DERIVATION AND ANALYSIS OF SHALLOW WATER-TYPE MODEL WITH VERTICAL PROFILE

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The propagation of surface waves is an essential issue for many applications such as harbor planning, tsunami propagation or marine energies. The dynamic of an incompressible, homogeneous, inviscid fluid is governed by the free surface Euler model (E), also called water waves equations. Unfortunately, (E) is too complex to be simulated for geophysical applications and reduced models are required. The simpler and the most commonly used for applications nowadays is the non-linear Shallow Water model (SW). The main advantage of (SW) is its mathematical structure, i.e. hyperbolic, which has led to the design of robust and accurate numerical strategies. However, it is well-known that (SW) is not a satisfactory model for the propagation of the waves. More precisely, the main assumptions to derive (SW) from (E) are:

H^p_{yp}) the so-called hydrostatic pressure,
H^u_{yp}) the homogeneity of the horizontal velocity in the column of water.

To skirt the second H^u_{yp}), the layerwise semi-discrete shallow water model (SW_L) was proposed [1]. The first part of the talk will be devoted to the derivation of this model and its complete analysis in the case of two layers. Due to the vertical exchange, several additional difficulties appear such as nonconservative products, resonances and coalescence of waves.

The second part of the talk will be devoted to the formal derivation of a layerwise version of the Green-Naghdi model (GN_L), i.e. skirting H^p_{yp}). The dispersion law of (GN_L) converge to the dispersion law of (E) when the number of layer becomes large which acts as a weak argument of consistence. Some numerical simulations of (GN_L) will be shown to illustrate the impact of the layerwise discretization.

References

[1] Audusse, E., Bristeau, M.-O., Perthame, B., and Sainte-Marie, J. A multilayer Saint-Venant system with mass exchanges for shallow water flows. Derivation and numerical validation. *ESAIM: M2AN 45*, 1 (2011), 169–200.

[2] N. Aguillon, E. Audusse, E. Godlewski, and M. Parisot, *Analysis of the Riemann Problem for a shallow water model with two velocities.* working paper or preprint, hal-01618722.

[3] Fernandez-Nieto, E. D., Parisot, M., Penel, Y., and Sainte-Marie, J. Layer-averaged approximation of Euler equations for free surface flows with a non-hydrostatic pressure. working paper or preprint, hal-01324012.

[4] Parisot, M. Entropy-satisfying scheme for a hierarchy of dispersive reduced models of free surface flow. working paper or preprint, hal-01242128.

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