Internship Proposal 2014

ZAMOLODCHIKOV TETRAHEDRA EQUATIONS
AND CATEGORIFICATION

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Duration. Three months.

Subject. Given a complex vector space $V$, a linear operator $R : V \otimes V \rightarrow V \otimes V$ is called a Yang-Baxter operator, if the equation

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$$

holds in $\text{End}(V \otimes V \otimes V)$, where $R_{12} = R \otimes 1_{V}$, etc. This equation, called the Yang-Baxter equation, or the triangle equation, has many interesting applications in mathematics, in particular for classical and quantum integrable systems.

The Zamolodchikov tetrahedra equation is a three-dimensional generalisation of the Yang-Baxter equation leading to a construction of three-dimensional classical and quantum integrable systems. In [KV94], Kapranov and Voevodsky give an overview of the Yang-Baxter and Zamolodchikov equations and develop a conceptual framework underlying Zamolodchikov equations using the theory of 2-categories.

The aim of this internship is to understand the article [KV94] and the necessary background, in particular

- the notions of Yang-Baxter and Zamolodchikov equations and their geometric interpretation via statistical models,
- the interpretation of the Zamolodchikov equation in terms of 2-category,
- the formulation of the Zamolodchikov equation using the notion of 2-vector spaces, a categorification of the notion of vector space as a state-space of a system.

Expected skills. Basic knowledge in general algebra (groups, rings, fields), in linear algebra and category theory (see [ML98] for example).

References.

[KV94] M. M. Kapranov and V. A. Voevodsky. 2-categories and Zamolodchikov tetra-
hedra equations. In Algebraic groups and their generalizations: quantum and
infinite-dimensional methods (University Park, PA, 1991), volume 56 of Proc.

[ML98] S. Mac Lane. Categories for the working mathematician, volume 5 of Graduate