Report on the paper "Breakdown for the Camassa-Holm equation using decay criteria and persistence in weighted spaces"

(paper submitted by L. Brandolese to *IMRN*)

The Camassa-Holm (CH) model, arising in the context of shallow water waves, marked an important recent development in nonlinear dynamics. This equation is of great current interest and several aspects very thoroughly investigated. The present paper offers a refreshingly new point of view by exploring the insight that can be provided by working in weighted spaces. The author addresses the issue of well-posedness and, more interestingly, he shows that if the initial data decay faster at infinity than the solitons, then wave breaking occurs. This last result was anticipated by some earlier partial results but in its present form is new and sheds light on the qualitative properties of the equation. The technical level is very good and the blow-up result is excellent. There are a few items that need to be improved in the presentation (a list of suggestions is provided below) but they are all minor.

In conclusion, I **strongly recommend acceptance for publication after a minor revision** (see below).

Comments and Suggestions

A few relatively minor points should be addressed before publication:

- The statements about the present state of existence, uniqueness and well-posedness made on lines -4 to -1 of page 1 are either not up-to-date or slightly misleading:

  - In [14, 6] the authors do not deal with well-posedness but with a uniform well-posedness. This is a rather special requirement that is not really typical, so this statement should be less emphasized.

  - The most general result for lower regularity is not that in [7]. For initial data in H^1 the existence and uniqueness of global weak solutions was proved in A. Bressan and A. Constantin, Global dissipative solutions of the Camassa-Holm equation, Anal. Appl. 5 (2007), 1–27; and in A. Bressan and A. Constantin, Global conservative solutions of the Camassa-Holm equation, Arch. Ration. Mech. Anal. 183 (2007), 215–239.

- On page 2, lines 6-8, the author discusses the peakons. It is worth pointing out that solutions of this type are not mere abstractizations: the peakons replicate a feature that is characteristic for the waves of great height — waves of largest amplitude that are exact solutions of the governing equations for irrotational water waves cf. the discussions in the papers A. Constantin and J. Escher, Particle trajectories in solitary water waves, Bull. Amer. Math. Soc. 44 (2007), 423–431; and J. F. Toland, Stokes waves, Topol. Methods Nonlinear Anal. 7 (1996), 1–48. Moreover, these solutions are orbitally stable — that is, their shape is stable under small perturbations and therefore these waves are recognizable physically cf. the paper A. Constantin and W. Strauss, Stability of peakons, Comm. Pure Appl. Math. 53 (2000), 603–610.

- On line 2 of page 12, in addition to [4, 15], one could also mention the paper D. Henry, Compactly supported solutions of the Camassa-Holm equation, J. Nonlinear Math. Phys. 12 (2005), 342–347.

- A typographical error on page 12: "breakes" instead of "brakes" in the statement of Theorem 6.1.

- In the list of references, some first names are written in full, while others are not. The author should stick to the standard procedure of writing just initials.

- An update of [18] is possible?