CC2 – Math2 18 mars 2015 — Corrigé — A. Frabetti

5pts **Exo 1** $W(R,I,t) = R I^2 t$, $R \geq 0$, $I \geq 0$, $t \geq 0$

1. $\frac{\partial W}{\partial R}(R,I,t) = I^2 t$ 1pt $\frac{\partial W}{\partial I}(R,I,t) = 2RIt$ 1pt $\frac{\partial W}{\partial t}(R,I,t) = R I^2$ 1pt

2. $\vec{\nabla} W(R,I,t) = \begin{pmatrix} I^2 t \\ 2RIt \\ RI^2 \end{pmatrix}$ ← 1pt

3. $dW_{(R,I,t)} = I^2 t \, dR + 2RIt \, dI + R I^2 \, dt$ ← 1pt

4. $dW_{(2,300,20)} = 300^2 \cdot 20 \, dR + 2 \times 2 \times 300 \times 20 \, dI + 2 \times 300^2 \, dt$
$= 1800000 \, dR + 24000 \, dI + 180000 \, dt$ 1pt

Exo 2 $h(x,y,z) = \left(x^2 z, \frac{x}{y}\right)$ 6pts

1. $D_h = \{(x,y,z) \mid y \neq 0\} = \mathbb{R}^3$ privé du plan $\{y=0\}$

← 1pt si apparait (x,y,z) ou le fait que $D_h \subset \mathbb{R}^3$
← 1pt si $y \neq 0$

2. $J_h(x,y,z) = \begin{pmatrix} 2xz & 0 & x^2 \\ \frac{1}{y} & -\frac{x}{y^2} & 0 \end{pmatrix}$

← 1pt pour bonne formule $\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$
← 0,5pt par dérivée

Exo 3 $\frac{\partial f}{\partial x}(x,y) = \frac{2y}{(x+y)^2}$ $\frac{\partial f}{\partial y}(x,y) = -\frac{2x}{(x+y)^2}$ 8 pts

1. $F(\rho,\varphi) = f(\rho\cos\varphi, \rho\sin\varphi) \Rightarrow x = \rho\cos\varphi$ et $y = \rho\sin\varphi$

$$\frac{\partial F}{\partial \rho} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial \rho} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \rho} = \frac{2\cancel{\rho}\sin\varphi}{\rho^2(\cos\varphi+\sin\varphi)^2} \cdot \cos\varphi - \frac{2\cancel{\rho}\cos\varphi}{\rho^2(\cos\varphi+\sin\varphi)^2} \cdot \sin\varphi = 0$$

$$\frac{\partial F}{\partial \varphi} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial \varphi} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial \varphi} = \frac{2\sin\varphi}{\cancel{\rho}(\cos\varphi+\sin\varphi)^2} \times (-\cancel{\rho}\sin\varphi) - \frac{2\cos\varphi}{\cancel{\rho}(\cos\varphi+\sin\varphi)^2} \times \cancel{\rho}\cos\varphi$$

$$= \frac{-2(\sin^2\varphi + \cos^2\varphi)}{(\cos\varphi+\sin\varphi)^2} = \frac{-2}{(\cos\varphi+\sin\varphi)^2}$$

Pour chaque dérivée : 1 pt. formule
+ 1,5 pts calculs

2. $G(t) = f(t, t^3) \Rightarrow x = t, \; y = t^3$

$$G'(t) = \frac{\partial f}{\partial x} \cdot x'(t) + \frac{\partial f}{\partial y} \cdot y'(t)$$
$$= \frac{2t^3}{(t+t^3)^2} \times 1 - \frac{2t}{(t+t^3)^2} \times 3t^2 = \frac{2t^3 - 6t^3}{t^2(1+t^2)^2} = \frac{-4t^{\cancel{3}}}{\cancel{t^2}(1+t^2)^2}$$
$$= -\frac{4t}{(1+t^2)^2}.$$

1 pt formule
+ 2 pts calculs