Discrete transparent boundary conditions for the linearised Green-Naghdi equations

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We consider the classical linearised Green-Naghdi one-dimensional system of equations which models the small amplitude and long waves propagation on the free water surface:

$$\begin{cases} \eta_t + w_x = 0, \\ w_t + \eta_x - \varepsilon w_{txx} = 0, \end{cases} \qquad x \in \mathbb{R}, t > 0$$

here ε is a parameter of dispersion. The system describes the evolution of free surface elevation η and fluid velocity w.

While the original system is set on the whole space, usually for practical applications the area of study is restricted and we should fix the artificial boundary conditions. What type of the conditions should be imposed is a key question for the analysis and mainly for numerical simulations of the dispersive equations. Moreover these boundary conditions should not affect on the solution. It needs to be taken into account that the energy of exact solution for the problem set to the whole space is conserved whereas the energy of restricted solution should decrease. The purpose of presented study is to find the suitable boundary conditions, which absorb some energy at the boundaries and lead to the well-posed initial boundary value problem.

According to techniques presented in [1]-[3] we will derive the continuous and discrete conditions to the system of equations. In continuous case it is required to compute time convolutions and invert the Laplace transform of an analytic function and it can be done explicitly. In contrast the construction of discrete conditions involve the non-explicit inversion of the Z-transform of an holomorphic function. And we propose the stable numerical procedure for this inversion. The dissipative property of proposed conditions is proved and the consistence of discrete conditions with the continuous ones is shown.

References

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