FORMULAIRE SUR LES CHAMPS VECTORIELS DE $\mathbb{R}^3$

	Coordonnées cartesiennes (x, y, z)	**Coordonnées cylindriques** (ρ, θ, z)	**Coordonnées sphériques** (r, θ, φ)
Champ de vecteurs $\vec{V}$	$V_x\ \vec{i} + V_y\ \vec{j} + V_z\ \vec{k}$	$V_\rho\ \vec{e_\rho} + V_\theta\ \vec{e_\theta} + V_z\ \vec{k}$	$V_r\ \vec{e_r} + V_\theta\ \vec{e_\theta} + V_\varphi\ \vec{e_\varphi}$
Gradient $\overrightarrow{\text{grad}}\, f = \vec{\nabla} f$	$\frac{\partial f}{\partial x}\ \vec{i} + \frac{\partial f}{\partial y}\ \vec{j} + \frac{\partial f}{\partial z}\ \vec{k}$	$\frac{\partial f}{\partial \rho}\ \vec{e_\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \theta}\ \vec{e_\theta} + \frac{\partial f}{\partial z}\ \vec{k}$	$\frac{\partial f}{\partial r}\ \vec{e_r} + \frac{1}{r}\frac{\partial f}{\partial \varphi}\ \vec{e_\varphi} + \frac{1}{r\cos\varphi}\frac{\partial f}{\partial \theta}\ \vec{e_\theta}$
Divergence $\text{div}\, \vec{V} = \nabla \cdot \vec{V}$	$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho V_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 V_r)}{\partial r} + \frac{1}{r\cos\varphi}\frac{\partial(\cos\varphi\ V_\varphi)}{\partial \varphi} + \frac{1}{r\cos\varphi}\frac{\partial V_\theta}{\partial \theta}$
Rotationnel $\overrightarrow{\text{rot}}\, \vec{V} = \vec{\nabla} \times \vec{V}$	$\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\ \vec{i} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\ \vec{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\ \vec{k}$	$\left(\frac{1}{\rho}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}\right)\ \vec{e_\rho} + \left(\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial \rho}\right)\ \vec{e_\theta} + \frac{1}{\rho}\left(\frac{\partial(\rho V_\theta)}{\partial \rho} - \frac{\partial V_\rho}{\partial \theta}\right)\ \vec{k}$	$\frac{1}{r\ \cos\varphi}\left(\frac{\partial(\cos\varphi\ V_\theta)}{\partial \varphi} - \frac{\partial V_\varphi}{\partial \theta}\right)\ \vec{e_r} + \frac{1}{r}\left(\frac{1}{\cos\varphi}\frac{\partial V_r}{\partial \theta} - \frac{\partial(r V_\theta)}{\partial r}\right)\ \vec{e_\varphi} + \frac{1}{r}\left(\frac{\partial(r V_\varphi)}{\partial r} - \frac{\partial V_r}{\partial \varphi}\right)\ \vec{e_\theta}$
Laplacien $\Delta f = \vec{\nabla} \cdot \vec{\nabla} f = \text{div}\,(\overrightarrow{\text{grad}}\, f)$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\cos\varphi}\frac{\partial}{\partial \varphi}\left(\cos\varphi\ \frac{\partial f}{\partial \varphi}\right) + \frac{1}{r^2\cos^2\varphi}\frac{\partial^2 f}{\partial \theta^2}$
Laplacien vectoriel $\Delta\vec{V}$	$\Delta V_x\ \vec{i} + \Delta V_y\ \vec{j} + \Delta V_z\ \vec{k}$	(affreux...)	(horrible !)

Identités : $\overrightarrow{\text{rot}}\,(\overrightarrow{\text{grad}}\, f) = \vec{\nabla} \times (\vec{\nabla} f) = \vec{0}$ $\qquad$ $\text{div}\,(\overrightarrow{\text{rot}}\, \vec{V}) = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{V}) = 0$

$$\overrightarrow{\text{rot}}\,(\overrightarrow{\text{rot}}\, \vec{V}) = \vec{\nabla} \times (\vec{\nabla} \times \vec{V}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{V}) - \Delta\vec{V} = \overrightarrow{\text{grad}}\,(\text{div}\, \vec{V}) - \Delta\vec{V}$$

Théorème de Poincaré : Sur $D \subset \mathbb{R}^3$ *simplement connexe* : $\quad \vec{V} = \overrightarrow{\text{grad}}\, f \iff \overrightarrow{\text{rot}}\, \vec{V} = 0$

Notations : $\vec{\nabla} = \frac{\partial}{\partial x}\ \vec{i} + \frac{\partial}{\partial y}\ \vec{j} + \frac{\partial}{\partial z}\ \vec{k}$ $\qquad$ $\Delta \equiv \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ $\qquad$ $\vec{\nabla} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$

$\vec{V} \cdot \vec{\nabla} = V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y} + V_z\frac{\partial}{\partial z}$ $\qquad$ $(\vec{V} \cdot \vec{\nabla})\vec{U} = V_x\frac{\partial U_x}{\partial x}\vec{i} + V_y\frac{\partial U_y}{\partial y}\vec{j} + V_z\frac{\partial U_z}{\partial z}\vec{k}$

Proprietés : $\overrightarrow{\text{grad}}\,(fg) = (\overrightarrow{\text{grad}}\, f)g + f(\overrightarrow{\text{grad}}\, g)$ $\qquad$ $\overrightarrow{\text{grad}}\,(\vec{U} \cdot \vec{V}) = (\vec{U} \cdot \vec{\nabla})\vec{V} + (\vec{V} \cdot \vec{\nabla})\vec{U} + \vec{U} \wedge \overrightarrow{\text{rot}}\, \vec{V} + \vec{V} \wedge \overrightarrow{\text{rot}}\, \vec{U}$

$\text{div}\,(f\vec{V}) = (\overrightarrow{\text{grad}}\, f)\vec{V} + f(\text{div}\, \vec{V})$ $\qquad$ $\text{div}\,(\vec{U} \wedge \vec{V}) = (\overrightarrow{\text{rot}}\, \vec{U}) \cdot \vec{V} - \vec{U} \cdot (\overrightarrow{\text{rot}}\, \vec{V})$

$\overrightarrow{\text{rot}}\,(f\vec{V}) = (\overrightarrow{\text{grad}}\, f) \wedge \vec{V} + f\ (\overrightarrow{\text{rot}}\, \vec{V})$ $\qquad$ $\overrightarrow{\text{rot}}\,(\vec{U} \wedge \vec{V}) = (\text{div}\, \vec{V})\ \vec{U} - \vec{V}\ (\text{div}\, \vec{U}) + (\vec{V} \cdot \vec{\nabla})\ \vec{U} - (\vec{U} \cdot \vec{\nabla})\ \vec{V}$

$\Delta(fg) = f\,\Delta g + 2\,\overrightarrow{\text{grad}}\, f \cdot \overrightarrow{\text{grad}}\, g + \Delta f\ g$