Abstracts, Résumés

Mini-cours

Walter MAZORCHUK, Uppsala University, Sweden, *Introduction to the category O.*

Abstract :

The aim of this series of lectures is to give a concise introduction into the algebraic aspects of the Bernstein-Gelfand-Gelfand category O associated with a triangular decomposition of a semi-simple complex Lie algebra. The main focus will be on its highest weight structure, Koszulity and its role in higher representation theory which manifests via a number of interesting functorial actions on this category and its derived category. A more detailed series of 32 lectures on category O can be found on the

`https://www.youtube.com/@uppsalaalgebra6902/videos`

Simon RICHE, Université Clermont-Auvergne, *The geometric Satake equivalence and some applications.*

Abstract :

In this course we will start by reviewing the statement of the geometric Satake equivalence, and give an overview of its proof for general coefficients. We will then explain the "Iwahori-Whittaker model" for this category, and explain how it can be used to study the representation theory of reductive algebraic groups over fields of positive characteristic. Finally we will explain the construction of Gaitsgory's central functor, and discuss its use in the construction of the Arkhipov-Bezrukavnikov equivalence.

Prerequisites : We will assume that the structure theory of reductive algebraic groups over algebraically closed fields is known, as well as the basic theory of constructible complexes on algebraic varieties and of the perverse t-structure. References for the first subject include the classical books of Borel, Humphreys and Springer; for the second subject we recommend the book "Perverse Sheaves and Applications to Representation Theory" by P. N. Achar. (Shorter overviews of these theories can also easily be found online.)

Bibliography:

(1) Historical references:

S. Arkhipov and R. Bezrukavnikov, *Perverse sheaves on affine flags and Langlands dual group*, Israel J. Math. 170 (2009), 135-183.

R. Bezrukavnikov, D. Gaitsgory, I. Mirkovic, S. Riche, and L. Rider, *An Iwahori-Whittaker model for the Satake category*, J. Ec. polytech. Math. 6 (2019), 707-735.

D. Gaitsgory, *Construction of central elements in the affine Hecke algebra via nearby cycles*, Invent. Math. 144 (2001), 253-280.

I. Mirkovic and K. Vilonen, *Geometric Langlands duality and representations of algebraic groups over commutative rings*, Ann. of Math. 166 (2007), 95-143.

S. Riche and G. Williamson, *Smith-Treumann theory and the linkage principle*, Publ. Math. Inst. Hautes Etudes Sci. 136 (2022), 225-292.

(2) More detailed accounts of some of these constructions:

P. Achar and S. Riche, *Central sheaves on affine flag varieties*, preliminary version of a book, `https://lmbp.uca.fr/~riche/central.pdf`

P. Baumann and S. Riche, *Notes on the geometric Satake equivalence*, in Relative aspects in representation theory, Langlands functoriality and automorphic forms, 1-134, Lecture Notes in Math., 2221, CIRM Jean-Morlet Ser., Springer, Cham, 2018.

T. Richarz, *Basics on affine Grassmannians*, notes, `https://timo-richarz.com/wp-content/uploads/2020/02/BoAG_02.pdf`.

Exposés.

Raphaël BEUZART-PLESSIS, Aix-Marseille Université, *On the formal degree conjecture for classical groups.*

Abstract :

The local Langlands correspondence gives a parametrization of irreducible representations of real or p-adic reductive groups in terms of arithmetic data (essentially morphisms from the Weil or Weil-Deligne group of the local field to the dual L-group). It is natural to ask whether we can read through this correspondence some simple invariants attached to irreducible representations. In this talk, I will focus on a conjecture of Hiraga, Ichino and Ikeda expressing the formal degree of a discrete series in terms of adjoint gamma factors. For classical groups over a p-adic field, this conjecture has been established for odd orthogonal and unitary groups by two completely different methods. I shall explain a new proof that also works for even orthogonal and symplectic groups. This uses the theory of twisted endoscopy as well as basic objects from harmonic analysis (orbital integrals and Plancherel formulas) and builds on previous ideas of Shahidi.

Magdalena BOOS, Ruhr University Bochum, *Symmetric quiver representations and beyond.*

Abstract :

The notion of a symmetric quiver was introduced by Derksen and Weyman in 2002. Symmetric quiver representations are collected in so-called symmetric representation varieties which are acted on by reductive groups via change of basis. They can be seen as classical type B, C and D analogues of quiver representation varieties. We give an introduction to symmetric quiver representations, motivate our interest in the theory and show first results on the mentioned group actions. This is joint work with G. Cerulli Irelli.

Elyes BOUGHATTAS, Université Sorbonne Paris-Nord, *Problème de Galois inverse et principes locaux-globaux.*

Résumé :

L'étude de l'arithmétique des espaces homogènes de SL_n apporte un angle d'attaque prometteur au problème de Galois inverse. Après avoir donné un aperçu historique de cette approche, initiée par Noether, je me concentrerai sur les développements récents concernant un fermé remarquable de

l'ensemble des points adéliques : l'ensemble de Brauer-Manin. Si le temps le permet, j'esquisserai comment obtenir une réponse positive au problème de Galois inverse pour de nouvelles familles de groupes non résolubles.

Adrien BROCHIER, Université Paris Cité, *Harish-Chandra bimodules and quantum exponentiation of Hamiltonian Poisson varieties.*

Abstract :

Let G be a complex reductive algebraic group. Various authors have developed notions of (quasi-) G-Poisson varieties equipped with a multiplicative version of a moment map valued in the group G. One of the main motivations for that formalism is that it provides a nice combinatorial description of the canonical Atiyah–Bott Poisson structure on character varieties of surfaces. A crucial feature of these structures is that they come equipped with certain operations (fusion and Hamiltonian reduction) which in particular expresses the compatibility of the Poisson structure on character varieties with cutting and gluing.

Exponentiation is a procedure taking an ordinary Hamiltonian variety (i.e. equipped with a moment map into the dual of the Lie algebra of G) and turning it (formally) into a "multiplicative" one. Crucially, this operation is compatible with fusion and Hamiltonian reduction on both sides, showing that the character variety of a surface is formally Poisson isomorphic to a much simpler Poisson variety.

In this talk, I'll describe a quantization of this construction which is essentially given by pulling back along a certain monoidal functor from quantum to classical Harish-Chandra bimodules. This construction is compatible with categorical/quantum analogs of fusion and Hamiltonian reduction. Time permitting, I'll explain some applications of that construction in representation theory.

Giovanna CARNOVALE, Université de Padova, *The exotic and enhanced Jordan stratifications.*

Abstract :

This is part of an ongoing project with Filippo Ambrosio, Francesco Esposito, Lewis Topley and Neil Saunders.

Let K be an algebraically closed field. Syu Kato has studied the exotic module $\wedge^2 K^{2n} + K^{2n}$ for $\mathrm{Sp}_{2n}(K)$ and used its nilpotent cone to establish a relation between Springer correspondence for $\mathfrak{sp}_{2n}$ over the complex numbers with the one in characteristic 2. It was proved by Achar-Henderson that the poset of exotic nilpotent orbits is isomorphic to the poset of $\mathrm{GL}_{2n}(K)$-orbits

in the nilpotent cone of the enhanced module $\mathfrak{gl}_n(K) + K^n$ for $\mathrm{GL}_n(K)$. We construct an enhanced and an exotic version of the Jordan stratification and show how these stratifications are related with each other and with the corresponding nilpotent cones.

Arnaud ETEVE, Sorbonne Université, *Géométrie de la représentation de Gelfand-Graev; Geometry of the Gelfand-Graev representation.*

Résumé-Abstract :

Soit G un groupe réductif sur un corps fini $\mathbb{F}_q$ et $B = TU$ une paire de Borel de G. La représentation de Gelfand-Graev Γ_ψ est une représentation du groupe fini $G(\mathbb{F}_q)$ obtenue comme l'induite d'un caractère non dégénéré ψ de $U(\mathbb{F}_q)$. Un problème fondamental est la description de l'algèbre de ses endomorphismes. Un théorème de Li et Shotton-Li donne une description de cette algèbre en terme du groupe de Langlands dual. Dans cette exposé, j'expliquerai une approche géométrique de cette description.

Let G be a reductive group over a finite field $\mathbb{F}_q$ and let $B = TU$ be a Borel pair of G. The Gelfand-Graev representation Γ_ψ of the finite group $G(\mathbb{F}_q)$ is constructed by inducing a generic character ψ of the finite group $U(\mathbb{F}_q)$. A fundamental problem is the description of the endomorphism algebra of this representation. A theorem of Li and Shotton-Li yields an isomorphism between this endomorphism algebra and an algebra depending only on the Langlands dual group. In this talk, I will discuss a geometric approach of this theorem.

Enrica FLORIS, Université de Poitiers, *Normal split subvarieties of a homogeneous manifold.*

Abstract :

Van de Ven proved in 1959 that the subvarieties of the projective space whose normal short exact sequence splits are linear subspaces. In this talk I will explain a generalisation of this result to rational homogeneous varieties: a subvariety of a rational homogeneous variety whose normal exact sequence splits is a rational homogeneous variety. This is a joint work with Andreas Hoering.

Simon JACQUES, Université Clermont-Auvergne, *Orbit closures in flag varieties for the centralizer of an order-two nilpotent element : normality and resolutions for types A, B, D.*

Abstract :

We present the main part of https://arxiv.org/abs/2207.09123. Let G be a reductive algebraic group in classical types A, B, D and e be an element of its Lie algebra with Z its centraliser in G for the adjoint action. We suppose that e identifies with an nilpotent matrix of order two, which guarantees the number of Z-orbits in the flag variety of G is finite. We show that any closure Y of such orbit is normal. We also prove that Y is Cohen-Macaulay with rational singularities provided that the base field is of characteristic zero. For this purpose, we exhibit a birational, rational morphism onto Y built on Schubert varieties and symmetric subgroups. We also use a Frobenius splitting theorem due to X.He and J.F.Thomsen, and an inductive result inspired by M.Brion and S.Kumar, developped by N.Perrin and E.Smirnov in https://smf.emath.fr/publications/les-composantes-de-fibre-de-springer-dans-le-cas-de-deux-colonnes-de-types-et-d-sont/-. Our work generalizes a result of the latter article on Springer fibers.

Martina LANINI, Universtà di Roma "Tor Vergata", *GKM-Theory for cyclic quiver Grassmannians.*

Abstract :

After recalling some background on Goresky-Kottwitz-MacPherson (GKM) version of the Localization Theorem for equivariant cohomology, and some of the applications of such a result to (equivariant) Schubert calculus and geometric representation theory, I will explain how it is possible -and why it is desirable- to extend such techniques to the quiver Grassmannian setting. This is joint work with Alex Puetz.

João LOURENÇO, WWU Münster, *Variétés des drapeaux et ±régularité globale.*

Résumé :

En théorie des groupes et des représentations, on s'intéresse typiquement aux variétés classifiantes des drapeaux (comme par exemple le quotient d'un groupe réductif par un borélien) et aussi à leurs sous-variétés équivariantes (typiquement appelées de Schubert). On sait d'après Lauritzen-Raben-Pedersen-Thomsen et Cass que tous les recouvrements finis des variétés de Schubert scindent dans la catégorie des modules (c'est-à-dire, elles sont globalement ±régulières). Comme application, ils ont classifié les D-modules/mod p faisceaux étales équivariants simples des variétés de drapeaux. Néanmoins, leur preuve repose sur le critère de Mehta–Ramanathan pour le scindage du frobenius, qui est peu adapté à certaines grassmanniennes affines. On en donne

une nouvelle preuve en s'appuyant sur un théorème d'annulation à la Kodaira de Bhatt et l'inversion de l'adjunction pour des variétés à bord.

Il s'agit d'un travail en commun avec R. Cass.

Matilde MACCAN, Universités Rennes 1 et Grenoble Alpes, *Parabolic subgroup schemes with maximal reduced part in characteristic 2 and 3.*

Abstract :

Classifying homogeneous projective varieties with Picard group isomorphic to $\mathbb{Z}$ leads to the study of parabolic subgroup schemes of (semi)simple algebraic groups, having reduced part which is maximal. Over an algebraically closed field of positive characteristic, any such subgroup is obtained by fattening with the kernel of a purely inseparable isogeny, in all types but G_2 in characteristic 2. This generalizes previous works of Haboush-Lauritzen and Wenzel. We start by giving a classification of isogenies with simply connected source, based on ideas from Borel-Tits, Conrad-Gabber-Prasad. We then sketch a proof of the main result, in which the quotient $Lie(G)/Lie(P_{red})$ is seen as a representation of a Levi subgroup of P_{red}. We exhibit a concrete counterexample in characteristic 2 : a variety which is built seeing G_2 as the automorphism group of an octonion algebra. If time permits, I will explain why this is the only counterexample and conclude the classification in type G_2.

Timothée MARQUIS, Université Catholique de Louvain, *On the centre of Iwahori-Hecke algebras.*

Abstract :

Iwahori-Hecke algebras $C_q(W)$ are deformations of the group algebra of a Coxeter group W. They are intimately related with the representation theory of groups with a BN-pair whose associated building is locally finite (such as Kac- Moody groups over finite fields). We recently proved that the centre of $C_q(W)$ is trivial (in the sense that it only consists of the constant functions) whenever W is of (irreducible) indefinite type – when W is of finite or affine type, this is not true anymore. This is a consequence of a purely Coxeter group-theoretic result on conjugacy classes of W.

After reviewing the context and motivation for this latter result, I will explain the key ideas behind its (partly geometric, partly combinatorial) proof. This is joint work with Sven Raum.

Emily NORTON, University of Kent, *Parabolic Kazhdan-Lusztig polynomials and oriented Temperley-Lieb algebras.*

Abstract :

Work by Brundan and Stroppel in the 2010s showed that parabolic Kazhdan-Lusztig polynomials in type A arise from diagrammatic algebras they called extended Khovanov arc algebras. These polynomials appear all over the place in representation theory. I will talk about another place they seem to crop up – work in progress with Olivier Dudas. And I will discuss how to explain the diagrammatic rule for computing them using an oriented version of the Temperley-Lieb algebra – work with Chris Bowman, Maud De Visscher, Niamh Farrell, and Amit Hazi.

Margherita PAOLINI, Università de L'Aquila, *Integral form of Affine Lie Algebras.*

Abstract :

The study of integral forms for the universal enveloping algebra of a complex finite-dimensional simple Lie algebra goes back to the work of Kostant and Chevalley. Their construction was later generalized to the non-twisted affine case by Garland and to the twisted affine case by Mitzman. The main difficulty in the affine case is to produce the integral basis for the subalgebra of the enveloping algebra generated by the imaginary root vectors. In my work with Ilaria Damiani we studied the analogous integral form, defined by working with divided powers of the Drinfeld generators (rather than the usual Chevalley generators as in the work of Garland-Mitzman). In this talk we will examine the structures involved in describing these two integral forms; special attention will be given to the case where $\mathfrak{g}$ is an algebra of type $A_{2n}^{(2)}$.

Théo PINET, Université de Paris (Paris 7), *R-matrices in the category $\mathcal{O}$ of Borel quantum loop algebras.*

Abstract :

Quantum loop algebras are infinite-dimensional algebras which are naturally involved in the theory of integrable systems as well as in the study of cluster algebras, quiver varieties and Coulomb branches. The category $\mathcal{C}$ of finite-dimensional modules over such an algebra $U_q(\mathfrak{g})$ is an interesting example of a non-semisimple category with generic braidings; that is, isomorphisms of $V \otimes W$ onto $W \otimes V$ for "generic" simple objects V and W. Recently, Hernandez constructed similar isomorphisms for a subcategory $\mathcal{O}^-$ of the category of representations of the Borel subalgebra $U_q(\mathfrak{b})$ of $U_q(\mathfrak{g})$. This subcategory $\mathcal{O}^-$, defined by Hernandez–Leclerc in the context of monoidal categorifications of cluster algebras, has the same Grothendieck

ring as that of another subcategory $\mathcal{O}^+$ of modules over $U_q(\mathfrak{b})$. This last observation raises the following questions:

Is the category $\mathcal{O}^+$ generically braided? Can we lift the isomorphism between the Grothendieck rings of $\mathcal{O}^+$ and $\mathcal{O}^-$ into an isomorphism of categories?

In this talk, we will answer the above questions positively and give, if time permits, potential applications of our results to generalized quantum affine Schur–Weyl dualities (in the spirit of Kang–Kashiwara–Kim–Oh–Park) and to the representation theory of shifted quantum affine algebras.

Timo RICHARZ, TU Darmstadt, *Ramified geometric Satake equivalence.*

Abstract :

The geometric Satake equivalence relates certain sheaves on the affine Grassmannian with representations of the Langlands dual group. By construction the equivalence depends on the choice of a split reductive group. For coefficients in a field of characteristic 0, Xinwen Zhu extended the aforementioned equivalence to include non-split reductive groups. This is referred to as the ramified geometric Satake equivalence. In my talk I speak about joint work with Pramod Achar, João Lourenço and Simon Riche where we extend the ramified geometric Satake equivalence from rational coefficients to integral and modular coefficients.

David STEWART, University of Manchester, *Absolute rigidity of simple modules for algebraic groups (jt with Michael Bate).*

Abstract :

Let k be a field and let G be a smooth affine k-group of finite type. We show that the simple G-modules are absolutely rigid; that is, if V is a simple G-module, then the socle and radical series for VE as a GE-module coincide for any field extension E/k. Central to the proof is the authors' recent classification of simple modules for connected G by highest weight. We use this to describe the endomorphism ring of simple G-modules and thereby give a dimension formula. For most of our results, the key case to consider is when G is a pseudo-split pseudo-reductive group and E/k is finite and purely inseparable.

Emmanuel WAGNER, Université Paris Cité, *Une action de $\mathfrak{sl}(2)$ sur les mousses et les homologies d'entrelacs $\mathfrak{gl}(N)$; $\mathfrak{gl}(N)$ link homologies and foams as $\mathfrak{sl}(2)$ representations.*

Résumé, Abstract :

Dans cet exposé, j'expliquerai comment les mousses (certains CW complexes de dimension 2) permettent de donner une définition complètement combinatoire des homologies $\mathfrak{gl}(N)$, le cas $N = 2$ correspond à la célébrée homologie de Khovanov. Cette description permet de construire une action de $\mathfrak{sl}(2)$ sur ces groupes d'homologie. Nous verrons aussi comment étendre fonctoriellement l'action précédente. Il s'agit d'un travail en commun avec You Qi, Louis-Hadrien Robert et Joshua Sussan.

In this talk, I will explain how foams (some 2-dimensional CW complexes) allow to give a purely combinatorial definition of $\mathfrak{gl}(N)$ link homologies, the case $N = 2$ being the celebrated Khovanov homology. This construction allows to see an $\mathfrak{sl}(2)$ action on these homologies. We will also see how to extend the previous action fonctorially. Joint work with You QI, Louis-Hadrien Robert and Joshua Sussan.

Fang YANG, Tsinghua University, *Quantum cluster algebras associated to weighted projective lines.*

Abstract :

Fan Qin used quantum Caldero–Chapoton characters of representations of acyclic quivers to categorify the quantum cluster algebras of corresponding quivers. Bernstein–Rupel showed that quantum cluster characters are homomorphisms from Hall algebras to the specialized quantum cluster algebras of acyclic quivers.

In this talk, we will construct such quantum cluster characters from Hall algebras of weighted projective lines to quantum torus, and show that the image of this map admits a quantum cluster structure. When the weighted projective line is of domestic type, the image is precisely the quantum cluster algebras of an acyclic quiver of affine type. This talk is based on a joint work with Fan Xu.

Emilien ZABETH, Université Clermont-Auvergne, *Block decomposition via the geometric Satake equivalence.*

Abstract :

Let G be a reductive algebraic group over a field of prime characteristic. In 2020, S.Riche and G.Williamson gave a new proof of the Linkage principle by using the geometry of the affine Grassmannian via the geometric Satake equivalence. This principle, which is a fundamental result in the theory of modular algebraic representations, allows one to decompose the category of (algebraic, finite dimensional) representations $Rep(G)$ into a direct sum of

subcategories, parametrized by certain orbits of the affine Weyl group. It turns out that some of these subcategories (labelled by 'special' orbits) are still decomposable, and the true decomposition of $Rep(G)$ into (indecomposable) blocks is due to Donkin (1980). In this talk, I will explain how to recover Donkin's description geometrically by extending Riche-Williamson's result.

Kirill ZAINOULLINE, University of Ottawa, *From structure algebras of moment graphs to generalized cohomology of a group.*

Abstract :

(This is a joint work with Martina Lanini and Rui Xiong.)

We prove that the structure algebra of a Bruhat moment graph of a finite real root system is a Hopf algebroid (in the language of 2-monoidal categories) with respect to the Hecke and the Weyl actions. In particular, it implies that the natural Hopf-algebra structure on the algebraic oriented cohomology $A(G)$ of Levine-Morel of a split semi-simple linear algebraic group G can be lifted to a 'bi-Hopf' structure on the T-equivariant algebraic oriented cohomology of the complete flag variety. We discuss various applications of this result to generalized Schubert calculus, to Coxeter groups and finite real root systems.

Yifei ZHAO, University of Münster, *What are covering groups?*

Abstract :

In the study of automorphic forms, one encounters certain topological covers of reductive groups which are not linear algebraic groups themselves, and yet share many features of reductive groups. In this talk, I will explain the geometric origin of these covering groups and discuss some form of Langlands duality for them.
