FORMULAIRE SUR LES CHAMPS VECTORIELS DE $\mathbb{R}^3$

	Coordonnées cartesiennes (x,y,z)	**Coordonnées cylindriques** (ρ,θ,z)	**Coordonnées sphériques** (r,θ,φ)
Champ de vecteurs $\vec{V}$	$V_x\ \vec{i}+V_y\ \vec{j}+V_z\ \vec{k}$	$V_\rho\ \vec{e_\rho}+V_\theta\ \vec{e_\theta}+V_z\ \vec{k}$	$V_r\ \vec{e_r}+V_\theta\ \vec{e_\theta}+V_\varphi\ \vec{e_\varphi}$
Gradient $\overrightarrow{\text{grad}}\, f=\vec{\nabla} f$	$\frac{\partial f}{\partial x}\ \vec{i}+\frac{\partial f}{\partial y}\ \vec{j}+\frac{\partial f}{\partial z}\ \vec{k}$	$\frac{\partial f}{\partial \rho}\ \vec{e_\rho}+\frac{1}{\rho}\frac{\partial f}{\partial \theta}\ \vec{e_\theta}+\frac{\partial f}{\partial z}\ \vec{k}$	$\frac{\partial f}{\partial r}\ \vec{e_r}+\frac{1}{r}\frac{\partial f}{\partial \varphi}\ \vec{e_\varphi}+\frac{1}{r\cos\varphi}\frac{\partial f}{\partial \theta}\ \vec{e_\theta}$
Divergence $\text{div}\,\vec{V}=\nabla\cdot\vec{V}$	$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho V_\rho)}{\partial \rho}+\frac{1}{\rho}\frac{\partial V_\theta}{\partial \theta}+\frac{\partial V_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 V_r)}{\partial r}+\frac{1}{r\cos\varphi}\frac{\partial(\cos\varphi\ V_\varphi)}{\partial \varphi}+\frac{1}{r\cos\varphi}\frac{\partial V_\theta}{\partial \theta}$
Rotationnel $\overrightarrow{\text{rot}}\,\vec{V}=\vec{\nabla}\times\vec{V}$	$\left(\frac{\partial V_z}{\partial y}-\frac{\partial V_y}{\partial z}\right)\vec{i}+\left(\frac{\partial V_x}{\partial z}-\frac{\partial V_z}{\partial x}\right)\vec{j}+\left(\frac{\partial V_y}{\partial x}-\frac{\partial V_x}{\partial y}\right)\vec{k}$	$\left(\frac{1}{\rho}\frac{\partial V_z}{\partial \theta}-\frac{\partial V_\theta}{\partial z}\right)\vec{e_\rho}+\left(\frac{\partial V_\rho}{\partial z}-\frac{\partial V_z}{\partial \rho}\right)\vec{e_\theta}+\frac{1}{\rho}\left(\frac{\partial(\rho V_\theta)}{\partial \rho}-\frac{\partial V_\rho}{\partial \theta}\right)\vec{k}$	$\frac{1}{r\ \cos\varphi}\left(\frac{\partial(\cos\varphi\ V_\theta)}{\partial \varphi}-\frac{\partial V_\varphi}{\partial \theta}\right)\vec{e_r}+\frac{1}{r}\left(\frac{1}{\cos\varphi}\frac{\partial V_r}{\partial \theta}-\frac{\partial(rV_\theta)}{\partial r}\right)\vec{e_\varphi}+\frac{1}{r}\left(\frac{\partial(rV_\varphi)}{\partial r}-\frac{\partial V_r}{\partial \varphi}\right)\vec{e_\theta}$
Laplacien $\Delta f=\vec{\nabla}\cdot\vec{\nabla} f$ $=\text{div}\,(\overrightarrow{\text{grad}}\, f)$	$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 f}{\partial \theta^2}+\frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)+\frac{1}{r^2\cos\varphi}\frac{\partial}{\partial \varphi}\left(\cos\varphi\ \frac{\partial f}{\partial \varphi}\right)+\frac{1}{r^2\cos^2\varphi}\frac{\partial^2 f}{\partial \theta^2}$
Laplacien vectoriel $\Delta\vec{V}$	$\Delta V_x\ \vec{i}+\Delta V_y\ \vec{j}+\Delta V_z\ \vec{k}$	(affreux...)	(horrible !)

Notations : $\vec{\nabla}=\frac{\partial}{\partial x}\ \vec{i}+\frac{\partial}{\partial y}\ \vec{j}+\frac{\partial}{\partial z}\ \vec{k}$ $\qquad \Delta\equiv\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ $\qquad \vec{\nabla}\times\vec{V}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ V_x & V_y & V_z\end{vmatrix}$

$\vec{V}\cdot\vec{\nabla}=V_x\frac{\partial}{\partial x}+V_y\frac{\partial}{\partial y}+V_z\frac{\partial}{\partial z}$ $\qquad (\vec{V}\cdot\vec{\nabla})\vec{U}=V_x\frac{\partial U_x}{\partial x}\ \vec{i}+V_y\frac{\partial U_y}{\partial y}\ \vec{j}+V_z\frac{\partial U_z}{\partial z}\ \vec{k}$

Proprietés :

$\overrightarrow{\text{grad}}\,(fg)=(\overrightarrow{\text{grad}}\, f)g+f(\overrightarrow{\text{grad}}\, g)$ $\qquad \overrightarrow{\text{grad}}\,(\vec{U}\cdot\vec{V})=(\vec{U}\cdot\vec{\nabla})\vec{V}+(\vec{V}\cdot\vec{\nabla})\vec{U}+\vec{U}\wedge\overrightarrow{\text{rot}}\,\vec{V}+\vec{V}\wedge\overrightarrow{\text{rot}}\,\vec{U}$

$\text{div}\,(f\vec{V})=(\overrightarrow{\text{grad}}\, f)\vec{V}+f(\text{div}\,\vec{V})$ $\qquad \text{div}\,(\vec{U}\wedge\vec{V})=(\overrightarrow{\text{rot}}\,\vec{U})\cdot\vec{V}-\vec{U}\cdot(\overrightarrow{\text{rot}}\,\vec{V})$

$\overrightarrow{\text{rot}}\,(f\vec{V})=(\overrightarrow{\text{grad}}\, f)\wedge\vec{V}+f\ (\overrightarrow{\text{rot}}\,\vec{V})$ $\qquad \overrightarrow{\text{rot}}\,(\vec{U}\wedge\vec{V})=(\text{div}\,\vec{V})\ \vec{U}-\vec{V}\ (\text{div}\,\vec{U})+(\vec{V}\cdot\vec{\nabla})\ \vec{U}-(\vec{U}\cdot\vec{\nabla})\ \vec{V}$

$\Delta(fg)=f\,\Delta g+2\,\overrightarrow{\text{grad}}\, f\cdot\overrightarrow{\text{grad}}\, g+\Delta f\ g$

Identités : $\overrightarrow{\text{rot}}\,(\overrightarrow{\text{grad}}\, f)=\vec{\nabla}\times(\vec{\nabla} f)=\vec{0}$ $\qquad \text{div}\,(\overrightarrow{\text{rot}}\,\vec{V})=\vec{\nabla}\cdot(\vec{\nabla}\times\vec{V})=0$

$\overrightarrow{\text{rot}}\,(\overrightarrow{\text{rot}}\,\vec{V})=\vec{\nabla}\times(\vec{\nabla}\times\vec{V})=\vec{\nabla}(\vec{\nabla}\cdot\vec{V})-\Delta\vec{V}=\overrightarrow{\text{grad}}\,(\text{div}\,\vec{V})-\Delta\vec{V}$

Théorème de Poincaré : Sur $D\subset\mathbb{R}^3$ *simplement connexe* : $\vec{V}=\overrightarrow{\text{grad}}\, f \iff \overrightarrow{\text{rot}}\,\vec{V}=0$

Sur $D\subset\mathbb{R}^3$ *contractile* : $\vec{V}=\overrightarrow{\text{rot}}\,\vec{U} \iff \text{div}\,\vec{V}=0$

Théorème de Stokes : Si $\vec{V}=\overrightarrow{\text{rot}}\,\vec{U}$: $\iint_{S^+}\vec{V}\cdot\vec{dS}=\oint_{\partial S^+}\vec{U}\cdot\vec{d\ell}$

Théorème de Ostrogradski : Si $S^+=\partial\Omega$ est une surface fermée : $\oiint_{S^+}\vec{V}\cdot\vec{dS}=\iiint_\Omega \text{div}\,\vec{V}\ dx\ dy\ dz$

Corollaires : Si $\vec{V}=\overrightarrow{\text{grad}}\, f$ et C^+ est une courbe fermée : $\oint_{C^+}\vec{V}\cdot\vec{d\ell}=0$

Si $\vec{V}=\overrightarrow{\text{grad}}\, f$ et C^+ est une courbe qui relie A à B : $\int_{C^+}\vec{V}\cdot\vec{d\ell}=f(B)-f(A)$