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PCSI L1 UE Math2
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FORMULAIRE SUR LES REPÈRES MOBILES DE $\mathbb{R}^3$

	Coordonnées cartesiennes	Coordonnées cylindriques	Coordonnées sphériques
Point P	$(x,y,z) \quad x,y,z \in \mathbb{R}$	$(\rho,\theta,z) \begin{bmatrix} \rho \geq 0 \\ \theta \in [0,2\pi[\\ z \in \mathbb{R} \end{bmatrix}$	$(r,\theta,\varphi) \begin{bmatrix} r \geq 0 \\ \theta \in [0,2\pi[\\ \varphi \in]-\frac{\pi}{2},\frac{\pi}{2}[\end{bmatrix}$
Chang. coordonnées	$\begin{cases} \rho = \sqrt{x^2+y^2} \\ \theta = \begin{cases} \arctan\frac{x}{y} & \text{si } x>0 \\ \frac{\pi}{2} & \text{si } x=0 \\ \arctan\frac{x}{y}+\pi & \text{si } x<0 \end{cases} \\ z = z \end{cases}$ $\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \text{idem} \\ \varphi = \arcsin\frac{z}{\sqrt{x^2+y^2+z^2}} \end{cases}$	$\begin{cases} x = \rho\,\cos\theta \\ y = \rho\,\sin\theta \\ z = z \end{cases}$ $\begin{cases} r = \sqrt{\rho^2+z^2} \\ \theta = \theta \\ \varphi = \arcsin\frac{z}{\sqrt{\rho^2+z^2}} \end{cases}$	$\begin{cases} x = r\,\cos\theta\,\cos\varphi \\ y = r\,\sin\theta\,\cos\varphi \\ z = r\,\sin\varphi \end{cases}$ $\begin{cases} \rho = r\,\cos\varphi \\ \theta = \theta \\ z = r\,\sin\varphi \end{cases}$
Repère en P	$(P;\ \vec{i},\vec{j},\vec{k}) \begin{bmatrix} \vec{i} = \dfrac{\partial}{\partial x} \\ \vec{j} = \dfrac{\partial}{\partial y} \\ \vec{k} = \dfrac{\partial}{\partial z} \end{bmatrix}$	$(P;\ \vec{e_\rho},\vec{e_\theta},\vec{k}) \begin{bmatrix} \vec{e_\rho} = \dfrac{\partial}{\partial \rho} \\ \vec{e_\theta} = \dfrac{1}{\rho}\dfrac{\partial}{\partial \theta} \\ \vec{k} = \dfrac{\partial}{\partial z} \end{bmatrix}$	$(P;\ \vec{e_r},\vec{e_\theta},\vec{e_\varphi}) \begin{bmatrix} \vec{e_r} = \dfrac{\partial}{\partial r} \\ \vec{e_\theta} = \dfrac{1}{r\cos\varphi}\dfrac{\partial}{\partial \theta} \\ \vec{e_\varphi} = \dfrac{1}{r}\dfrac{\partial}{\partial \varphi} \end{bmatrix}$
Chang. repères	$\begin{cases} \vec{e_\rho} = \cos\theta\ \vec{i} + \sin\theta\ \vec{j} \\ \vec{e_\theta} = -\sin\theta\ \vec{i} + \cos\theta\ \vec{j} \\ \vec{k} = \vec{k} \end{cases}$ $\begin{cases} \vec{e_r} = \cos\theta\ \cos\varphi\ \vec{i} + \sin\theta\ \cos\varphi\ \vec{j} \\ \qquad + \sin\varphi\ \vec{k} \\ \vec{e_\theta} = -\sin\theta\ \vec{i} + \cos\theta\ \vec{j} \\ \vec{e_\varphi} = -\cos\theta\ \sin\varphi\ \vec{i} - \sin\theta\ \sin\varphi\ \vec{j} \\ \qquad + \cos\varphi\ \vec{k} \end{cases}$	$\begin{cases} \vec{i} = \cos\theta\ \vec{e_\rho} - \sin\theta\ \vec{e_\theta} \\ \vec{j} = \sin\theta\ \vec{e_\rho} + \cos\theta\ \vec{e_\theta} \\ \vec{k} = \vec{k} \end{cases}$ $\begin{cases} \vec{e_r} = \cos\varphi\ \vec{e_\rho} + \sin\varphi\ \vec{k} \\ \vec{e_\theta} = \vec{e_\theta} \\ \vec{e_\varphi} = -\sin\varphi\ \vec{e_\rho} + \cos\varphi\ \vec{k} \end{cases}$	$\begin{cases} \vec{i} = \cos\theta\cos\varphi\ \vec{e_r} - \sin\theta\ \vec{e_\theta} \\ \qquad - \cos\theta\sin\varphi\ \vec{e_\varphi} \\ \vec{j} = \sin\theta\cos\varphi\ \vec{e_r} + \cos\theta\ \vec{e_\theta} \\ \qquad - \sin\theta\sin\varphi\ \vec{e_\varphi} \\ \vec{k} = \sin\varphi\ \vec{e_r} + \cos\varphi\ \vec{e_\varphi} \end{cases}$ $\begin{cases} \vec{e_\rho} = \cos\varphi\ \vec{e_r} - \sin\varphi\ \vec{e_\varphi} \\ \vec{e_\theta} = \vec{e_\theta} \\ \vec{k} = \sin\varphi\ \vec{e_r} + \cos\varphi\ \vec{e_\varphi} \end{cases}$