Theorem 6. *The category $T_{\mathfrak{g}}$ is canonically antiequivalent to the category of locally unitary finite-dimensional $\mathcal{A}_{\mathfrak{g}}$-modules, where $\mathcal{A}_{\mathfrak{g}} = \varinjlim \mathcal{A}^r_{\mathfrak{g}}$.*

Theorem 7. *The algebras $\mathcal{A}_{o(\infty)}$ and $\mathcal{A}_{sp(\infty)}$ are isomorphic, therefore the categories $T_{o(\infty)}$ and $T_{sp(\infty)}$ are equivalent.*

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On conjugacy of MADs in *k*-loop algebras

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(joint work with P. Gille, A. Pianzola)

Throughout k will denote a field of characteristic 0. For integers $n \geq 0$ we set $R = R_n = k[t_1^{\pm 1}, \dots, t_n^{\pm 1}]$. We let $\dot{\mathfrak{g}}$ denote a split simple finite dimensional Lie algebra over k and $\dot{\mathbf{G}}$ the corresponding simple simply connected algebraic group over k. Recall that a Lie algebra $\mathfrak{g}$ over R is called a form of $\dot{\mathfrak{g}} \otimes_k R$ if there exists a faithfully flat R-algebra S such that

$$\mathfrak{g} \otimes_R S \simeq (\dot{\mathfrak{g}} \otimes_k R) \otimes_R S \simeq \dot{\mathfrak{g}} \otimes_k S.$$

Our form $\mathfrak{g}$ can also be viewed as a Lie algebra over k (which is infinite dimensional if $n \geq 1$). In the theory of affine Kac-Moody algebras, or more generally for extended affine Lie algebras, the emphasis is in viewing the relevant objects over k (and not R). We now introduce the most relevant k-objects attached to $\mathfrak{g}$ in this work.

A subalgebra $\mathfrak{m}$ of the k-Lie algebra $\mathfrak{g}$ is called an AD subalgebra if $\mathfrak{g}$ admits a k-basis consisting of simultaneous eigenvectors of $\mathfrak{m}$, i.e. there exists a family (λ_i) of functionals $\lambda_i \in \mathfrak{m}^*$, and a k-basis $\{ v_i \}_{i \in I}$ of $\mathfrak{g}$ such that

$$[h, v_i] = \langle \lambda_i, h \rangle \, v_i \quad \forall h \in \mathfrak{m}.$$

It is not difficult to see that any such $\mathfrak{m}$ is necessarily abelian, so AD can be thought as shorthand for abelian k-diagonalizable or ad-k-diagonalizable. An AD subalgebra which is maximal (in the sense that it is not properly included in another AD) is called a MAD.

In infinite dimensional Lie theory $\mathfrak{m}$ plays the role which a Cartan subalgebra $\mathfrak{h}$ plays for $\dot{\mathfrak{g}}$ in the classical theory. One of the central theorems of classical

Lie theory is that all split Cartan subalgebras of $\dot{\mathfrak{g}}$ are conjugate under $\dot{\mathbf{G}}(k)$, a theorem of Chevalley. This result yields the most elegant proof that the type of the root system of $(\dot{\mathfrak{g}}, \dot{\mathfrak{h}})$ is an invariant of $\dot{\mathfrak{g}}$. The main thrust of our work is to investigate the question of conjugacy of MADs of $\mathfrak{g}$. Our first result says that the conjugacy of MADs is equivalent to conjugacy of maximal split tori in a simple simply connected group scheme over R corresponding to $\mathfrak{g}$.

Theorem. Let $\mathbf{G}$ be a simple simply connected group scheme over R and

$$\mathfrak{g} = \mathrm{Lie}\,(\mathbf{G}) = \underline{\mathrm{Lie}}\,(\mathbf{G})(R).$$

(1) If $\mathbf{S}$ is a maximal split torus of $\mathbf{G}$ then its Lie algebra $\mathrm{Lie}\,(\mathbf{S}) \subset \mathfrak{g}$ contains a unique MAD $\mathfrak{m} = \mathfrak{m}(\mathbf{S})$ of $\mathfrak{g}$.

(2) Let $\mathfrak{m}$ be a MAD of $\mathfrak{g}$. Then $Z_{\mathbf{G}}(R\mathfrak{m}) := H$ is a reductive R-group. Its radical contains a unique maximal split torus $\mathbf{S}\,(\mathfrak{m})$ of $\mathbf{G}$.

(3) The process $\mathfrak{m} \to \mathbf{S}\,(\mathfrak{m})$ and $\mathbf{S} \to \mathfrak{m}\,(\mathbf{S})$ described above gives a bijection between the set of MADs of $\mathfrak{g}$ and the set of maximal split tori of $\mathbf{G}$.

From the way we constructed the above bijective correspondence it follows that the conjugacy of two maximal k-diagonalizable subalgebras in $\mathfrak{g}$ is equivalent to conjugacy of the corresponding maximal R-split tori in $\mathbf{G}$. The following example shows that in general case maximal R-split tori are not necessary conjugate.

Example. Let D be a quaternion algebra over $R = k[t_1^{\pm 1}, t_2^{\pm 1}]$ with generators T_1, T_2 and relations $T_1^2 = t_1,\ T_2^2 = t_2$ and $T_2T_1 = -T_1T_2$ and let $A = M_2(D)$. We may view A as a D-endomorphism algebra of a 2-dimensional space $V = D \oplus D$ over D where D acts on V on the right. Let $\mathbf{G} = \mathrm{SL}\,(1, A)$. It contains an R-split torus $\mathbf{S}_1$ whose R-points are matrices of the form

$$\begin{pmatrix} x & o \\ 0 & x^{-1} \end{pmatrix}$$

where $x \in R^{\times}$. Let $K = k(t_1, \dots, t_n)$. Since K-rank of $\mathbf{G}$ is equal to 1, the torus $\mathbf{S}_1$ is a maximal split in $\mathbf{G}$.

Consider now a D-linear map $f : V = D \oplus D \to D$ given by

$$(u, v) \to (1 + T_1)u - (1 + T_2)v.$$

Let $\mathcal{L}$ be its kernel. One can show that $\mathcal{L}$ is a projective D-module which is not free. Since f is split, we have another decomposition $V \simeq \mathcal{L} \oplus D$. Let $\mathbf{S}_2$ be an R-split torus in $\mathbf{G}$ consisting of linear transformations acting on the first summand $\mathcal{L}$ by multiplication $x \in R^{\times}$ and on the second summand by x^{-1}. As above, $\mathbf{S}_2$ is a maximal R-split torus in $\mathbf{G}$. We claim that $\mathbf{S}_1$ and $\mathbf{S}_2$ are not conjugate. To see this we note that given $\mathbf{S}_1$ we can restore two summands in the decomposition $V = D \oplus D$ as two subspaces in V consisting of eigenvectors of elements $s \in \mathbf{S}_1(R)$. Similarly, we can uniquely restore two summands in the decompositions $V = \mathcal{L} \oplus D$ out of $\mathbf{S}_2$. Assuming now that $\mathbf{S}_1$ and $\mathbf{S}_2$ are conjugate by an element in $\mathbf{G}(R)$ we obtained immediately that the subspace $\mathcal{L}$ in V is isomorphic to one of the components of $V = D \oplus D$, in particular $\mathcal{L}$ is free – a contradiction.

Thus for twisted forms of a split group scheme over R the conjugacy fails in general case. However for a large class of Lie algebras called multiloop algebras and for the corresponding group schemes we do have conjugacy. We recall that a group scheme $\mathbf{G}$ and its Lie algebra $\mathfrak{g}$ are called multiloop if $\mathbf{G}$ is a twisted form of a split $\dot{\mathbf{G}}$ by a cocycle with coefficients in $\mathrm{Aut}\,(\dot{\mathbf{G}})(\bar{k})$.

Theorem. Let $\mathbf{G}$ be a simple simply connected multiloop R-group scheme. Then two maximal R-split tori $\mathbf{S}_1$, $\mathbf{S}_2$ in $\mathbf{G}$ such that their centralizers in $C_{\mathbf{G}}(S_i)$ are multiloop are conjugate by an element in $\mathbf{G}(R)$.

Remark. The above counter-example shows that the assumptions that $\mathbf{G}$ and $C_{\mathbf{G}}(S_i)$ are multiloop cannot be dropped in general case.

The main ingredient of the proof of conjugacy is the following result on torsors over R which provides us with the classification of multiloop group schemes and their Lie algebras.

Theorem. Let G be an algebraic group over k. Let $F = k((t_1))\cdots((t_n))$. Then a canonical mapping $H^1_{loop}(R, G) \to H^1(F, G)$ is bijective.

Gerbes, gerbal representations and 3-cocycles

Jouko Mickelsson

In this talk I will explain relations between on one hand the recent discussion on 3-cocycles and categorical aspects of representation theory, [FZ], and on the other hand gauge anomalies, gauge group extensions and 3-cocycles in quantum field theory, [CGRS].

The set up for categorical representation theory consists of an abelian category C, a group G, and a map F which associates to each $g \in G$ a functor F_g in the category C such that for any pair $g, h \in G$ there is an isomorphism

$$i_{g,h} : F_g \circ F_h \to F_{gh}.$$

For a triple $g, h, k \in G$ we have a pair of isomorphisms $i_{g,hk} \circ i_{h,k}$ and $i_{gh,k} \circ i_{g,h}$ from $F_g \circ F_h \circ F_k$ to F_{ghk} :

They are not necessarily equal; one can have a *central extension* (with values in an abelian group)

$$i_{g,hk} \circ i_{h,k} = \alpha(g, h, k) i_{gh,k} \circ i_{g,h}$$

with $\alpha(g, h, k) \in \mathbf{C}^\times$ a 3-cocycle.

The smooth loop group LG (G compact,simple) has a central extension defined by a (local) 2-cocycle. According to Frenkel and Zhu, increase the cohomoligal degree by one unit by going to the double loop group $L(LG)$. They do this algebraically, utilizing the idea of A. Pressley and G. Segal by embedding the loop group LG (actually, its Lie algebra) to an appropriate universal group $U(\infty)$ (or its Lie algebra). The point of this talk is to show how this is done in the smooth