Confluentes Mathematici, Vol. 4, No. 1 (2012) 1203001 (2 pages)

DOI: 10.1142/S1793744212030016

INTRODUCTION

Renormalization theory lies on the edge of mathematics and physics. It appeared in the middle of the 20th century as a computational tool used in quantum field theory and statistical physics, and nowadays appears to be deeply connected to a wide variety of areas of pure mathematics, including algebra (Hopf algebras, operads), combinatorics (graphs, trees), number theory (multiple zeta functions, motives), differential and algebraic geometry (geometric quantization, Poisson structures), analysis (mould calculus, nonlinear partial differential equations), dynamical systems (KAM theory), probability (Brownian and stochastic processes) and statistics (scaling relations of critical phenomena).

This volume is partially based on the lectures given at the conference "Renormalization: Algebraic, Geometric and Probabilistic Aspects" held at the University of Lyon (France) on June 16–18, 2010. It offers a glimpse of some recent developments of renormalization theory and their application to mathematical physics, namely

- connections between different renormalization groups in quantum field theory;
- constructive field theory and its links with rough paths;
- the Hopf algebra approach to renormalization;
- renormalization within the Batalin–Vilkovisky formalism.

Each article is self-contained and can be read independently of the others. The first one by Michael Dütsch establishes a precise connection between the renormalization groups as defined by Stückelberg–Petermann and by Wilson. The Stückelberg–Petermann renormalization group is the group of finite renormalizations of the S-matrix in the framework of causal perturbation theory. The Wilson renormalization group describes the dependence of the theory on a UV-cutoff. In the framework of causal perturbation theory, the author introduces an ultraviolet cutoff, constructs a regularized S-matrix and proves a pertinent flow equation for the effective potential, which can be compared to a version of the Wilson renormalization group.

The second article by Jérémie Unterberger offers an introduction to constructive field theory in the Bosonic case, intended for mathematicians or mathematical physicists knowing the basic arguments of quantum field theory, and desiring to discover a general framework in which this topic can be made rigorous. The author then applies the renormalization tools to the construction of rough paths and fractionary stochastic calculus.

The article by Dominique Manchon describes various classes of bialgebras and Hopf algebras of oriented graphs. In particular, the author constructs two commutative connected graded Hopf algebras, such that one is a comodule-coalgebra on the other. These algebras provide a generalization of some Hopf algebras of rooted trees extensively used within the Connes–Kreimer approach to the combinatorics of perturbative renormalization.

The last article by Dmitry Tamarkin presents a mathematically consistent procedure to construct the Batalin–Vilkovisky bracket on the space of observables for a free scalar Boson in a Euclidean space. The author follows the approach of Cattaneo and Felder for sigma models, but within the framework of $\mathcal{D}$-modules. He then interprets renormalization as the way to treat the ambiguity in the definition of a deformation of the Poisson algebra of observables.

We are indebted to various organizations for the financial support of the conference "Renormalization: Algebraic, Geometric and Probabilistic Aspects". Let us first thank the GDR (groupement de recherche) Renormalization, funded by the CNRS, without which this conference would not have taken place. We are also very grateful to the Institut Camille Jordan for its financial help and for kindly putting at our disposal all the facilities needed for the favourable progress of the conference. We also acknowledge the support offered by the University of Lyon and the Mathematics Department of Lyon 1 University. Special thanks to Sylvie Paycha who initiated the GDR Renormalization, and to Monique Gaffier for her constant and invaluable help.

D. Calaque, *A. Frabetti*, *D. Perrot*
and F. Vignes-Tourneret