DIFFERENTIAL SPLIT THERMODYNAMIC MODEL FOR GAS-LIQUID COMPOSITIONAL FLOW

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- INTRODUCTION
- MATH. FORMULATION
- HT–SPLITTING
- OPEN THERMODYNAMICS
- CONCLUSIONS

INTRODUCTION

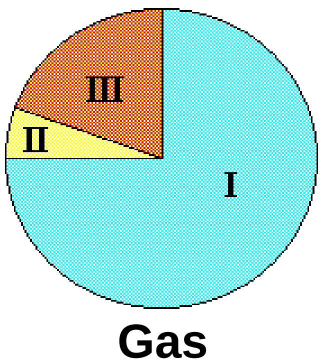
FLUID CHARACTERIZATION

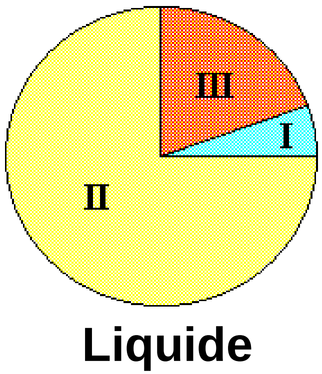
$$\mathbf{I}: \left\{\begin{matrix} N_2 \\ CH_4 \\ C_2H_6 \end{matrix}\right\} \text{light components}$$

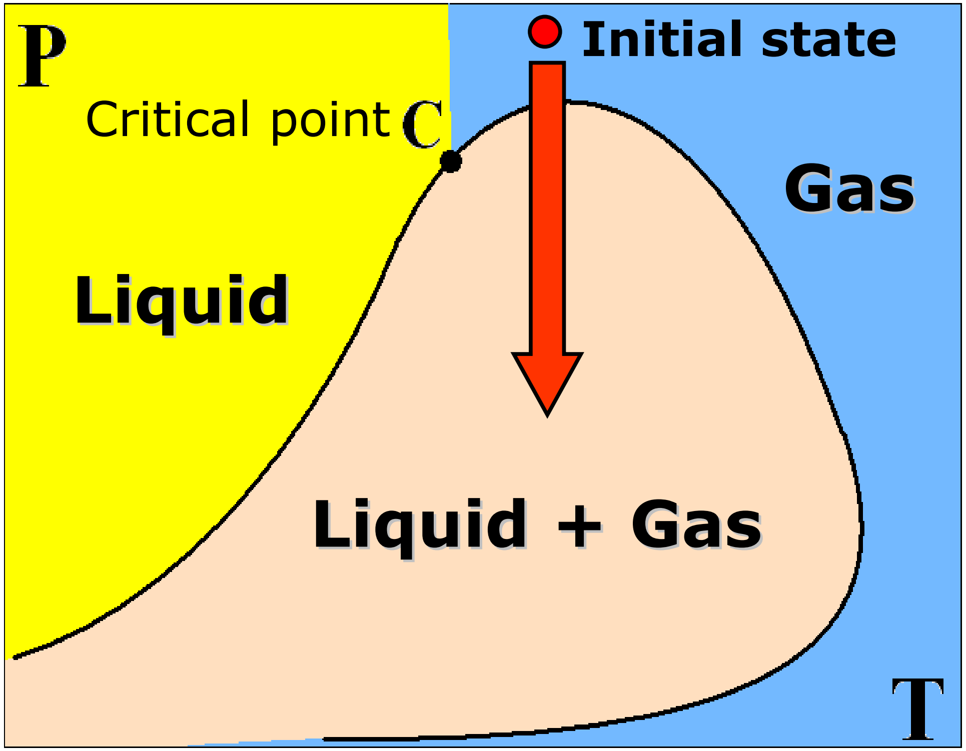
$$\mathbf{II}: \left\{\begin{matrix} C_5H_{12} \\ C_6H_{14} \\ \vdots \\ C_{20}H_{42} \end{matrix}\right\} \text{heavy components}$$

$$\mathbf{III}: \left\{\begin{matrix} C_3H_8 \\ C_4H_{10} \end{matrix}\right\} \text{neutral components}$$

- N components
- Two-phase
- Phase Exchange

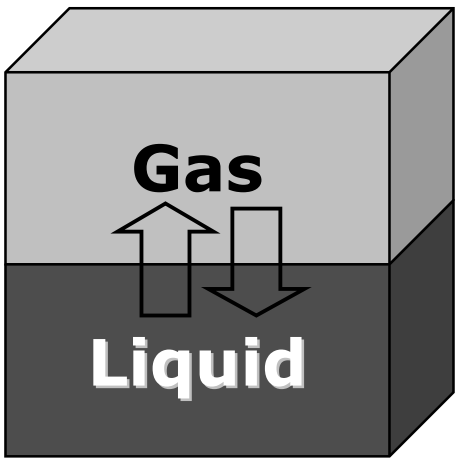






THERMODYNAMIC BEHAVIOURS

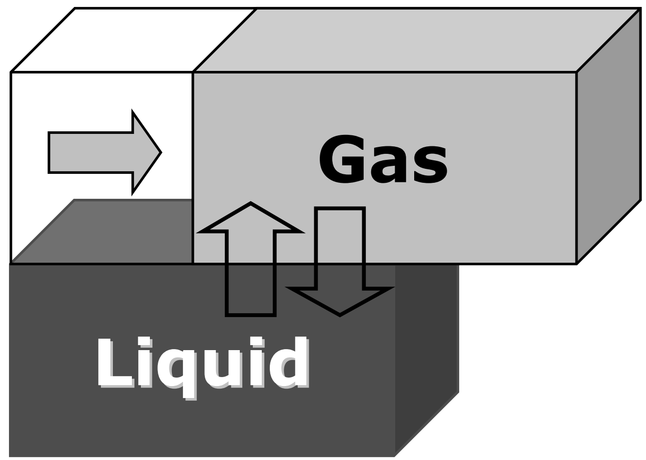
Close Thermodynamic System



Individual Volums

Phase Exchange

Open Thermodynamic System



Individual Volums

Phase Exchange + Transport

MATHEMATICAL FORMULATION

COMPOSITIONAL MODEL

H

Mass balance for each chemical component *k* :

$$\phi \frac{\partial}{\partial t}\left(\rho_l c_l^{(k)} s + \rho_g c_g^{(k)}\left[1-s\right]\right) + \mathrm{div}\left(\rho_l c_l^{(k)} \mathbf{V}_l + \rho_g c_g^{(k)} \mathbf{V}_g\right) = 0, \qquad k = 1, \ldots, N$$

Momentum balance for each phase (the Darcy law)

$$\mathbf{V}_l = -\frac{K k_l}{\mu_l} \, grad\left(P_l + \rho_l g z\right)$$

$$\mathbf{V}_g = -\frac{K k_g}{\mu_g} \, grad\left(P_g + \rho_g g z\right)$$

COMPOSITIONAL MODEL

Phase equilibrium :

$$\nu_g^k\left(P,\left\{c_g^q\right\}_{q=1}^N\right)=\nu_l^k\left(P,\left\{c_l^q\right\}_{q=1}^N\right) \quad k=1,\dots,N$$

(ν_i^k - the chemical potential)

$P_g = P_l$ or $P_g = P_l + P_c(s)$

Phase state :

$$\rho_g = \rho_g\left(P,\left\{c_g^q\right\}_{q=1}^N\right),$$

$$\rho_l = \rho_l\left(P,\left\{c_l^q\right\}_{q=1}^N\right)$$

$$k=1,\dots,N$$

COMPOSITIONAL MODEL

Gibbs "Rule of Phases" T=const

H ⇒ Number of equations = N+2
Number of variables = 2N-2+3 = 2N+1

Difference = N-1 = v_T = Thermodynamic Variance

T ⇒

$$\rho_i = \rho_i(P), \quad i = g, l$$

$$c_i^{(k)} = c_i^{(k)}(P), \quad k = 1, \ldots, N$$

v_T = N-1

H & T Coupled System

Huge Nonlinear Problem

HT-SPLITTING

H T - S P L I T T I N G

Dimensionless canonical form

H

$$\varepsilon \frac{\partial}{\partial \tau}\left(\varphi_l \overline{\rho} s + \varphi_g (1-s)\right) = \mathrm{div}\left(\left[\psi_g k_g + \omega \psi_l k_l\right] \mathrm{grad} p\right),$$

gas flow

$$\varepsilon\left(\overline{\rho}\frac{\partial\left(\varphi_l s\right)}{\partial \tau} + \overline{\rho}\varphi_l s \frac{\partial \zeta_l^{(N)}}{\partial \tau} + \varphi_g (1-s)\frac{\partial \zeta_g^{(N)}}{\partial \tau}\right) = \omega \,\mathrm{div}\left(\psi_l k_l \mathrm{grad} p\right) +$$

$$\omega \psi_l k_l \mathrm{grad} p \cdot \mathrm{grad} \zeta_l^{(N)} + \psi_g k_g \mathrm{grad} p \cdot \mathrm{grad} \zeta_g^{(N)},$$

liquid flow

$$\varepsilon\left[\overline{\rho}\varphi_l s \frac{\partial}{\partial \tau}\left(\zeta_l^{(k)} - \zeta_l^{(N)}\right) + \varphi_g (1-s)\frac{\partial}{\partial \tau}\left(\zeta_g^{(k)} - \zeta_g^{(N)}\right)\right] =$$

$$\omega \psi_l k_l \mathrm{grad} p \cdot \mathrm{grad}\left(\zeta_l^{(k)} - \zeta_l^{(N)}\right) + \psi_g k_g \mathrm{grad} p \cdot \mathrm{grad}\left(\zeta_g^{(k)} - \zeta_g^{(N)}\right),$$

$$k = 1, \ldots, N-2$$

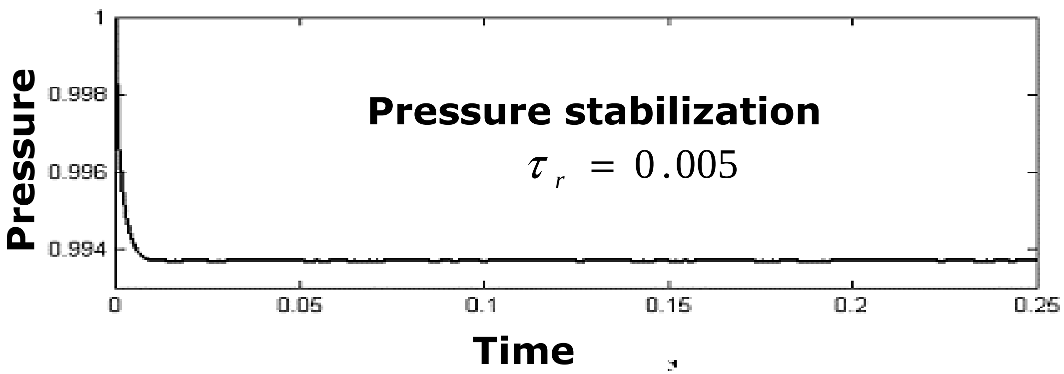
transport of basic components

HT-SPLITTING

Characteristic parameters of the system

Perturbation parameter:

$$\varepsilon = \frac{\text{Perturbation propagation time}}{\text{Reservoir depletion time}} \sim 10^{-3} - 10^{-2}$$



$$\varepsilon, \omega << 1$$

$$\varepsilon \sim \omega$$

Relative phase mobility parameter:

$$\omega = \frac{\text{liquid mobility}}{\text{gas mobility}} = \frac{\langle \rho_l \mu_g \rangle}{\langle \rho_g \mu_l \rangle} \sim 10^{-3} - 10^{-2}$$

HT-SPLITTING

Limit behavior of the compositional model $\omega, \varepsilon \to 0$

H

$$0 = \operatorname{div}\left(\psi_g k_g \operatorname{grad} p\right);$$

gas flow

$$\varepsilon\left(\bar{\rho}\frac{\partial(\varphi_l s)}{\partial\tau} + \bar{\rho}\varphi_l s\frac{\partial\varsigma_l^{(N)}}{\partial\tau} + \varphi_g(1-s)\frac{\partial\varsigma_g^{(N)}}{\partial\tau}\right) =$$

$$\omega \operatorname{div}\left(\psi_l k_l \operatorname{grad} p\right) + \psi_g k_g \operatorname{grad} p \cdot \operatorname{grad}\varsigma_g^{(N)};$$

liquid flow

$$0 = \psi_g k_g \operatorname{grad} p \cdot \operatorname{grad}\left(\varsigma_g^{(k)} - \varsigma_g^{(N)}\right), \quad k = 1, \dots, N-2$$

transport of basic components

This subsystem can be integrated along streamlines

HT – SPLITTING

Transformation of the transport sub-system

$$0 = \psi_g k_g \,\text{grad}\, p \cdot \text{grad}\left(\varsigma_g^{(k)} - \varsigma_g^{(N)}\right), \quad k = 1,\ldots, N-2$$

Along a streamline *l* :

$$0 = \psi_g k_g \frac{\partial p}{\partial l} \cdot \frac{\partial}{\partial l}\left(\varsigma_g^{(k)} - \varsigma_g^{(N)}\right) \Rightarrow \frac{\partial}{\partial l}\left(\varsigma_g^{(k)} - \varsigma_g^{(N)}\right) = 0 \Rightarrow \frac{\partial \varsigma_g^{(k)}}{\partial l} = \frac{\partial \varsigma_g^{(N)}}{\partial l}$$

$$\Rightarrow \frac{1}{\Delta c^{(k)}} \frac{\partial c_g^{(k)}}{\partial l} = \frac{1}{\Delta c^{(k)}} \frac{\partial c_g^{(N)}}{\partial l} \Rightarrow$$

HT-SPLITTING

$$\sim \sum_{k=1}^{N-2}\left(\frac{1}{\Delta c^{(q)}}\delta_{qk}-\frac{1}{\Delta c^{(q)}}\frac{\partial c_g^{(N)}}{\partial \chi^{(q)}}\right)\frac{\partial \chi^{(q)}}{\partial l}-\frac{1}{\Delta c^{(N)}}\frac{\partial c_g^{(N)}}{\partial p}\frac{\partial p}{\partial l}=0$$

$$\Rightarrow \frac{\partial}{\partial l}\Phi\left(p,\chi^{(1)},...,\chi^{(N-2)}\right)=\frac{\partial \Phi^{(k)}}{\partial p}\frac{\partial p}{\partial l}+\frac{\partial \Phi^{(k)}}{\partial \chi^{(1)}}\frac{\partial \chi^{(1)}}{\partial l}+...+\frac{\partial \Phi^{(k)}}{\partial \chi^{(N-2)}}\frac{\partial \chi^{(N-2)}}{\partial l}=0$$

$$\Rightarrow \Phi^{(k)}\left(p,\chi^{(1)},...,\chi^{(N-2)}\right)=const^{(k)},\quad k=1,...,N-2$$

$$\Rightarrow \chi^{(k)}=\chi^{(k)}(p),\quad k=1,...,N-2$$

$$\Rightarrow \sum_{k=1}^{N-2}\left(\frac{1}{\Delta c^{(q)}}\delta_{qk}-\frac{1}{\Delta c^{(q)}}\frac{\partial c_g^{(N)}}{\partial \chi^{(q)}}\right)\frac{d\chi^{(q)}}{dp}-\frac{1}{\Delta c^{(N)}}\frac{\partial c_g^{(N)}}{\partial p}=0$$

$$\Rightarrow \boxed{\frac{1}{\Delta c^{(q)}}\frac{dc_g^{(k)}}{dp}=\frac{1}{\Delta c^{(N)}}\frac{dc_g^{(N)}}{dp},\qquad k=1,...,N-2}$$

A differential thermodynamic system

This subsystem is transformed into a thermodynamic one along streamlines

HT – SPLITTING

Hydrodynamic subsystem

H

$$\begin{cases} \operatorname{div}\left(\psi_g k_g \operatorname{grad} p\right) = 0, \\ \overline{\rho}\varphi_l \dfrac{\partial s}{\partial \tau} + \overline{\rho}\varphi_l s \dfrac{\partial \zeta_l^{(N)}}{\partial \tau} + \varphi_g (1-s) \dfrac{\partial \zeta_g^{(N)}}{\partial \tau} = \dfrac{\omega}{\varepsilon} \operatorname{div}\left(\psi_l k_l \operatorname{grad} p\right) + \\ \qquad \dfrac{1}{\varepsilon} \psi_g k_g \operatorname{grad} p \cdot \operatorname{grad} \zeta_g^{(N)} \end{cases}$$

$\Rightarrow$ p, s

Thermodynamic subsystem

T

$$\begin{cases} \nu_g^{(k)}\left(p, \left\{c_g^{(q)}\right\}_{q=1}^{N}\right) = \nu_l^{(k)}\left(p, \left\{c_l^{(q)}\right\}_{q=1}^{N}\right), \quad k = 1, \ldots, N \\ \rho_g = \rho_g\left(p, \left\{c_g^{(q)}\right\}_{q=1}^{N}\right), \qquad \rho_l = \rho_l\left(p, \left\{c_l^{(q)}\right\}_{q=1}^{N}\right) \\ \sum_{k=1}^{N} c_g^{(k)} = 1, \qquad \sum_{k=1}^{N} c_l^{(k)} = 1 \\ \boxed{\dfrac{1}{\Delta c^{(q)}} \dfrac{dc_g^{(k)}}{dp} = \dfrac{1}{\Delta c^{(N)}} \dfrac{dc_g^{(N)}}{dp}, \quad k = 1, \ldots, N-2} \end{cases}$$

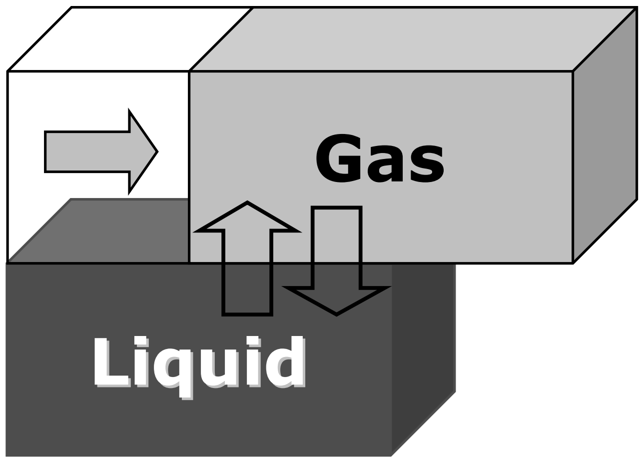
"Delta-law"

$\Rightarrow$ $c_g^{(k)}$, $c_l^{(k)}$

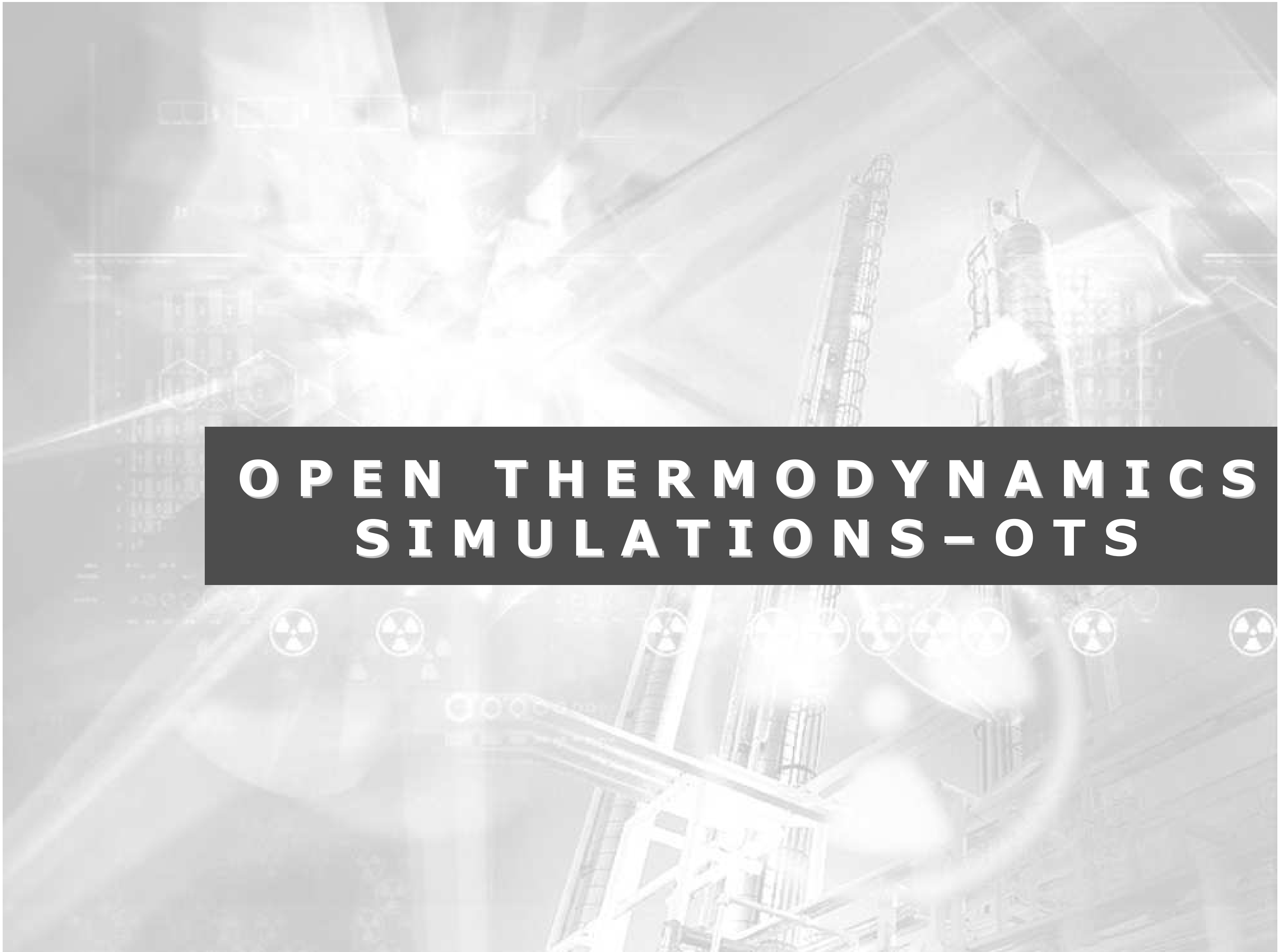
Interpretation of the New Delta-Law

$$\frac{1}{\Delta c^{(q)}}\frac{dc_g^{(k)}}{dp}=\frac{1}{\Delta c^{(N)}}\frac{dc_g^{(N)}}{dp},\qquad k=1,\dots,N-2$$

Open Thermodynamic System



Depend on the pressure only !



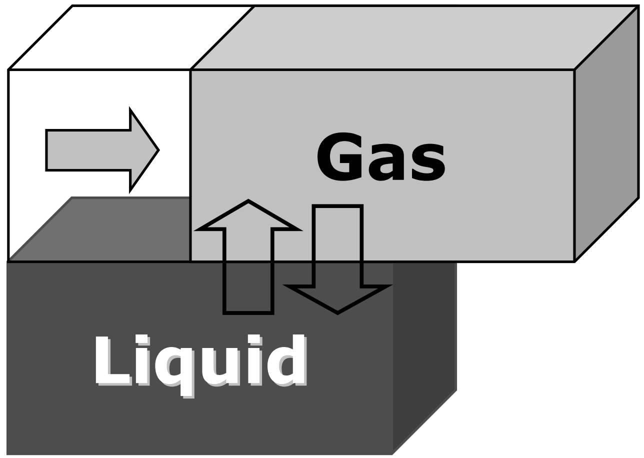
OPEN THERMODYNAMICS SIMULATIONS–OTS

Thermodynamic System Behaviours

Close or Open ?

Close System

Phase Exchange



Open System

Phase Exchange + Transport

Delta Law for an Open System

$$\frac{1}{\Delta c^{(k)}}\frac{dc_g{}^{(k)}}{dp}=\frac{1}{\Delta c^{(N)}}\frac{dc_g{}^{(N)}}{dp},\qquad k=1,\dots,N-2$$

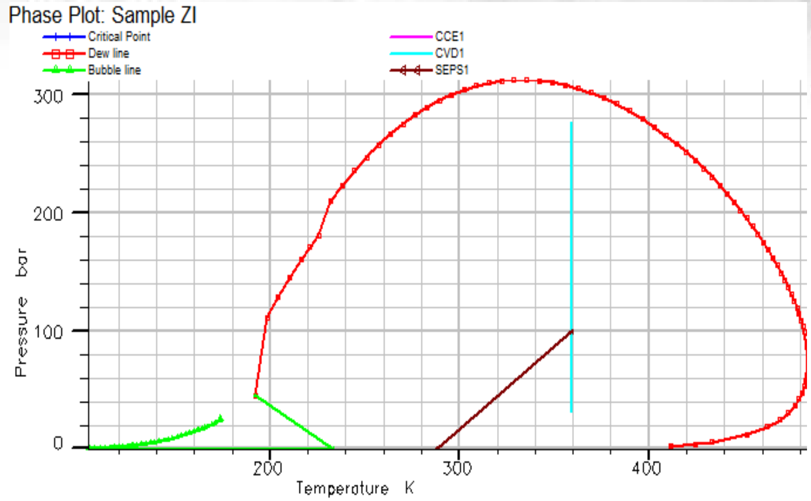
$\Rightarrow$

$$V(P)=\frac{\sum_{i=1}^{N-2}\frac{1}{\Delta c^{(k)}}\frac{dc_g{}^{(k)}}{dp}}{(N-2)\frac{1}{\Delta c^{(N)}}\frac{dc_g{}^{(N)}}{dp}}=1$$

Case Test 1: Urengoy Reservoir

8 Components Mixture:
CO_2, N_2, C1, C2, C3, C4, F1, F2

Specified temperature: 359 K



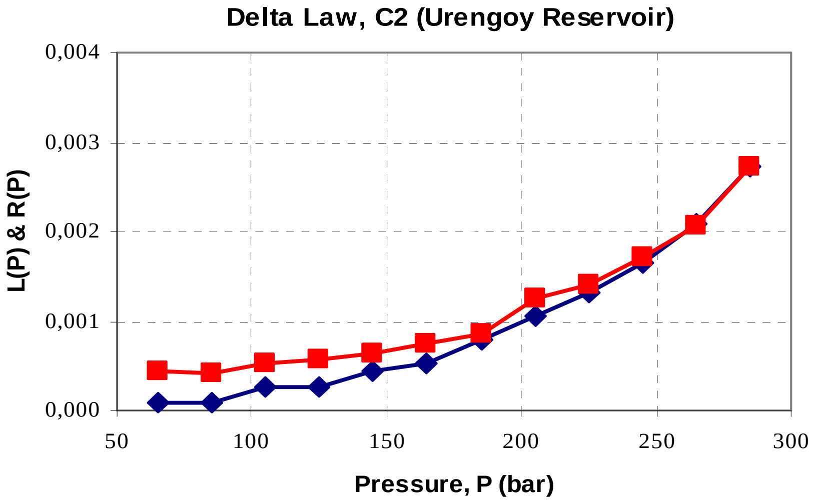
Data: ECLIPCE Close Thermodynamics (PVTi)

Thermodynamic System Behaviours

Delta Law

?

$$\frac{1}{\Delta c^{(k)}}\frac{dc_g^{(k)}}{dp}=\frac{1}{\Delta c^{(N)}}\frac{dc_g^{(N)}}{dp}$$



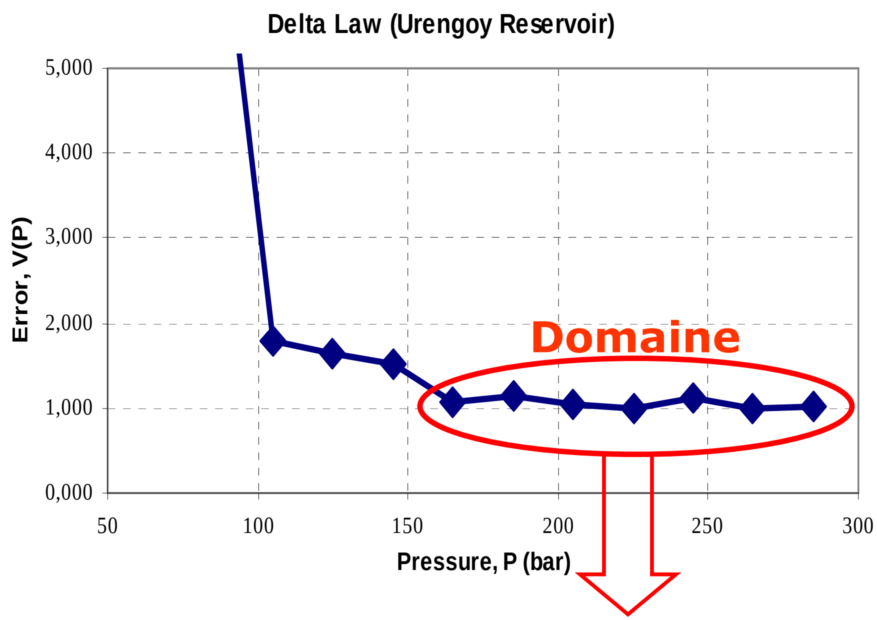
Case Test 1: Urengoy Reservoir

Thermodynamic System Behaviours

Delta Law

?

$$V(P) = \frac{\sum_{i=1}^{N-2} \frac{1}{\Delta c^{(k)}} \frac{dc_g{}^{(k)}}{dp}}{(N-2)\frac{1}{\Delta c^{(N)}} \frac{dc_g{}^{(N)}}{dp}} = 1$$

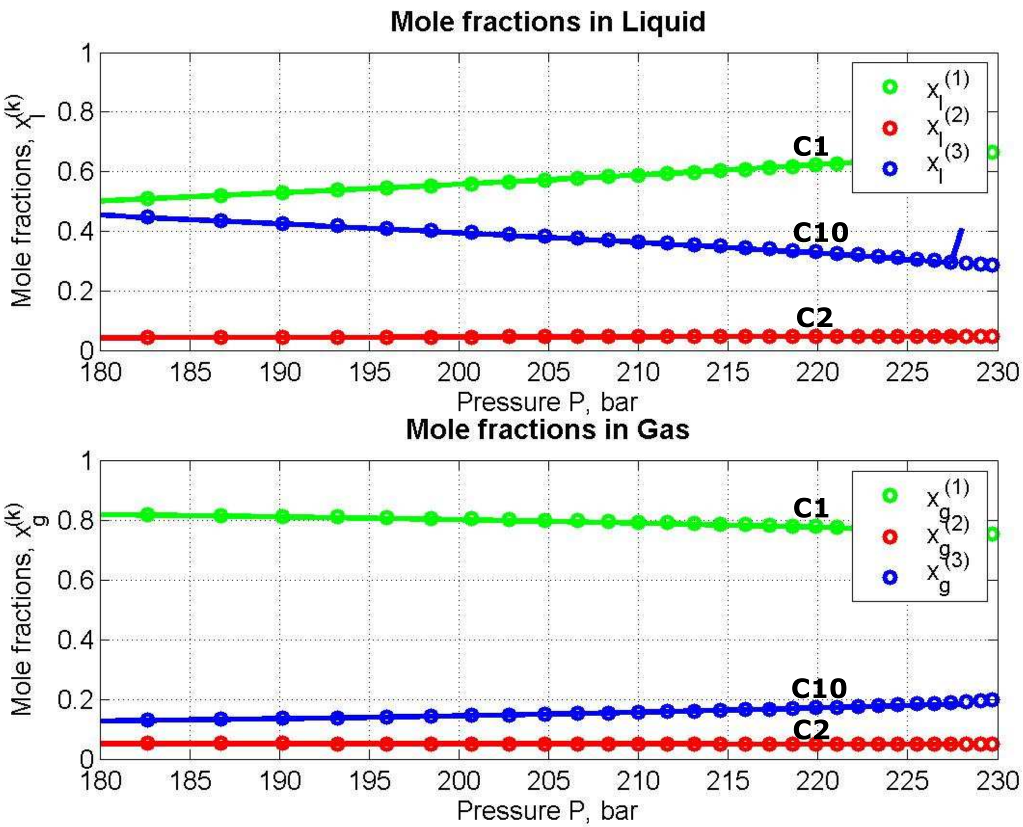


Close ~ Open

New O T S. Case Test 2:

3 Components Mixture:
CH_4, C_2H_6, $C_{10}H_{22}$

Open Thermodynamic System – Lines (NEW)
Thermodynamic Behaviours in an Open System – Circles (ECLIPCE 300)

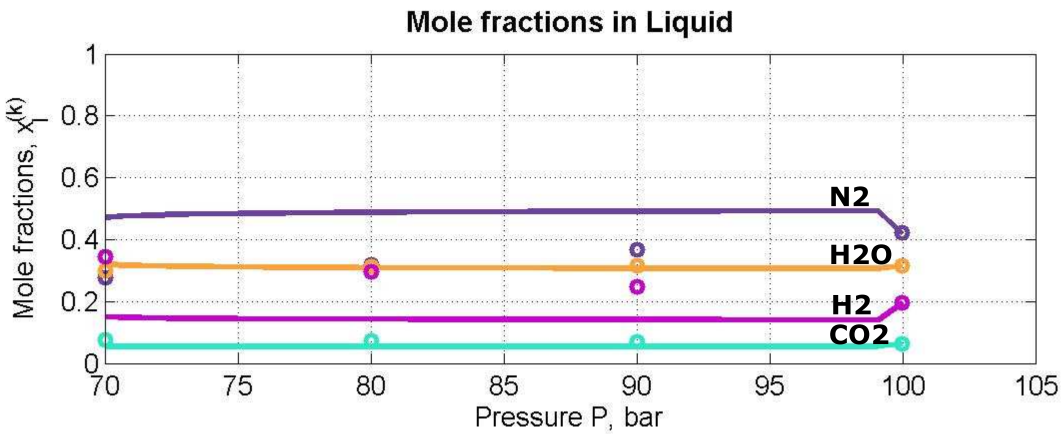


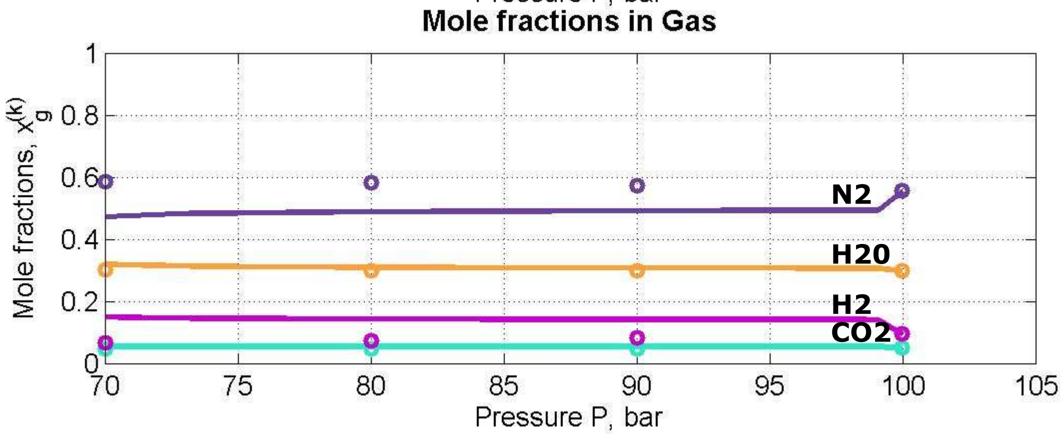
New O T S. Case Test 4: Radioactive Waste Storage

4 Components Mixture:
H_2, H_2O, N_2, CO_2

Specified temperature: 298 K

Open Thermodynamic System – Lines (NEW)
Close Thermodynamic System – Circles (ECLIPCE PVTi)





CONCLUSIONS

CONCLUSIONS

- **H-T Splitting along streamlines**
  - Contrast & Stabilisation

- **New Open Thermodynamic System**
  - Independent New Differential T. System
  - New OT Simulator

- **Hydrodynamic Compositional System consists of 2 Equations**
  -Pressure & Saturation

APPLICATIONS

- Gas-Condensate Well Representation

- Streamline Simulator for the 3D Dynamic Analysis of the Compositional Flows in Oil Reservoirs

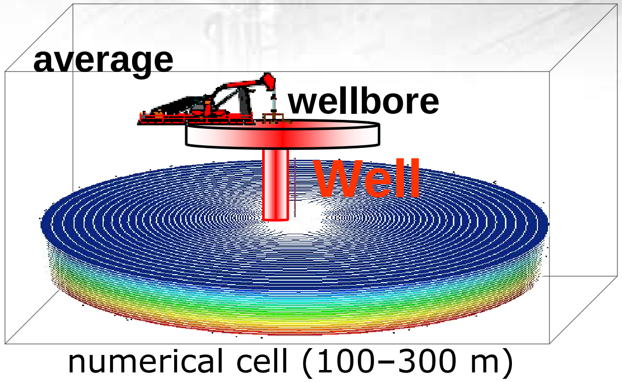
- Transfer of the Gas Around Storage of Radioactive Waste

Perspective

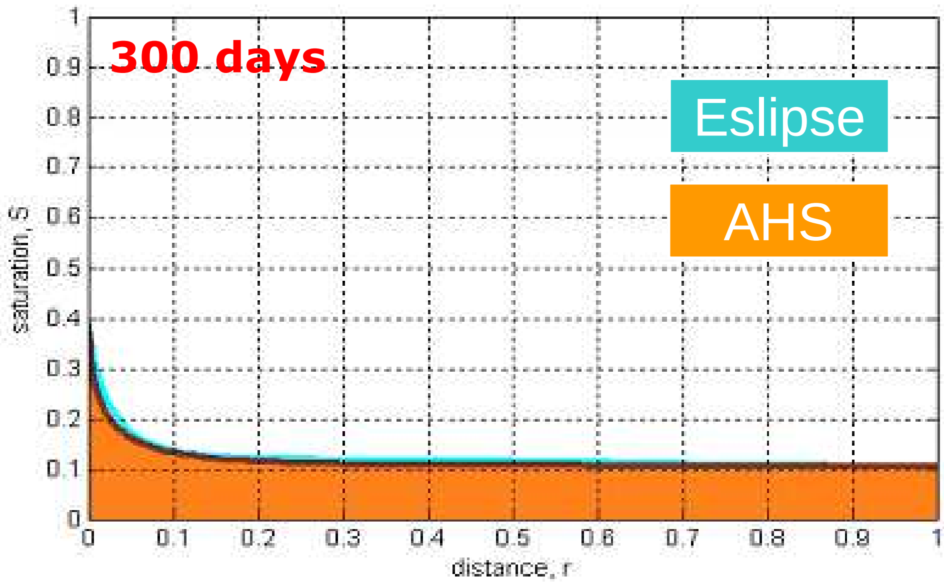
- Enhanced Oil Recovery

APPLICATIONS

Gas-Condensate Well Representation

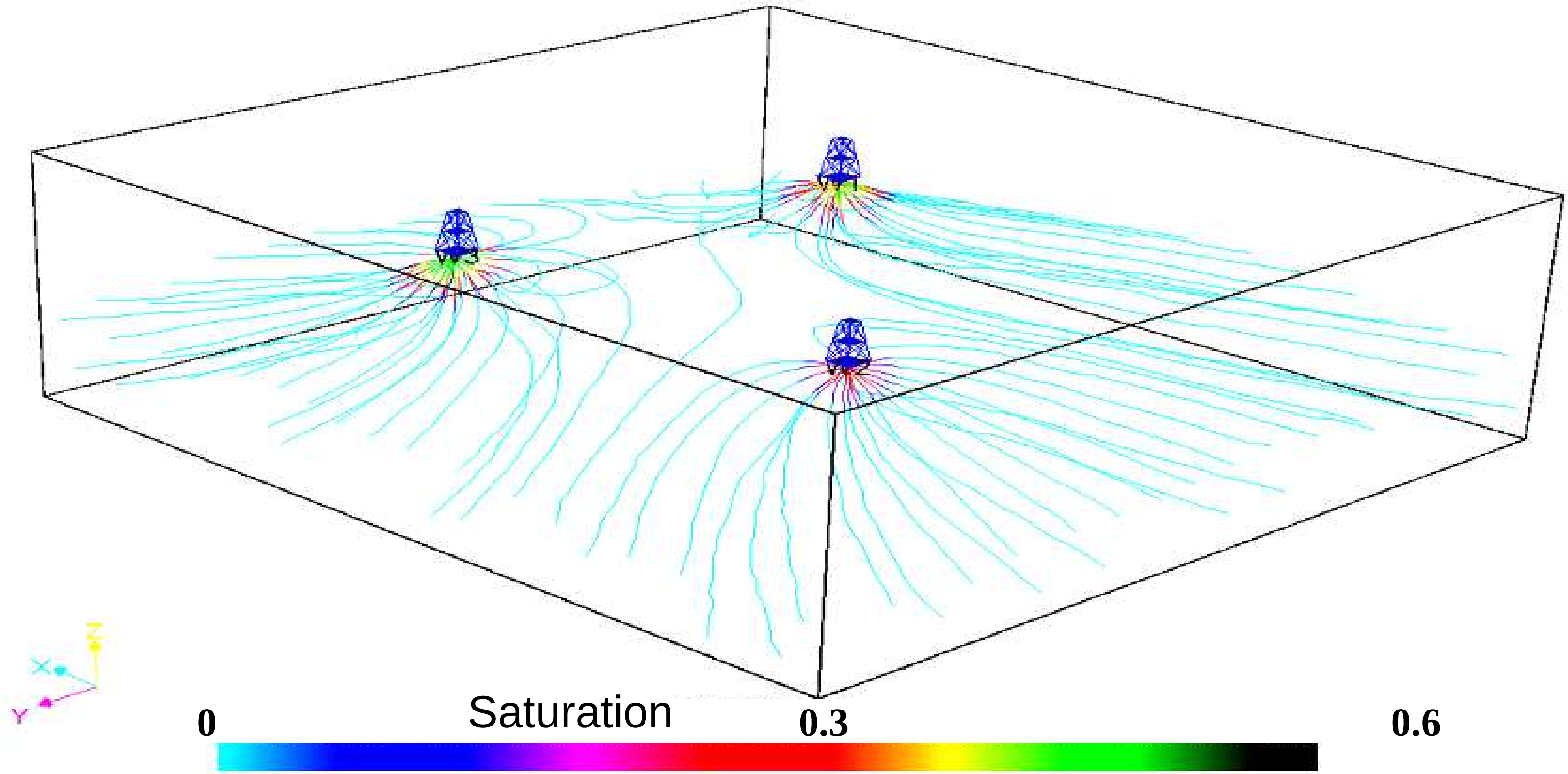


Saturation



APPLICATIONS

Streamline Simulator for the 3D Dynamic Analysis of the Compositional Flows in Oil Reservoirs



Saturation variation along the streamlines
during the natural depletion of the gas-condensate reservoir

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