Errata of the book
Analysis and Geometry of Markov Diffusion Operators
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- Page 35, line 10 : instead of
$$\sum_{y\in E}\mu(x)\mathrm{L}(x,y)=0$$
read
$$\sum_{x\in E}\mu(x)\mathrm{L}(x,y)=0,$$
thanks to Michał Strzelecki.

- Page 36, line 4 : instead of $\hat{L}$ read K, thanks again to Michał Strzelecki.

- Page 39, line -1 : instead of $F(x,T)$ read $F(T,x)$, thanks to Michał Strzelecki.

- Page 42, line -4 : the matrix $\mathfrak{g}$ is supposed to be also definite-positive, thanks to Michał Strzelecki.

- Page 49, line -13 : instead of $g=\psi'(f)$ read $g=\psi(f)$, thanks to Michał Strzelecki.

- Page 79, line -5 : instead of
$$\sum_{i,j=1}^n(\partial_{ij}f)^2\geqslant\frac{1}{n}(\sum_{i=1}^n\partial_i^2 f)^2$$
read
$$\sum_{i,j=1}^n(\partial_{ij}^2 f)^2\geqslant\frac{1}{n}(\sum_{i=1}^n\partial_{ii}^2 f)^2,$$
thanks to Michał Strzelecki.

- Page 90, line 11 : instead of
$$k_3(t,d)=\frac{1}{4\pi t)^{3/2}}\,\frac{d}{\sinh(d)}\,\exp\Big(-t-\frac{d^2}{4t}\Big)$$

read (add a ()

$$k_3(t,d) = \frac{1}{(4\pi t)^{3/2}}\,\frac{d}{\sinh(d)}\,\exp\Big(-t-\frac{d^2}{4t}\Big)$$

- Page 129, line -1 : instead of

$$\Gamma(f)(x) = \lim_{k\to\infty}\Big(\frac{1}{2t_k}P_{t_k}(f^2)(x) - P_{t_k}(f)(x)^2\Big)$$

read

$$\Gamma(f)(x) = \lim_{k\to\infty}\frac{1}{2t_k}\Big(P_{t_k}(f^2)(x) - P_{t_k}(f)^2(x)\Big)$$

thanks to Michał Strzelecki.

- Page 156, line 17 : instead of $f-g$ read $f+g$ (three times), thanks to Michał Strzelecki.
- Page 158, line 13 : instead of

$$H(f)(g,h) = \frac{1}{2}\Big[\Gamma\big(g,\Gamma(f,h)\big) + \Gamma\big(h,\Gamma(f,g)\big) - \Gamma\big(f,\Gamma(g,h)\Big].$$

read (add a))

$$H(f)(g,h) = \frac{1}{2}\Big[\Gamma\big(g,\Gamma(f,h)\big) + \Gamma\big(h,\Gamma(f,g)\big) - \Gamma\big(f,\Gamma(g,h)\big)\Big].$$

- Page 170, line -6: remove $L^*(f)$ at the beginning of the formula.
- Page 200, line -8 to the end of the page. Replace the paragraph by the following:

The second set *(ii)* of inequalities, without any boundary condition, appears as a consequence of *(iii)* by symmetrization and periodization (for $f:[0,1]\to\mathbb{R}$ arbitrary, define $g:[-1,+1]\to\mathbb{R}$ by $g(x)=f(x)$ for $x\in[0,+1]$, $g(x)=f(-x)$ for $x\in[-1,0]$, and apply *(iii)* to g on the interval $[-1,+1]$ after re-scaling).

Finally *(i)* is a consequence of *(iii)* by anti-symmetrization and periodization. For $f:[0,1]\to\mathbb{R}$ such that $f(0)=f(1)=0$, define $g:[-1,+1]\to\mathbb{R}$ by $g(x)=f(x)$ for $x\in[0,+1]$, $g(x)=-f(-x)$ for $x\in[-1,0]$. Then

$$\int_{[0,1]} f^2dx = \int_{[-1,1]} g^2\frac{dx}{2} - \Big(\int_{[-1,1]} g\frac{dx}{2}\Big)^2 \leq \frac{1}{\pi^2}\int_{[-1,1]} g'^2\frac{dx}{2} = \frac{1}{\pi^2}\int_{[0,1]} f'^2dx,$$

where *(iii)* has been applied to the probability measure $1_{[-1,1]}\frac{dx}{2}$ with the optimal constant $1/\pi^2$. The function $f(x)=\sin(\pi x)$ is an optimal function from a direct computation.

- Page 201, line -7 : instead of

$$\int_K (f_\ell^2 - \frac{1}{\mu(K)}\int_K f_\ell d\mu)^2 d\mu,$$

read

$$\int_K (f_\ell - \frac{1}{\mu(K)}\int_K f_\ell d\mu)^2 d\mu,$$

(thanks to Arnak Dalalyan).

- Page 205, Proposition 4.6.4 : instead of

$$C_{K\cup L} \leq \frac{\mu(K\cap L)}{\mu(K\cup L)} \max(C_K, C_L),$$

read

$$C_{K\cup L} \leq 2\frac{\mu(K\cap L)}{\mu(K\cup L)} \max(C_K, C_L),$$

(thanks to Michał Strzeleck).

- Page 240, line -1 : instead of $s = \int_E f d\mu$ read $s = f$, thanks to Michał Strzelecki.
- Page 249, Proposition 5.2.7 : instead of $\mathbb{E}_1, \mathbb{E}_2$ read E_1, E_2.
- Page 251, formula (5.3.2), read $(q-1)^{k/2}$ instead of $(q-1)^k$, thanks to Max Fathi.
- Page 263, line 11 : instead of $\Lambda^{q-1}(s)$ in the LHS, read

$$\frac{q^2}{q'}\Lambda^{q-1}(s)\Lambda'(s),$$

moreover the function q is decreasing, thanks to Michał Strzelecki.

- Page 267, line -7 : instead of

$$\frac{d(x,y)}{2t}$$

in the RHS, read

$$\frac{d(x,y)}{2\sqrt{t}},$$

thanks to Michał Strzelecki.

- Page 298, line -8: instead of

$$P_t(f\log f) - P_t f \log P_t f \leq t\Delta P_t f + \frac{n}{2}\log(1 - \frac{2t}{n}\frac{P_t(f\Delta(\log f))}{P_t f}),$$

read

$$P_t(f\log f) - P_t f \log P_t f \leq t\Delta P_t f + \frac{n}{2}P_t f\log(1 - \frac{2t}{n}\frac{P_t(f\Delta(\log f))}{P_t f}).$$

- Page 301, line 11 : instead of

$$\Lambda''(s) \geqslant \frac{2[LP_t f - \Lambda'(s)]^2}{nP_t f} + \rho\Lambda'(s),$$

read

$$\Lambda''(s) \geqslant \frac{2[LP_t f - \Lambda'(s)]^2}{nP_t f} + 2\rho\Lambda'(s).$$

- Page 315, formula (6.9.2) : instead $\hat{L}$, read $\hat{L}(f)$.

- Page 338, line -1: instead of $I(u)$, read $I_{\mu,F}(u)$.
- Page 364, line -6: The sentence starting by *In the finite measure case...* is not correct. It has to be replaced by the following one : In the finite measure case, the tight Nash inequality (3.2.3), p. 281, correspond to a function Φ which is the inverse function of $(1,+\infty) \ni x \mapsto (x^{1+2/n} - x)/C$.
- Page 372, line -5: instead of $e^{-C/t}$, read $e^{-t/C}$.
- Page 373, line -13: instead of $w(x) = p(x)^{1/2}(1+x^2)^{-\beta}$, read $w(x) = p(x)^{-1/2}(1+x^2)^{-\beta}$ (thanks to Persi Diaconis).
- Page 425, Theorem 8.6.3: the set A_{d_t} should be here the d_t-closed neighborhood of A instead the open one ($A_{d_t} = \{x \in E\,;\, d(x,A) \leq d_t\}$ instead $A_{d_t} = \{x \in E\,;\, d(x,A) < d_t\}$).
- Page 448, line -7: (the line before formula (9.3.5)) the integration is w.r.t. the measure $u^{1-1/n}dx$ instead udx (thanks to Emanuel Milman).
- Page 464, formula (9.7.4) should be

  $$W_2^2(P_t f\mu, P_t g\mu) \leq W_2^2(f\mu, g\mu) + 2n(\sqrt{t} - \sqrt{s})^2,$$

  thanks to Luigia Ripani.