

# Branching Law for the Finite Subgroups of $\mathbf{SL}_4\mathbb{C}$

## Example — Type *II*

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The group of type *II* is the subgroup  $\langle F_1, F'_2, F'_3 \rangle$  of  $\mathbf{SL}_4\mathbb{C}$ , with

$$F_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & j^2 \end{pmatrix}, \quad F'_2 = \frac{1}{3} \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 2 & 2 \\ 0 & 2 & -1 & 2 \\ 0 & 2 & 2 & -1 \end{pmatrix}, \quad F'_3 = \frac{1}{4} \begin{pmatrix} -1 & \sqrt{15} & 0 & 0 \\ \sqrt{15} & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \end{pmatrix}.$$

Here  $l+1 = 5$ ,  $\text{rank}(A^{(1)}) = \text{rank}(A^{(2)}) = 4$ .

$\Theta^{(1)} = \overline{\Theta^{(3)}} = (4, 0, -1, 1, -1)$ ,  $\Theta^{(2)} = (6, -2, 1, 0, 1)$ ,  $p = 4$ , and  $\tau_0 = s_0s_1$ ,  $\tau_1 = s_2$ ,  $\tau_2 = s_3$ ,  $\tau_3 = s_4$ .

$$A^{(1)} = A^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 2 \end{pmatrix}, \quad A^{(2)} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 & 2 \\ 0 & 2 & 2 & 2 & 2 \end{pmatrix}, \quad C_{II} = \begin{pmatrix} 2 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -2 & -2 \\ 0 & -2 & 2 & -2 & -2 \\ -2 & -2 & -2 & 2 & -2 \\ 0 & -2 & -2 & -2 & 2 \end{pmatrix}.$$

$$\begin{aligned} D_{II}(t, u, w) &= (w-1)^4 (u+1)^3 (u-1)^5 (t-1)^4 (w^2+w+1) (w^4+w^3+w^2+w+1) \\ &\quad (w+1)^2 (u^4+u^3+u^2+u+1) (u^2+u+1)^2 (t^2+t+1) (t^4+t^3+t^2+t+1) (t+1)^2 \\ &= (u-1)(u+1)(u^2+u+1)\tilde{D}_{II}(t)\tilde{D}_{II}(u)\tilde{D}_{II}(w) \end{aligned}$$

with  $\tilde{D}_{II}(t) = (t-1)^4(t+1)^2(t^2+t+1)(t^4+t^3+t^2+t+1)$

$$\hat{P}_{II}(t) = \frac{t^8 - t^6 + t^4 - t^2 + 1}{t^{12} - 2t^{10} - t^9 + t^8 + t^7 + t^5 + t^4 - t^3 - 2t^2 + 1}$$

$$\begin{aligned} N_{II}(t, u, w)_0 &= -t^{10}u^{14}w^{10} - t^{10}u^{13}w^{10} + t^{10}u^{14}w^8 + t^{10}u^{12}w^{10} + t^8u^{14}w^{10} + t^{10}u^{13}w^8 - t^{10}u^{12}w^9 + 3t^{10}u^{11}w^{10} - t^9u^{14}w^8 - \\ & t^9u^{12}w^{10} - t^8u^{14}w^9 + t^8u^{13}w^{10} - t^{10}u^{14}w^6 - t^{10}u^{13}w^7 - 4t^{10}u^{12}w^8 - 4t^{10}u^{11}w^9 - t^{10}u^{10}w^{10} - t^9u^{14}w^7 - 3t^9u^{13}w^8 - \\ & 3t^9u^{12}w^9 - 4t^9u^{11}w^{10} - 2t^8u^{14}w^8 - 3t^8u^{13}w^9 - 4t^8u^{12}w^{10} - t^7u^{14}w^9 - t^7u^{13}w^{10} - t^6u^{14}w^{10} - 2t^{10}u^{13}w^6 - t^{10}u^{12}w^7 - \\ & 9t^{10}u^{11}w^8 - 5t^{10}u^{10}w^9 - 4t^{10}u^9w^{10} - 5t^9u^{13}w^7 - 4t^9u^{12}w^8 - 9t^9u^{11}w^9 - 5t^9u^{10}w^{10} - 6t^8u^{13}w^8 - 4t^8u^{12}w^9 - 9t^8u^{11}w^{10} - \\ & 5t^7u^{13}w^9 - t^7u^{12}w^{10} - 2t^6u^{13}w^{10} + t^{10}u^{14}w^4 - 2t^{10}u^{13}w^5 + t^{10}u^{12}w^6 - t^{10}u^{11}w^7 - 4t^{10}u^{10}w^8 - 2t^{10}u^9w^9 - 6t^{10}u^8w^{10} + \\ & t^9u^{14}w^5 - 2t^9u^{13}w^6 - 5t^9u^{12}w^7 - 3t^9u^{11}w^8 - 11t^9u^{10}w^9 - 2t^9u^9w^{10} - 3t^8u^{13}w^7 - 2t^8u^{12}w^8 - 3t^8u^{11}w^9 - 4t^8u^{10}w^{10} - \\ & 3t^7u^{13}w^8 - 5t^7u^{12}w^9 - t^7u^{11}w^{10} - 2t^6u^{13}w^9 + t^6u^{12}w^{10} + t^5u^{14}w^9 - 2t^5u^{13}w^{10} + t^4u^{14}w^{10} - 3t^{10}u^{12}w^5 + 4t^{10}u^{11}w^6 - \\ & 2t^{10}u^{10}w^7 + 2t^{10}u^9w^8 - 2t^{10}u^8w^9 - 4t^{10}u^7w^{10} - t^9u^{14}w^4 + t^9u^{13}w^5 - 5t^9u^{12}w^6 + 2t^9u^{11}w^7 - 5t^9u^{10}w^8 - 9t^9u^9w^9 - \\ & 2t^9u^8w^{10} - t^8u^{14}w^5 - t^8u^{13}w^6 - 8t^8u^{12}w^7 + 4t^8u^{11}w^8 - 5t^8u^{10}w^9 + 2t^8u^9w^{10} - 2t^7u^{14}w^6 + 2t^7u^{13}w^7 - 8t^7u^{12}w^8 + \\ & 2t^7u^{11}w^9 - 2t^7u^{10}w^{10} - 2t^6u^{14}w^7 - t^6u^{13}w^8 - 5t^6u^{12}w^9 + 4t^6u^{11}w^{10} - t^5u^{14}w^8 + t^5u^{13}w^9 - 3t^5u^{12}w^{10} - t^4u^{14}w^9 - \\ & t^{10}u^{14}w^2 + t^{10}u^{13}w^3 - 4t^{10}u^{12}w^4 + 2t^{10}u^{11}w^5 + t^{10}u^{10}w^6 - 5t^{10}u^9w^7 + 5t^{10}u^8w^8 - 5t^{10}u^7w^9 - t^{10}u^6w^{10} - 2t^9u^{14}w^3 - \\ & t^9u^{13}w^4 - 4t^9u^{12}w^5 - 3t^9u^{11}w^6 + 5t^9u^{10}w^7 - 11t^9u^9w^8 - 7t^9u^8w^9 - 5t^9u^7w^{10} - 3t^8u^{14}w^4 + t^8u^{13}w^5 - 7t^8u^{12}w^6 - \end{aligned}$$

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$$\begin{aligned}
&4t^2u^2w^2 + 3tu^5 + 3tu^4w - 2tw^3w^2 - tu^2w^3 + 3u^6 + 3u^5w - 2u^3w^3 + t^2u^3 + 2t^2u^2w + 2t^2uw^2 + 3tu^4 + 4tu^3w + 2tu^2w^2 + \\
&2u^5 + 3u^4w + u^3w^2 - 2t^2u^2 + t^2uw - t^2w^2 + 3tu^3 + 4tu^2w + tuw^2 + 2u^4 + 3u^3w - 2u^2w^2 + t^2u + t^2w + 2tu^2 + 2tuw + tw^2 + \\
&2u^3 + 2u^2w + uw^2 + t^2 + 2tu + tw + 3u^2 + 2uw + w^2) (w^4 + w^3 + w^2 + w + 1) (u^4 + u^3 + u^2 + u + 1) (t^4 + t^3 + t^2 + t + 1)
\end{aligned}$$