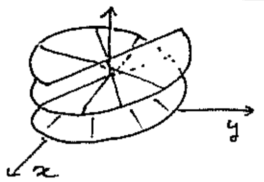


<u>Hélicoïde</u> $S = \{ f(u,v) = (v\cos u,\ v\sin u,\ au) \mid u,v \in \mathbb{R} \}$

$$I_f(u,v) = \begin{pmatrix} a^2+v^2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$II_f(u,v) = \frac{a}{\sqrt{a^2+v^2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Longrightarrow -dN = \begin{pmatrix} 0 & \frac{a}{(a^2+v^2)^{3/2}} \\ \frac{a}{\sqrt{a^2+v^2}} & 0 \end{pmatrix}$$

$K = -\frac{a^2}{(a^2+v^2)^2} < 0$, $H = 0 \Rightarrow$ surface minimale

courbures principales : $k_+ = \frac{a}{a^2+v^2}$, $k_- = -\frac{a}{a^2+v^2}$

directions principales : $\vec{v}_+ = f_u + \sqrt{a^2+v^2}\, f_v$, $\vec{v}_- = f_u - \sqrt{a^2+v^2}\, f_v$

<u>lignes de courbure</u> : $\gamma_\pm(t) = f(u(t), v(t))$ t.q. $\gamma_\pm'(t) = \vec{v}_\pm = f_u \pm \sqrt{a^2+v^2}\, f_v$

$$\Leftrightarrow \begin{cases} u'(t) = 1 \\ v'(t) = \pm\sqrt{a^2+v^2} \end{cases} \Leftrightarrow \begin{cases} u(t) = t + u_0 & \text{(en reparamétrisant : } u_0 = 0) \\ \frac{v'(t)}{\sqrt{a^2+v(t)^2}} = \pm 1 \end{cases} \Leftrightarrow$$

$$\pm\int_0^t dt = \int_0^t \frac{v'(t)\,dt}{\sqrt{a^2+v(t)^2}} = \int_{v_0}^{v(t)} \frac{\frac{1}{a}\,dv}{\sqrt{1+\left(\frac{v}{a}\right)^2}} = \Big[\operatorname{argsh} v\Big]_{v_0}^{v(t)} = \operatorname{argsh} v(t) - \operatorname{argsh} v_0$$

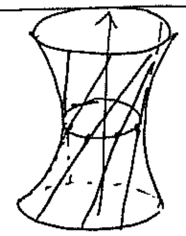
$$\Leftrightarrow \operatorname{argsh} v(t) = \operatorname{argsh} v_0 \pm t \Leftrightarrow v(t) = \operatorname{sh}(\pm t + \operatorname{argsh} v_0) = v_0 \operatorname{ch} t \pm \sqrt{1+v_0^2}\, \operatorname{sh} t.$$

$$\Rightarrow \gamma_\pm(t) = f(u(t), v_\pm(t)) = (\pm \operatorname{sh} t \cos t,\ \pm \operatorname{sh} t \sin t,\ at).$$

<u>Courbes asymptotiques</u> : $\gamma(t) = f(u(t), v(t))$ t.q. $II(\gamma'(t), \gamma'(t)) = 0 \Leftrightarrow u'(t)\,v'(t) = 0$

1) $u'(t) = 0, \forall v(t) \Rightarrow \gamma(t) = f(t, v(t)) = (v(t)\cos t,\ v(t)\sin t,\ at)$

2) $v'(t) = 0, \forall u(t) \Rightarrow \gamma(t) = f(u(t), t) = (t\cos u(t),\ t\sin u(t),\ a\,u(t))$.



<u>Hyperboloïde à une nappe</u> $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 - z^2 = 1\}$

$S = \{ f(u,v) = (\cos u - v\sin u,\ \sin u + v\cos u,\ v) \mid u \in [0, 2\pi[,\ v \in \mathbb{R} \}$

$$I_f(u,v) = \begin{pmatrix} 1+v^2 & 1 \\ 1 & 2 \end{pmatrix}, \quad II_f(u,v) = -\frac{1}{\sqrt{1+2v^2}} \begin{pmatrix} 1+v^2 & 1 \\ 1 & 0 \end{pmatrix}$$

$$-dN = -\frac{1}{(1+2v^2)^{3/2}} \begin{pmatrix} 1+2v^2 & 2 \\ 0 & -1 \end{pmatrix}$$

$K = -\frac{1}{(1+2v^2)^2} < 0$, $H = -\frac{v^2}{(1+2v^2)^{3/2}} \neq 0$ hors cercle central.

Courbures principales : $k_1 = -\frac{1}{\sqrt{1+2v^2}} < 0$, $k_2 = \frac{1}{(1+2v^2)^{3/2}} > 0$

Dir. principales : $\vec{v}_1 = f_u$, $\vec{v}_2 = f_u - (1+v^2) f_v$.

Lignes de courbure : 1) $\gamma(t) = f(t, v_0)$

2) $\gamma(t) = f(u(t), v(t))$ t.q. $\begin{cases} u'(t) = 1 \\ v'(t) = -(1+v^2) \end{cases} \Leftrightarrow \begin{cases} u(t) = t + u_0 & (u_0 = 0) \\ \arctan v(t) = \arctan v_0 - t \end{cases}$

$$\Leftrightarrow v(t) = \tan(\arctan v_0 - t) = \frac{v_0 - \tan t}{1 + v_0 \tan t}.$$

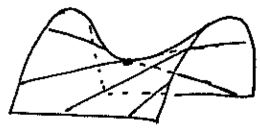
Courbes asymptotiques : $\gamma(t) = f(u(t), v(t))$ t.q. $II(\gamma'(t), \gamma'(t)) = 0 \Leftrightarrow u'(t)\left[(1+v(t)^2)u'(t) + 2v'(t)\right] = 0$

1) $u'(t) = 0,\ \forall v(t) \Rightarrow \gamma(t) = f(t, v(t))$.

2) $\frac{v'(t)}{1+v^2(t)} = -\frac{1}{2} u'(t) \Leftrightarrow \arctan v(t) = \arctan v_0 - \frac{1}{2}(u(t) - u_0) \Leftrightarrow v(t) = \tan\left(\frac{-u(t) + u_0 + 2\arctan v_0}{2}\right)$

etc.

2) <u>Paraboloïde hyperbolique</u> $S = \{(x,y,z) \in \mathbb{R}^3 \mid z = xy\}$



$S = \{ f(u,v) = (u, v, uv) \mid u,v \in \mathbb{R} \}$

$$I_f(u,v) = \begin{pmatrix} 1+v^2 & uv \\ uv & 1+u^2 \end{pmatrix} , \quad II_f(u,v) = \begin{pmatrix} 0 & \frac{1}{\sqrt{1+u^2+v^2}} \\ \frac{1}{\sqrt{1+u^2+v^2}} & 0 \end{pmatrix}$$

$$-dN(u,v) = \frac{1}{(1+u^2+v^2)^{3/2}} \begin{pmatrix} -uv & 1+u^2 \\ 1+v^2 & -uv \end{pmatrix} \Rightarrow K = -\frac{1}{(1+u^2+v^2)} < 0 , \quad H = -\frac{uv}{(1+u^2+v^2)^{3/2}}.$$

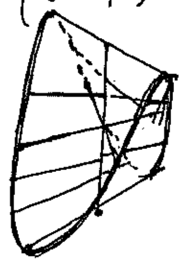
courbures principales : $k_\pm = \dfrac{-uv \pm \sqrt{(1+u^2)(1+v^2)}}{(1+u^2+v^2)^{3/2}}$ $\Rightarrow$ lignes de courbure horribles.

courbes asymptotiques : $\gamma(t) = f(u(t), v(t))$ t.q. $u'(t)\, v'(t) = 0 \Leftrightarrow$

1) $u'(t) = 0$, $\forall v(t) \Rightarrow \gamma(t) = f(t, v(t)) = (t, v(t), t v(t))$, e.g. $v(t) = v_0 \Rightarrow$ droite $\{y = v_0 , z = v_0 x\}$

2) $v'(t) = 0$, $\forall u(t) \Rightarrow \gamma(t) = f(u(t), t) = (u(t), t, t u(t))$, e.g. $u(t) = u_0 \Rightarrow$ droite $\{x = u_0 , z = u_0 y\}$

<u>Conoïde de Plucker</u> $S = \left\{(x,y,z) \in \mathbb{R}^3 \mid z = \frac{xy}{x^2+y^2}\right\}$ ou bien $\{(x^2+y^2)z = xy\}$



$S = \left\{ f(u,v) = \left(v(1 \pm \sqrt{1-4u^2}), 2uv, u \right) \mid u \in [-1/2, 1/2], v \in \mathbb{R} \right\}$

$$I_f(u,v) = \begin{pmatrix} \frac{4v^2 - 4u^2 + 1}{1-4u^2} & \mp \frac{4uv}{\sqrt{1-4u^2}} \\ \mp \frac{4uv}{\sqrt{1-4u^2}} & 2(1 \pm \sqrt{1-4u^2}) \end{pmatrix}$$

$$II_f(u,v) = \frac{1}{\sqrt{\det I}} \begin{pmatrix} \frac{\pm 8uv}{(1-4u^2)^{3/2}} & \frac{\pm 8u^2}{\sqrt{1-4u^2}} + 2 \pm 2\sqrt{1-4u^2} \\ \text{idem} & 0 \end{pmatrix}$$

$-dN$, K, H, k_1, k_2 horribles....
