convex Integration Theory in a historical perspective in order to emphasize the key ideas behind its development.

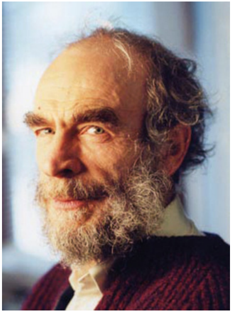
• These key ideas were **outrageously simple**... Sometimes, you do not need tons of abstract and conceptual theories to get a breakthrough.

• Lectures on the *h*-principle were already given in 2012 on this MA2 with a classical perspective. It is highly recommended to download the pdf notes on my webpage (section "Enseignement"), to take a look at them and possibly to do some of the exercises.

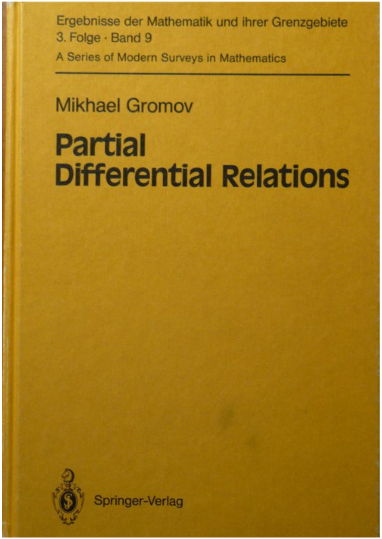
Content

- L1 : Nash-Kuiper Theorem
- L2 : From Nash-Kuiper to Gromov
- L3 : 1D Convex Integration
- L4 : Gromov Theorem on Ample Relations
- L5 : Constructions of C^1 isometric maps
- L6 : Theillière's Formula

Bibliography



Mikhail Gromov



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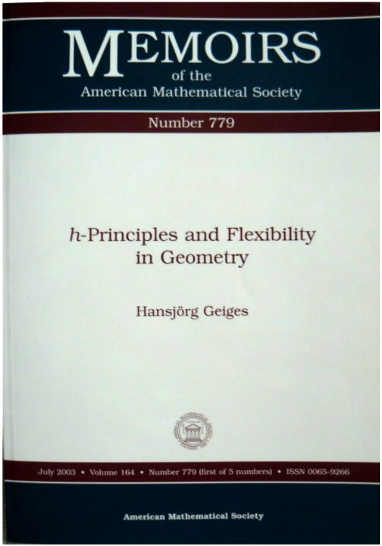
Hansjörg Geiges



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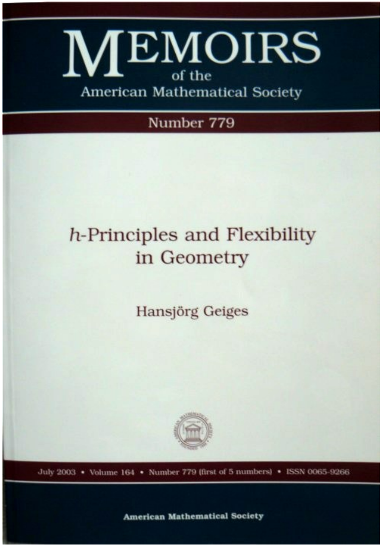
Hansjörg Geiges et Sinem Onaran



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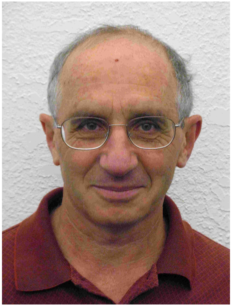
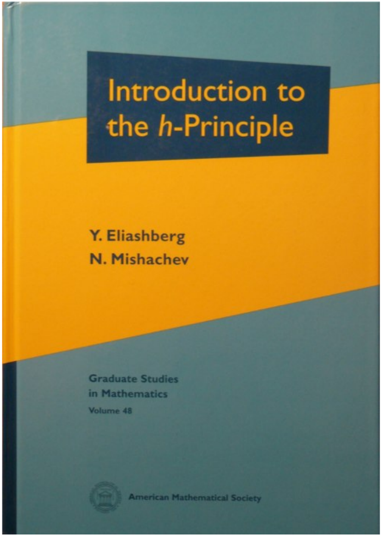


Yakov Eliashberg



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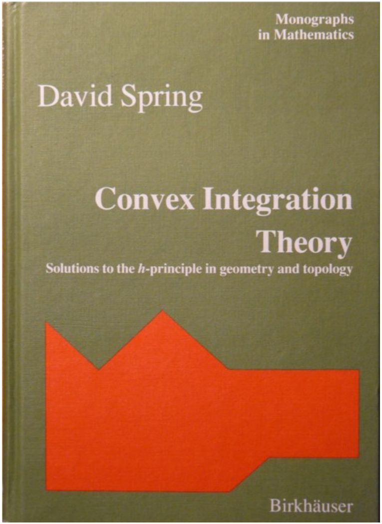
David Spring



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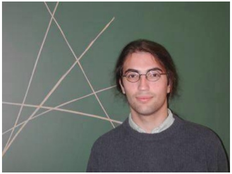
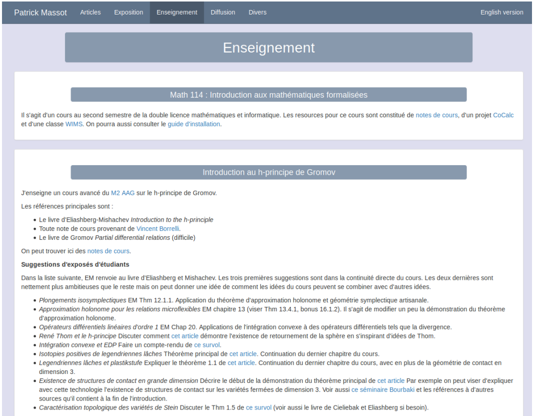


Patrick Massot



Patrick Massot Articles Exposition Enseignement Diffusion Divers English version

Enseignement

Math 114 : Introduction aux mathématiques formalisées

Il s'agit d'un cours au second semestre de la double licence mathématiques et informatique. Les resources pour ce cours sont constitué de notes de cours, d'un projet CoCalc et d'une classe WIMS. On pourra aussi consulter le guide d'installation.

Introduction au h-principe de Gromov

J'enseigne un cours avancé du M2 AAG sur le h-principe de Gromov.

Les références principales sont :

- Le livre d'Eliashberg-Mishachev *Introduction to the h-principle*
- Toute note de cours provenant de Vincent Borrelli.
- Le livre de Gromov *Partial differential relations* (difficile)

On peut trouver ici des notes de cours.

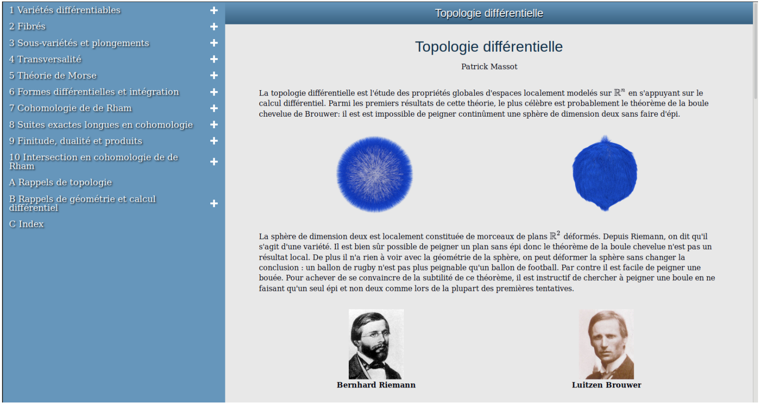
Suggestions d'exposés d'étudiants

Dans la liste suivante, EM renvoie au livre d'Eliashberg et Mishachev. Les trois premières suggestions sont dans la continuité directe du cours. Les deux dernières sont nettement plus ambitieuses que le reste mais on peut donner une idée de comment les idées du cours peuvent se combiner avec d'autres idées.

- *Plongements isosymplectiques* EM Thm 12.1.1. Application du théorème d'approximation holonome et géométrie symplectique artisanale.
- *Approximation holonome pour les relations microflexibles* EM chapitre 13 (viser Thm 13.4.1, bonus 16.1.2). Il s'agit de modifier un peu la démonstration du théorème d'approximation holonome.
- *Opérateurs différentiels linéaires d'ordre 1* EM Chap 20. Applications de l'intégration convexe à des opérateurs différentiels tels que la divergence.
- *René Thom et le h-principe* Discuter comment cet article démontre l'existence de retournement de la sphère en s'inspirant d'idées de Thom.
- *Intégration convexe et EDP* Faire un compte-rendu de ce survol.
- *Isotopies positives de legendriennes lâches* Théorème principal de cet article. Continuation du dernier chapitre du cours.
- *Legendriennes lâches et plastikstufe* Expliquer le théorème 1.1 de cet article. Continuation du dernier chapitre du cours, avec en plus de la géométrie de contact en dimension 3.
- *Existence de structures de contact en grande dimension* Décrire le début de la démonstration du théorème principal de cet article Par exemple on peut viser d'expliquer avec cette technologie l'existence de structures de contact sur les variétés fermées de dimension 3. Voir aussi ce séminaire Bourbaki et les références à d'autres sources qu'il contient à la fin de l'introduction.
- *Caractérisation topologique des variétés de Stein* Discuter le Thm 1.5 de ce survol (voir aussi le livre de Cieliebak et Eliashberg si besoin).

Pdf of the lectures "Introduction to the h-principle" on his web page

Bibliography



Patrick Massot : lectures on Differential Topology

Now to business !

We have some *Wrong Way* signs to explode with *h*-principle dynamites !